Journal of Lie Theory
Volume **22** (2012) 1109–1124

A Method of Proving Non-Unitarity of Representations of p-adic Groups

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Communicated by B. Ørsted

Abstract. In this paper the authors study irreducible subquotients of the certain class of induced representations of classical p-adic groups $SO(2n+1, F)$ and $Sp(2n, F)$. The induced representations in question are the ones which contain, as subquotients, generalized Steinberg and generalized trivial representation. We prove that the only unitarizable irreducible subquotients of the induced representations in question are precisely generalized Steinberg and generalized trivial representation, thus continuing the previous work of the first author and M. Tadić. This is, in a certain sense, a generalization of Casselman's results in the case of classical p-adic groups.
Mathematics Subject Classification 2000: 22E35, 22E50, 11F70.
Key Words and Phrases: Classical p-adic groups, generalized Steinberg representation, unitary representations, non-unitarity criterion.

1. Introduction

This paper continues the work begun in [H-T].

Let F be a p-adic field and G the F-points of a connected reductive group defined over F. Let B denote a minimal parabolic subgroup of G and δ the corresponding modular function. The induced representation $\mathrm{Ind}_B^G(\delta^{\frac{1}{2}})$ has the trivial representation as a quotient and the Steinberg representation as a subrepresentation ([Ca1]). In [Ca2], Casselman showed that none of the remaining irreducible subquotients of $\mathrm{Ind}_B^G(\delta^{\frac{1}{2}})$ is unitary. The aim of [H-T] and the present paper is to show the corresponding result for the inducing representation which gives rise to the generalized Steinberg representation (in the sense of [T5]) of

$$Sp(2n, F) \text{ and } SO(2n+1, F).$$

Let ρ be an irreducible unitary supercuspidal representation of some $GL(n_1, F)$ and σ an irreducible supercuspidal representation of $Sp(2n_0, F)$ or $SO(2n_0+1, F)$. If $\rho \not\cong \tilde{\rho}$, then the parabolically induced representation (see section 2 for more details) $|det|^x\rho \rtimes \sigma$ is irreducible for all $x \in \mathbb{R}$; if $\rho \cong \tilde{\rho}$, there is a unique $\alpha \geq 0$ such that $\nu^\alpha\rho \rtimes \sigma$ is reducible ([Sil2]). Based on certain conjectures, one expects

*supported in part by NSA grant H98230-10-1-0162.

ISSN 0949–5932 / \$2.50

$\alpha \in \frac{1}{2}\mathbb{Z}$; see [Mœ],[Zh] and [Sh1],[Sh2] for more on expected and known values of α. For $\alpha > 0$, the representation $|det|^{\alpha+m}\rho \times |det|^{\alpha+m-1}\rho \times \cdots \times |det|^{\alpha}\rho \rtimes \sigma$ has a unique irreducible subrepresentation, which is square-integrable. This subrepresentation is analogous to the Steinberg representation in a number of respects; we refer to it as a generalized Steinberg representation ([T5]). The analogue to Casselman's result is that the unique irreducible subrepresentation (generalized Steinberg representation) and unique irreducible quotient (which might be called a generalized trivial representation) are unitary; the remaining subquotients are not. The generalized Steinberg is square-integrable, hence unitary; the generalized trivial is dual to the generalized Steinberg (in the sense of [Aub], [S-S]) so unitary by [H]. The nonunitarity of approximately half the remaining subquotients was proved in [H-T]; this paper deals with the remaining cases.

The basic strategy for proving the non-unitarizability of a representation π remains the same as in [H-T]. We choose a suitable ζ, an irreducible unitary representation of a general linear group and consider the induced representation $\mathrm{Ind}_P^G(\zeta \otimes \pi)$. We then identify an irreducible subquotient π_1 of $\mathrm{Ind}_P^G(\zeta \otimes \pi)$. Were π unitary, $\mathrm{Ind}_P^G(\zeta \otimes \pi)$ would decompose as a direct sum, so π_1 would be a subrepresentation. In particular, the Jacquet module $r_{M,G}(\pi_1)$ would have to contain $\zeta \otimes \pi$ as a composition factor. However, by identifying π_1 as a subquotient of a different induced representation $\mathrm{Ind}_Q^G(\tau \otimes \theta)$, we can show that this is not the case. In particular, we use the μ^* structure of [T4] to show that $r_{M,G}(\tau \rtimes \theta)$ does not contain a copy of $\zeta \otimes \pi$. Thus $r_{M,G}(\pi_1)$ cannot contain a copy of $\zeta \otimes \pi$, so π_1 cannot appear as a subrepresentation of $\mathrm{Ind}_P^G(\zeta \otimes \pi)$. Therefore, π could not have been unitary.

We now give a brief overview of the paper. The next section reviews notation and background material needed in the remainder of the paper. It also contains a statement of the main result–Theorem 2.2. In the third section, we prove the non-unitarizability of $L(\delta(\Delta_1), \ldots, \delta(\Delta_k); \delta(\Delta_{k+1}; \sigma))$ (notation for the Langlands classification–cf. section 2) in the case where $|\Delta_{k+1}| > 1$ following the strategy discussed above. In the fourth section, we prove non-unitarizability in the case where $|\Delta_{k+1}| = 0$ and $|\Delta_k| = 1$ using the same basic strategy. However, to expedite matters, we show that this case is dual (in the sense of [Aub], [S-S]) to that of $|\Delta_{k+1}| > 1$. This allows us to transfer the results needed using duality rather than reproving them from scratch.

We would like to take this opportunity to thank the Erwin Schrödinger Institute for their hospitality during the authors' visits, where work on this paper began.

2. Notation and preliminaries

In this section, we review some notation and background material needed in the remainder of this paper. We largely retain the notation of [H-T].

As in [H-T], we have F a p-adic field with $char(F) \neq 2$ and $S_n(F) = Sp(2n, F)$ or $SO(2n+1, F)$. Recall that for $G = S_n(F)$ (resp., $G = GL(n, F)$), a standard parabolic subgroup of G has the form $P = MU$ with $M \cong GL(n_1, F) \times \cdots \times GL(n_k, F) \times S_{n_0}(F)$ (resp., $M \cong GL(n_1, F) \times \cdots \times GL(n_k, F)$ for $GL(n, F)$).

As in [B-Z], if $P = MU$ is a standard parabolic subgroup of G, we write $i_{G,M}$ and $r_{M,G}$ for the normalized induction and Jacquet functors. We interpret these as both induced representations, as well as their images in the corresponding Grothendieck group setting (i.e., their semisimplifications). Similarly, we interpret $\pi_1 \geq \pi_2$ in the Grothendieck group setting–the multiplicity of any irreducible θ in π_1 is at least that of θ in π_2.

We now recall some structure theory from [Z] and [T4]. First, let

$$R = \bigoplus_{n \geq 0} \mathcal{R}(GL(n,F)) \text{ and } R[S] = \bigoplus_{n \geq 0} \mathcal{R}(S_n(F)),$$

where $\mathcal{R}(G)$ denotes the Grothendieck group of the category of smooth finite-length representations of G. We define multiplication on R as follows: suppose ρ_1, ρ_2 are representations of $GL(n_1,F), GL(n_2,F)$, respectively. We have $M = GL(n_1,F) \times GL(n_2,F)$ is the Levi factor of a standard parabolic subgroup of $G = GL(n,F)$, where $n = n_1 + n_2$, and set $\tau_1 \times \tau_2 = i_{G,M}(\tau_1 \otimes \tau_2)$. This extends (after semisimplification) to give the multiplication $\times : R \times R \longrightarrow R$. To describe the comultiplication on R, let $M_{(i)}$ denote the standard Levi factor for $G = GL(n,F)$ having $M_{(i)} = GL(i,F) \times GL(n-i,F)$. For a representation τ of $GL(n,F)$, we define

$$m^*(\tau) = \sum_{i=0}^{n} r_{M_{(i)},G}\tau,$$

the sum of semisimplified Jacquet modules (lying in $R \otimes R$). This extends to a map $m^* : R \longrightarrow R \otimes R$. We note that with this multiplication and comultiplication (and antipode map given by the Zelevinsky involution, a special case of the general duality operator of [Aub],[S-S]), R is a Hopf algebra. Similarly, if one extends $\rtimes$ from above to a map $\rtimes : R \otimes R[S] \longrightarrow R[S]$, we have $R[S]$ as a module over R. Now, let $M_{(i)} = GL(i,F) \otimes S_{n-i}(F)$, a standard Levi factor for $G = S_n(F)$. For a representation π of $S_n(F)$, we define

$$\mu^*(\pi) = \sum_{i=0}^{n} r_{M_{(i)},G}\pi,$$

the sum of semisimplified Jacquet modules (lying in $R \otimes R[S]$). This extends to a map $\mu^* : R[S] \longrightarrow R \otimes R[S]$. This gives $R[S]$ the structure of an M^*-module over R ([T4]):

Theorem 2.1. *Define $M^* : R \longrightarrow R \otimes R$ by*

$$M^* = (m \otimes 1) \circ (\tilde{\ } \otimes m^*) \circ s \circ m^*,$$

where m denotes the multiplication $\times : R \otimes R \longrightarrow R$, $\tilde{\ }$ denotes contragredient, and $s : R \otimes R \longrightarrow R \otimes R$ the extension of the map defined on representations by $s : \tau_1 \otimes \tau_2 \longmapsto \tau_2 \otimes \tau_1$. Then

$$\mu^*(\tau \rtimes \pi) = M^*(\tau) \rtimes \mu^*(\pi),$$

where $\rtimes$ on the right hand side is determined by $(\tau_1 \otimes \tau_2) \rtimes (\tau \otimes \theta) = (\tau_1 \times \tau) \otimes (\tau_2 \rtimes \theta)$.

We now review the Langlands classification for general linear groups and the classical groups under consideration (see [B-W], [Sil], [K]; also [T3]). As in [H-T], we favor the quotient setting. However, in certain places we use the subrepresentation setting, as well as certain Jacquet module observations which follow directly from the subrepresentation setting, so review both. We start with general linear groups. Suppose $\tau_1, \ldots, \tau_k$ are irreducible tempered representations of general linear groups and τ an irreducible tempered representation of a symplectic or special odd-orthogonal group. If $x_1 > x_2 > \cdots > x_k > 0$ (resp., $x_1 > \cdots > x_k$, then $\nu^{x_1}\tau_1 \times \nu^{x_2}\tau_2 \times \cdots \times \nu^{x_k}\tau_k \rtimes \tau$ (resp., $\nu^{x_1}\tau_1 \times \nu^{x_2}\tau_2 \times \cdots \times \nu^{x_k}\tau_k$) has a unique irreducible quotient which we denote $L(\nu^{x_1}\tau_1, \ldots, \nu^{x_k}\tau_k; \tau)$ (resp., $\mathcal{L}(\nu^{x_1}\tau_1, \ldots, \nu^{x_k}\tau_k)$). Note that the switch from L to $\mathcal{L}$ to distinguish the case of general linear groups from that of classical groups represents a minor notational change from [H-T]. The Langlands classification for general linear groups is used more frequently in this paper; the change was made for added clarity. Every irreducible admissible representation of a symplectic or special odd-orthogonal group (resp., general linear group) may be written in this way, and the data $\nu^{x_1}\tau_1 \otimes \nu^{x_2}\tau_2 \otimes \cdots \otimes \nu^{x_k}\tau_k \otimes \tau$ (resp., $\nu^{x_1}\tau_1 \otimes \nu^{x_2}\tau_2 \otimes \cdots \otimes \nu^{x_k}\tau_k \otimes \tau$) are unique. Similarly, if $y_1 < y_2 < \cdots < y_k < 0$ (resp., $y_1 < \cdots < y_k$), then $\nu^{y_1}\tau_1 \times \nu^{y_2}\tau_2 \times \cdots \times \nu^{y_k}\tau_k \rtimes \tau$ (resp., $\nu^{y_1}\tau_1 \times \nu^{y_2}\tau_2 \times \cdots \times \nu^{y_k}\tau_k$) has a unique irreducible subrepresentation which we denote $L_{sub}(\nu^{y_1}\tau_1, \ldots, \nu^{y_k}\tau_k; \tau)$ (resp., $\mathcal{L}_{sub}(\nu^{y_1}\tau_1, \ldots, \nu^{y_k}\tau_k)$). Again, every irreducible admissible representation of a symplectic or special odd-orthogonal group (resp., general linear group) may be written in this way, and the data $\nu^{y_1}\tau_1 \otimes \nu^{y_2}\tau_2 \otimes \cdots \otimes \nu^{y_k}\tau_k \otimes \tau$ (resp., $\nu^{y_1}\tau_1 \otimes \nu^{y_2}\tau_2 \otimes \cdots \otimes \nu^{y_k}\tau_k$) are unique. These are related by

$$L(\nu^{x_1}\tau_1, \ldots, \nu^{x_k}\tau_k; \tau) \cong L_{sub}(\nu^{-x_1}\tilde{\tau}_1, \ldots, \nu^{-x_k}\tilde{\tau}_k; \tau).$$

and

$$\mathcal{L}(\nu^{x_1}\tau_1, \ldots, \nu^{x_k}\tau_k) \cong \mathcal{L}_{sub}(\nu^{x_k}\tau_k, \ldots, \nu^{x_1}\tau_1).$$

Note that it follows immediately that for the appropriate standard Levi factor M, one has

$$r_{M,G}L(\nu^{x_1}\tau_1, \ldots, \nu^{x_k}\tau_k; \tau) \geq \nu^{-x_1}\tilde{\tau}_1 \otimes \cdots \otimes \nu^{-x_k}\tilde{\tau}_k \otimes \tau,$$

and that $\nu^{-x_1}\tilde{\tau}_1 \otimes \cdots \otimes \nu^{-x_k}\tilde{\tau}_k \otimes \tau$ is the unique irreducible subquotient of $r_{M,G}(\nu^{x_1}\tau_1 \times \cdots \times \nu^{x_k} \rtimes \tau)$ having its central character (see Proposition 5.3 [B-J]). The corresponding claim also holds for general linear groups.

As in [Z], we let $\nu = |det|$. For ρ an irreducible supercuspidal representation of $GL(r, F)$, we let $[\nu^a\rho, \nu^\rho]$ denote the segment $\{\nu^a\rho, \nu^{a+1}\rho, \ldots, \nu^b\rho\}$ ($a \leq b$ with $b-a \in \mathbb{Z}$). The representation $\nu^b\rho \times \nu^{b-1}\rho \times \cdots \times \nu^a\rho$ has a unique irreducible subrepresentation $\delta([\nu^a\rho, \nu^b\rho])$ (which is essentially square-integrable) and a unique irreducible quotient $\mathfrak{s}([\nu^a\rho, \nu^b\rho])$. Note that $\mathfrak{s}([\nu^a\rho, \nu^b\rho]) = \mathcal{L}(\nu^b\rho, \nu^{b-1}\rho, \ldots, \nu^a\rho) = \delta(\widehat{[\nu^a\rho, \nu^b\rho]})$, where $\hat{}$ denotes the dual in the sense of [Aub], [S-S].
Suppose ρ is an irreducible unitary supercuspidal representation of $GL(r, F)$ with $\rho \cong \tilde{\rho}$. If σ is an irreducible supercuspidal representation of some $S_t(F)$, there is a unique $\alpha \geq 0$ such that $\nu^\alpha\rho \rtimes \sigma$ is reducible (if $\rho \ncong \tilde{\rho}$, then $\nu^x\rho \rtimes \sigma$ is irreducible for all $x \in \mathbb{R}$). For this α, the representation $\nu^{\alpha+n}\rho \times \nu^{\alpha+n-1}\rho \times \cdots \times \nu^\alpha\rho \rtimes \sigma$ has a unique irreducible subrepresentation–denoted $\delta([\nu^\alpha\rho, \nu^{\alpha+n}\rho]; \sigma)$. It

is regular and square-integrable ([T5]). The remaining irreducible subquotients of $\nu^{\alpha+n}\rho \times \nu^{\alpha+n-1}\rho \times \cdots \times \nu^{\alpha}\rho \rtimes \sigma$ have the form $L(\delta(\Delta_1), \ldots, \delta(\Delta_k); \delta(\Delta_{k+1}; \sigma))$, where the Δ_i are disjoint segments with $\cup_i \Delta_i = \{\nu^{\alpha}\rho, \nu^{\alpha+1}\rho, \ldots, \nu^{\alpha+n}\rho\}$ and the exponents appearing in Δ_i are greater than those appearing in Δ_{i+1}. The result we are after is the following:

Theorem 2.2. *With notation as above, an irreducible subquotient π of $\nu^{\alpha+n}\rho \times \nu^{\alpha+n-1}\rho \times \cdots \times \nu^{\alpha}\rho \rtimes \sigma$ is unitary if and only if $\pi = \delta([\nu^{\alpha}\rho, \nu^{\alpha+n}\rho]; \sigma)$ or $\pi = L(\nu^{\alpha+n}\rho, \nu^{\alpha+n-1}\rho, \ldots, \nu^{\alpha}\rho; \sigma)$.*

Proof. That $\pi = \delta([\nu^{\alpha}\rho, \nu^{\alpha+n}\rho]; \sigma)$ is unitary follows from the fact that it is square-integrable ([T5]). Next we note that $L(\nu^{\alpha+n}\rho, \nu^{\alpha+n-1}\rho, \ldots, \nu^{\alpha}\rho; \sigma)$ is dual to $\delta([\nu^{\alpha}\rho, \nu^{\alpha+n}\rho]; \sigma)$, hence unitary by [H]. If π is any other irreducible subquotient of $\nu^{\alpha+n}\rho \times \nu^{\alpha+n-1}\rho \times \cdots \times \nu^{\alpha}\rho \rtimes \sigma$, write $\pi = L(\delta(\Delta_1), \ldots, \delta(\Delta_k); \delta(\Delta_{k+1}; \sigma))$ with $k > 0$ and $|\Delta_i| > 1$ for at least one i. Nonunitarity when either (1) $\Delta_{k+1} = \emptyset$ and $|\Delta_k| > 1$, or (2) $|\Delta_{k+1}| = 1$ is proven in [H-T]. Nonunitarity when $|\Delta_{k+1}| > 1$ (resp., $\Delta_{k+1} = \emptyset$ and $|\Delta_k| = 1$) is Theorem 3.7 (resp., Theorem 4.2) of the present paper. ∎

3. The case $|\Delta_{k+1}| > 1$

In this section, we prove non-unitarizability in the case $|\Delta_{k+1}| > 1$.

Write

$$\pi = L(\delta(\Delta_1), \ldots, \delta(\Delta_k); \delta(\Delta_{k+1}; \sigma)) = L(a, \delta([\nu^{\alpha}\rho, \nu^{\alpha+m}\rho]; \sigma)).$$

If $\alpha \equiv 0 \bmod 1$, $\rho \rtimes \delta([\nu^{\alpha}\rho, \nu^{\alpha+m}\rho]; \sigma)$ is irreducible if $\alpha > 1$ and has two components if $\alpha = 1$ (e.g., see [Mu]). Write

$$\rho \rtimes \delta([\nu^{\alpha}\rho, \nu^{\alpha+m}\rho]; \sigma) = \begin{cases} T_m(\rho; \sigma) \text{ if } \alpha > 1, \\ T_m(\rho; \sigma) + T'_m(\rho; \sigma), \end{cases}$$

with $T_m(\rho; \sigma)$ having the smaller Jacquet module when $\alpha = 1$. Now, set

$$\pi_1 = \begin{cases} L(a, \nu^{\alpha+m}\rho \times \nu^{\alpha+m}\rho, \ldots, \nu^{\frac{1}{2}}\rho \times \nu^{\frac{1}{2}}\rho; \delta(\nu^{\alpha}\rho, \nu^{\alpha+m}\rho; \sigma)) \text{ if } \alpha \equiv \frac{1}{2} \bmod 1, \\ L(a, \nu^{\alpha+m}\rho \times \nu^{\alpha+m}\rho, \ldots, \nu\rho \times \nu\rho; T_m(\rho; \sigma)) \text{ if } \alpha \equiv 0 \bmod 1. \end{cases}$$

Lemma 3.1. $\pi_1 \leq \mathfrak{s}([\nu^{-\alpha-m}\rho, \nu^{\alpha+m}\rho]) \rtimes \pi$.

Proof. First, it follows directly from the Langlands classification that $\mu^*(\pi) \geq \mathcal{L}(-a) \otimes \delta([\nu^{\alpha}\rho, \nu^{\alpha+m}\rho]; \sigma)$, where $-a$ is the corresponding data in the subrepresentation setting of the Langlands classification (i.e., $L(a; \tau) = L_{sub}(-a; \tau)$). Further, since the supercuspidal support of $\mathcal{L}(-a)$ lies in $\{\nu^{-\alpha-m-1}\rho, \nu^{-\alpha-m-2}\rho, \ldots \nu^{-\alpha-n}\rho\}$, it follows that $\mathcal{L}(-a) \otimes \delta([\nu^{\alpha}\rho, \nu^{\alpha+m}\rho]; \sigma)$ is the only term in $\mu^*(\pi)$

having its central character (since, e.g., any term in $M^*(\mathcal{L}(-a))$ other than $\mathcal{L}(-a)\otimes 1$ involves sign changes). It then follows that

$$\mu^*(\mathfrak{s}([\nu^{-\alpha-m}\rho,\nu^{\alpha+m}\rho])\rtimes\pi)\geq \mathcal{L}(-a)\otimes\mathfrak{s}([\nu^{-\alpha-m}\rho,\nu^{\alpha+m}\rho])\rtimes\delta([\nu^{\alpha}\rho,\nu^{\alpha+m}\rho];\sigma),$$

and this is the only term in $\mu^*(\mathfrak{s}([\nu^{-\alpha-m}\rho,\nu^{\alpha+m}\rho])\rtimes\pi)$ having its central character.

Next, a straightforward μ^* argument tells us that if $\alpha\equiv\frac{1}{2}\operatorname{mod}1$,

$$\mu^*(\mathfrak{s}([\nu^{-\alpha-m}\rho,\nu^{\alpha+m}\rho])\rtimes\delta([\nu^{\alpha}\rho,\nu^{\alpha+m}\rho];\sigma))$$
$$\geq \mathcal{L}(\nu^{-\frac{1}{2}}\rho\times\nu^{-\frac{1}{2}}\rho,\dots,\nu^{-\alpha-m}\rho\times\nu^{-\alpha-m}\rho)\otimes\delta([\nu^{\alpha}\rho,\nu^{\alpha+m}\rho];\sigma),$$

and this is the only term in $\mu^*(\mathfrak{s}([\nu^{-\alpha-m}\rho,\nu^{\alpha+m}\rho])\rtimes\delta([\nu^{\alpha}\rho,\nu^{\alpha+m}\rho];\sigma))$ with its central character. Then, for a suitable standard Levi factor M, we have

$$r_{M,G}((\mathfrak{s}([\nu^{-\alpha-m}\rho,\nu^{\alpha+m}\rho])\rtimes\pi)\geq$$
$$\mathcal{L}(-a)\otimes\mathcal{L}(\nu^{-\frac{1}{2}}\rho\times\nu^{-\frac{1}{2}}\rho,\dots,\nu^{-\alpha-m}\rho\times\nu^{-\alpha-m}\rho)\otimes\delta([\nu^{\alpha}\rho,\nu^{\alpha+m}\rho];\sigma),$$

which implies (by central character considerations)

$$\pi_1'\hookrightarrow\mathcal{L}(-a)\times\mathcal{L}(\nu^{-\frac{1}{2}}\rho\times\nu^{-\frac{1}{2}}\rho,\dots,\nu^{-\alpha-m}\rho\times\nu^{-\alpha-m}\rho)\rtimes\delta([\nu^{\alpha}\rho,\nu^{\alpha+m}\rho];\sigma),$$

where π_1' is the irreducible subquotient of $\mathfrak{s}([\nu^{-\alpha-m}\rho,\nu^{\alpha+m}\rho])\rtimes\pi$ containing $\mathcal{L}(-a)\otimes\mathcal{L}(\nu^{-\frac{1}{2}}\rho\times\nu^{-\frac{1}{2}}\rho,\dots,\nu^{-\alpha-m}\rho\times\nu^{-\alpha-m}\rho)\otimes\delta([\nu^{\alpha}\rho,\nu^{\alpha+m}\rho];\sigma)$ in its Jacquet module. However, by the Langlands classification,

$$\pi_1\hookrightarrow\mathcal{L}(-a)\times\mathcal{L}(\nu^{-\frac{1}{2}}\rho\times\nu^{-\frac{1}{2}}\rho,\dots,\nu^{-\alpha-m}\rho\times\nu^{-\alpha-m}\rho)\rtimes\delta([\nu^{\alpha}\rho,\nu^{\alpha+m}\rho];\sigma)$$

as unique irreducible subrepresentation. Thus $\pi_1'\cong\pi_1$, as needed. This finishes the case $\alpha\equiv\frac{1}{2}\operatorname{mod}1$.

The argument when $\alpha\equiv 0\operatorname{mod}1$ is similar, but with a few minor changes. Again, we have

$$\mu^*(\mathfrak{s}([\nu^{-\alpha-m}\rho,\nu^{\alpha+m}\rho])\rtimes L(a,\delta([\nu^{\alpha}\rho,\nu^{\alpha+m}\rho];\sigma)))\geq$$
$$\mathcal{L}(-a)\times\mathcal{L}(\nu^{-1}\rho\times\nu^{-1}\rho,\dots,\nu^{-\alpha-m}\rho\times\nu^{-\alpha-m}\rho)\otimes(\rho\rtimes\delta([\nu^{\alpha}\rho,\nu^{\alpha+m}\rho];\sigma)),$$

and this is the only term in $\mu^*(\mathfrak{s}([\nu^{-\alpha-m}\rho,\nu^{\alpha+m}\rho])\rtimes\delta([\nu^{\alpha}\rho,\nu^{\alpha+m}\rho];\sigma))$ with its central character. If $\alpha\neq 1$, we have $\rho\rtimes\delta([\nu^{\alpha}\rho,\nu^{\alpha+m}\rho];\sigma)=T_m(\rho;\sigma)$ (irreducible) and the same basic argument as above applies. If $\alpha=1$, we have $\rho\rtimes\delta([\nu^{\alpha}\rho,\nu^{\alpha+m}\rho];\sigma)\cong T_m(\rho;\sigma)\oplus T_m'(\rho;\sigma)$. Then, we let π_1' be the irreducible subquotient of $(\mathfrak{s}([\nu^{-\alpha-m}\rho,\nu^{\alpha+m}\rho])\rtimes L(a,\delta([\nu^{\alpha}\rho,\nu^{\alpha+m}\rho];\sigma))$ containing

$$\mathcal{L}(-a)\otimes L(\nu^{-1}\rho\times\nu^{-1}\rho,\dots,\nu^{-\alpha-m}\rho\times\nu^{-\alpha-m}\rho)\otimes T_m(\rho;\sigma)$$

in its Jacquet module. An argument similar to that above then shows $\pi_1'=\pi_1$, as needed. ■

Lemma 3.2. *Let*

$$\alpha_0=\begin{cases}\frac{1}{2} \text{ if } \alpha\equiv\frac{1}{2} \operatorname{mod}1,\\[2ex] 1 \text{ if } \alpha\equiv 0 \operatorname{mod}1.\end{cases}$$

Then

$$\mathcal{L}(\nu^{-\alpha_0}\rho, \dots, \nu^{-\alpha-m}\rho, -a) \times \mathfrak{s}([\nu^{-\alpha-m}, \nu^{-\alpha_0}\rho]) = \mathcal{L}(\nu^{-\alpha_0}\rho \times \nu^{-\alpha_0}\rho, \dots, \nu^{-\alpha-m}\rho \times \nu^{-\alpha-m}\rho, -a),$$

in particular, is irreducible.

Proof. First, we show $\mathcal{L}(\nu^{-\alpha_0}\rho, \dots, \nu^{-\alpha-m}\rho, -a) \times \mathfrak{s}([\nu^{-\alpha-m}, \nu^{-\alpha_0}\rho])$ is irreducible, then address the Langlands data. By duality, it suffices to show the irreducibility of

$$\mathcal{L}(\nu^{-\alpha_0}\rho, \dots, \nu^{-\alpha-m}\rho, -a)\hat{} \times \delta([\nu^{-\alpha-m}, \nu^{-\alpha_0}\rho]).$$

By section 3.3 [J3], $\mathcal{L}(\nu^{-\alpha_0}\rho, \dots, \nu^{-\alpha-m}\rho, -a)\hat{} \cong \mathcal{L}(\delta([\nu^{-j+1-\alpha_0}\rho, \nu^{-\alpha_0}\rho]), a^*)$ with $j \geq m$ and suitable a^*. Irreducibility now follows from Lemma 1.3.3 [J3] (noting that

$$\mathcal{L}(\delta([\nu^{-j+1-\alpha_0}\rho, \nu^{-\alpha_0}\rho]), a^*) = \mathcal{L}_{sub}(-a^*, (\delta([\nu^{-j+1-\alpha_0}\rho, \nu^{-\alpha_0}\rho]),$$

and every $\delta([\nu^d\rho, \nu^c\rho])$ appearing in $-a^*$ has $c < -m-1$ so $\delta([\nu^d\rho, \nu^c\rho]) \times \delta([\nu^{-j+1-\alpha_0}\rho, \nu^{-\alpha_0}\rho])$ is irreducible).

Let $\tau = \mathcal{L}(\nu^{-\alpha_0}\rho, \dots, \nu^{-\alpha-m}\rho, -a) \times \mathfrak{s}([\nu^{-\alpha-m}, \nu^{-\alpha_0}\rho])$. Since

$$\mathfrak{s}([\nu^{-\alpha-m}, \nu^{-\alpha_0}\rho]) = L(\nu^{-\alpha_0}\rho, \dots, \nu^{-\alpha_0-m}\rho)$$

we have a surjection

$$\mathcal{L}(\nu^{-\alpha_0}\rho, \dots, \nu^{-\alpha_0-m}\rho) \times \mathcal{L}(\nu^{-\alpha_0}\rho, \dots, \nu^{-\alpha_0-m}\rho) \times \mathcal{L}(-a) \longrightarrow \tau.$$

By [Z], $\mathcal{L}(\nu^{-\alpha_0}\rho, \dots, \nu^{-\alpha_0-m}\rho) \times \mathcal{L}(\nu^{-\alpha_0}\rho, \dots, \nu^{-\alpha_0-m}\rho)$ is irreducible, hence is isomorphic to $\mathcal{L}(\nu^{-\alpha_0}\rho \times \nu^{-\alpha_0}\rho, \dots, \nu^{-\alpha_0-m}\rho \times \nu^{-\alpha_0-m}\rho)$. Now, we have a surjection

$$\begin{aligned}&(\nu^{-\alpha_0}\rho \times \nu^{-\alpha_0}\rho) \times \cdots \times (\nu^{-\alpha_0-m}\rho \times \nu^{-\alpha_0-m}\rho)) \times \mathcal{I}(-a)\\ &\longrightarrow \mathcal{L}(\nu^{-\alpha_0}\rho, \dots, \nu^{-\alpha_0-m}\rho) \times \mathcal{L}(\nu^{-\alpha_0}\rho, \dots, \nu^{-\alpha_0-m}\rho) \times \mathcal{L}(-a)\\ &\longrightarrow \tau,\end{aligned}$$

where $\mathcal{I}(-a)$ denotes the corresponding standard module (the induced representation having $\mathcal{L}(-a)$ as unique irreducible quotient). By the Langlands classification, $\nu^{-\alpha_0}\rho \times \nu^{-\alpha_0}\rho \times \cdots \times \nu^{-\alpha_0-m}\rho \times \nu^{-\alpha_0-m}\rho \times \mathcal{I}(-a)$ has

$$\mathcal{L}(\nu^{-\alpha_0}\rho \times \nu^{-\alpha_0}\rho, \dots, \nu^{-\alpha-m}\rho \times \nu^{-\alpha-m}\rho, -a)$$

as unique irreducible quotient so

$$\tau = \mathcal{L}(\nu^{-\alpha_0}\rho \times \nu^{-\alpha_0}\rho, \dots, \nu^{-\alpha-m}\rho \times \nu^{-\alpha-m}\rho, -a),$$

as needed. ∎

With α_0 as above, let

$$\pi_0 = \begin{cases} L(a, \nu^{\alpha+m}\rho, \dots, \nu^{\frac{1}{2}}\rho; \delta(\nu^{\alpha}\rho, \nu^{\alpha+m}\rho; \sigma)) \text{ if } \alpha \equiv \frac{1}{2} \bmod 1, \\ \\ L(a, \nu^{\alpha+m}\rho, \dots, \nu\rho; T_m(\rho; \sigma)) \text{ if } \alpha \equiv 0 \bmod 1. \end{cases}$$

Lemma 3.3.

$$\pi_1 \leq \mathfrak{s}([\nu^{-\alpha-m}\rho, \nu^{-\alpha_0}\rho]) \rtimes \pi_0.$$

Proof. We actually show more: $\pi_1 \hookrightarrow \mathfrak{s}([\nu^{-\alpha-m}\rho, \nu^{-\alpha_0}\rho]) \rtimes \pi_0$. We do the case $\alpha \equiv 0 \bmod 1$; the case $\alpha \equiv \frac{1}{2} \bmod 1$ is similar but slightly easier. Here, we work in the subrepresentation setting of the Langlands classification for much of the proof. First, observe that from Lemma 3.2,

$$\begin{aligned}\pi_1 &\hookrightarrow \mathcal{L}_{sub}(-a, \nu^{-\alpha-m}\rho \times \nu^{-\alpha-m}\rho, \ldots, \nu^{-1}\rho \times \nu^{-1}\rho) \rtimes T_m(\rho;\sigma) \\ &\cong \mathcal{L}_{sub}(\nu^{-\alpha-m}\rho, \ldots, \nu^{-1}\rho) \times \mathcal{L}_{sub}(-a, \nu^{-\alpha-m}\rho, \ldots, \nu^{-1}\rho) \rtimes T_m(\rho;\sigma),\end{aligned}$$

so, by Lemma 5.5 of [J2], it follows that

$$\pi_1 \hookrightarrow \mathcal{L}_{sub}(\nu^{-\alpha-m}\rho, \ldots, \nu^{-1}\rho) \rtimes \theta$$

for some irreducible $\theta \leq \mathcal{L}_{sub}(-a, \nu^{-\alpha-m}\rho, \ldots, \nu^{-1}\rho) \rtimes T_m(\rho;\sigma)$. Next, write $\mu^*(\theta) = \sum_i \tau_i \otimes \theta_i$. Then a straightforward calculation using Theorem 2.1 gives

$$\begin{aligned}\mu^*(\pi_1) &\leq M^*(\mathcal{L}_{sub}(\nu^{-\alpha-m}\rho, \ldots, \nu^{-1}\rho)) \rtimes \mu^*(\theta) \\ &= \sum_i \sum_{j=-\alpha-m-1}^{-1} \sum_{\ell=-\alpha-m-1}^{j} \mathcal{L}_{sub}(\nu\rho, \ldots, \nu^{-j-1}\rho) \times \mathcal{L}_{sub}(\nu^{-\alpha-m}\rho, \ldots, \nu^{\ell}\rho) \times \tau_i \\ &\qquad \otimes \mathcal{L}_{sub}(\nu^{\ell+1}\rho, \ldots, \nu^{j}\rho) \rtimes \theta_i.\end{aligned}$$

By Frobenius reciprocity, $\mu^*(\pi_1) \geq \mathcal{L}_{sub}(\nu^{-\alpha-m}\rho, \ldots, \nu^{-1}\rho) \otimes \theta$. To have a term of this form above, we must have $j = -1$. Further, since

$$\tau_i \otimes \theta_i \leq \mu^*(\theta) \leq \mu^*(\mathcal{L}_{sub}(-a, \nu^{-\alpha-m}\rho, \ldots, \nu^{-1}\rho) \rtimes T_m(\rho;\sigma)),$$

we see that any term of the form $\nu^x\rho \otimes \lambda \leq r_{M_{(r)},G}(\tau_i)$ must have

$$x \notin \{-\alpha - m, -\alpha - m + 1, \ldots, -1\}.$$

In particular, it follows that we must have $\tau_i = 1$. Therefore, the only term of the form $\mathcal{L}_{sub}(\nu^{-\alpha-m}\rho, \ldots, \nu^{-1}\rho) \otimes \theta'$ in $\mu^*(\pi_1)$ is $\mathcal{L}_{sub}(\nu^{-\alpha-m}\rho, \ldots, \nu^{-1}\rho) \otimes \theta$. Now, by Frobenius reciprocity, this implies (for a suitable standard Levi factor M)

$$\pi_1 \hookrightarrow \mathcal{L}_{sub}(\nu^{-\alpha-m}\rho, \ldots, \nu^{-1}\rho) \times \mathcal{L}_{sub}(-a, \nu^{-\alpha-m}\rho \ldots, \nu^{-1}\rho) \rtimes T_m(\rho;\sigma),$$

so that

$$r_{M,G}(\pi_1) \geq \mathcal{L}_{sub}(\nu^{-\alpha-m}\rho, \ldots, \nu^{-1}\rho) \otimes \mathcal{L}_{sub}(-a, \nu^{-\alpha-m}\rho \ldots, \nu^{-1}\rho) \otimes T_m(\rho;\sigma),$$

and, consequently,

$$\mu^*(\theta) \geq \mathcal{L}_{sub}(-a, \nu^{-\alpha-m}\rho \ldots, \nu^{-1}\rho) \otimes T_m(\rho;\sigma).$$

Since the only component of $\mathcal{L}_{sub}(-a, \nu^{-\alpha-m}\rho \ldots, \nu^{-1}\rho) \rtimes T_m(\rho;\sigma)$ having

$$\mathcal{L}_{sub}(-a, \nu^{-\alpha-m}\rho \ldots, \nu^{-1}\rho) \otimes T_m(\rho;\sigma)$$

in its Jacquet module is $L_{sub}(-a, \nu^{-\alpha-m}\rho, \ldots, \nu^{-1}\rho; T_m(\rho;\sigma))$ (a property of the Langlands classification–see [B-J]), it follows that

$$\theta = L_{sub}(-a, \nu^{-\alpha-m}\rho \ldots, \nu^{-1}\rho; T_m(\rho;\sigma)),$$

as needed. ∎

Lemma 3.4. *Suppose* $\nu^x\rho \otimes \theta \leq r_{M_{(r)},G}(\pi_0)$ *with* $x \in \{-\alpha - m, -\alpha - m + 1, \ldots, \alpha + m\}$.

1. *Suppose* $\alpha \cong \frac{1}{2}\, mod 1$.
   (a) *If* $\alpha = \frac{1}{2}$, *then* $x = \alpha + m$.
   (b) *If* $\alpha > \frac{1}{2}$, *then* $x \in \{\frac{1}{2}, \alpha + m\}$.
2. *If* $\alpha \cong 0\, mod 1$, *then* $x \in \{0, \alpha + m\}$.

Proof. For 1, observe that any $\nu^x\rho \otimes \tau \leq m^*(\mathcal{L}(\nu^{-\frac{1}{2}}\rho, \ldots, \nu^{-\alpha-m}\rho, -a))$ has $x < -\alpha - m$ (by Proposition 2.1.4 [J3]). Also, any

$$\tau \otimes \nu^x\rho \leq m^*(\mathcal{L}(\nu^{-\frac{1}{2}}\rho, \ldots, \nu^{-\alpha-m}\rho, -a))$$

has $x = -\frac{1}{2}$ or $x < -\alpha - m$ (by Proposition 2.4.3 [J3]). It then follows that any

$$\nu^x\rho \otimes \tau \leq M^*(\mathcal{L}(\nu^{-\frac{1}{2}}\rho, \ldots, \nu^{-\alpha-m}\rho, -a))$$

has $x = \frac{1}{2}$ or $|x| > \alpha + m$. Also, any $\nu^x\rho \otimes \lambda \leq \mu^*(\delta([\nu^\alpha\rho, \nu^{\alpha+m}\rho]; \sigma))$ has $x = \alpha + m$. Since

$$\pi_0 \leq \mathcal{L}(\nu^{-\frac{1}{2}}\rho, \ldots, \nu^{-\alpha-m}\rho, -a) \rtimes \delta([\nu^\alpha\rho, \nu^{\alpha+m}\rho]; \sigma),$$

it then follows from Theorem 2.1 that the only possible $\nu^x\rho \otimes \theta \leq r_{M_{(r)},G}(\pi_0)$ with $|x| \leq \alpha + m$ have $x = \frac{1}{2}$ or $\alpha + m$. This finishes 1(b). For 1(a), we must still rule out the possibility $x = \frac{1}{2}$. For this, observe that

$$\pi_0 \hookrightarrow \mathcal{L}_{sub}(a) \times \mathfrak{s}([\nu^{-\frac{1}{2}-m}\rho, \ldots \nu^{-\frac{3}{2}}\rho]) \rtimes (L(\nu^{\frac{1}{2}}\rho, \delta([\nu^{\frac{1}{2}}\rho, \nu^{\frac{1}{2}+m}\rho]; \sigma)).$$

It then suffices to show there are no terms of the form $\nu^{\frac{1}{2}}\rho \otimes \lambda$ in

$$r_{M_{(r)},G}(L(\nu^{\frac{1}{2}}\rho, \delta([\nu^{\frac{1}{2}}\rho, \nu^{\frac{1}{2}+m}\rho]; \sigma))).$$

To see that this is the case, observe that

$$L(\nu^{\frac{1}{2}}\rho; \delta([\nu^{\frac{1}{2}}\rho, \nu^{\frac{1}{2}+m}\rho]; \sigma)) \leq \nu^{\frac{1}{2}}\rho \rtimes \delta([\nu^{\frac{1}{2}}\rho, \nu^{\frac{1}{2}+m}\rho]; \sigma).$$

Therefore, its dual is a subquotient of the generalized degenerate principal series $\nu^{-\frac{1}{2}}\rho \rtimes L(\nu^{m+\frac{1}{2}}\rho, \ldots, \nu^{\frac{1}{2}}\rho; \sigma)$, analyzed in [J1]. It has two irreducible subquotients; only the one labeled π_2 in Theorem 6.1 of that paper contains a term of the form $\nu^{\frac{1}{2}}\rho \otimes \ldots$ in its Jacquet module, hence must be the dual of $L(\nu^{\frac{1}{2}}\rho; \delta([\nu^{\frac{1}{2}}\rho, \nu^{\frac{1}{2}+m}\rho]; \sigma))$. Since $r_{M_{(r)},G}(\pi_2)$ contains no term of the form $\nu^{-\frac{1}{2}}\rho \otimes \ldots$, it follows that $r_{M_{(r)},G}L(\nu^{\frac{1}{2}}\rho; \delta([\nu^{\frac{1}{2}}\rho, \nu^{\frac{1}{2}+m}\rho]; \sigma))$ contains no term of the form $\nu^{\frac{1}{2}}\rho \otimes \ldots$, as claimed. The argument for 2 is similar–observe that

$$\pi_0 \hookrightarrow \mathcal{L}(\nu^{-1}\rho, \ldots, \nu^{-\alpha-m}\rho, -a) \rtimes T_m(\rho; \sigma)$$

tells us the only possible values of x from the set are $x = 0, 1, \alpha + k$. To see that $x \neq 1$, observe that

$$\pi_0 \leq \mathcal{L}(-a) \times \mathfrak{s}([\nu^{-\alpha-m}\rho, \rho]) \rtimes \delta([\nu^\alpha\rho, \nu^{\alpha+m}\rho]; \sigma),$$

finishing 2. ∎

Lemma 3.5. *If $\alpha \cong 0 \, mod 1$, then*

$$L(\nu^{\alpha+m}\rho, \dots, \nu\rho; T_m(\rho;\sigma)) \leq \rho \rtimes L(\nu^{\alpha+m}\rho, \dots, \nu\rho; \delta([\nu^{\alpha}\rho, \nu^{\alpha+m}\rho];\sigma).$$

Proof. Observe that

$$s_{\nu^{-\alpha-m}\rho\otimes\cdots\otimes\nu^{-1}\rho}(\rho \rtimes L(\nu^{\alpha+m}\rho, \dots, \nu\rho; \delta([\nu^{\alpha}\rho, \nu^{\alpha+m}\rho];\sigma)) = \nu^{-\alpha-m}\rho \otimes \cdots \otimes \nu^{-1}\rho \otimes (\rho \rtimes \delta([\nu^{\alpha}\rho, \nu^{\alpha+m}\rho];\sigma)),$$

where $s_{\nu^{-\alpha-m}\rho\otimes\cdots\otimes\nu^{-1}\rho}$ denotes the sum of everything in the (appropriate) Jacquet module of the form $\nu^{-\alpha-m}\rho \otimes \cdots \otimes \nu^{-1}\rho \otimes \theta$ for some θ. Let π_0' denote the irreducible subquotient of $\rho \rtimes L(\nu^{\alpha+m}\rho, \dots, \nu\rho; \delta([\nu^{\alpha}\rho, \nu^{\alpha+m}\rho];\sigma))$ containing

$$\nu^{-\alpha-m}\rho \otimes \cdots \otimes \nu^{-1}\rho \otimes T_m(\rho;\sigma)$$

in its Jacquet module. If $\alpha > 1$, then $\rho \rtimes \delta([\nu^{\alpha}\rho, \nu^{\alpha+m}\rho];\sigma) \cong T_m(\rho;\sigma)$ is irreducible and it follows from central character considerations that $\pi_0' \hookrightarrow \nu^{-\alpha-m}\rho \times \cdots \times \nu^{-1}\rho \rtimes T_m(\rho;\sigma)$. It then follows immediately from the Langlands classification that $\pi_0' \cong L(\nu^{\alpha+m}\rho, \dots, \nu\rho; T_m(\rho;\sigma))$, as needed. The argument when $\alpha = 1$ is the same if $s_{\nu^{-\alpha-m}\rho\otimes\cdots\otimes\nu^{-1}\rho}(\pi_0') = \nu^{-\alpha-m}\rho \otimes \cdots \otimes \nu^{-1}\rho \otimes T_m(\rho;\sigma)$. Were it to also contain $\nu^{-\alpha-m}\rho \otimes \cdots \otimes \nu^{-1}\rho \otimes T_m'(\rho;\sigma)$, one might have

$$\pi_0' \hookrightarrow \nu^{-\alpha-m}\rho \times \cdots \times \nu^{-1}\rho \rtimes T_m'(\rho;\sigma)$$

instead. However, this would give $\pi_0' \cong L(\nu^{\alpha+m}\rho, \dots, \nu\rho; T_m'(\rho;\sigma))$, which does not contain $\nu^{-\alpha-m}\rho \otimes \cdots \otimes \nu^{-1}\rho \otimes T_m(\rho;\sigma)$ in its Jacquet module (by Proposition 5.3 [B-J]), a contradiction. Thus we must have $\pi_0' \hookrightarrow \nu^{-\alpha-m}\rho \times \cdots \times \nu^{-1}\rho \rtimes T_m(\rho;\sigma)$, hence $\pi_0' \cong L(\nu^{\alpha+m}\rho, \dots, \nu\rho; T_m(\rho;\sigma))$, as needed. ∎

Lemma 3.6. *1. Suppose $\alpha \cong \frac{1}{2} \, mod 1$. Then $\mu^*(\pi_0)$ contains no terms of the form $\mathfrak{s}([\nu^{\frac{1}{2}}\rho, \nu^{\alpha+m}\rho]) \otimes \theta$.*

2. Suppose $\alpha \cong 0 \, mod 1$. Then $\mu^*(\pi_0)$ contains no terms of the form

$$\mathfrak{s}([\rho, \nu^{\alpha+m}\rho]) \otimes \theta.$$

Proof. We first address the case $\alpha \equiv \frac{1}{2} \mod 1$. If $\alpha = \frac{1}{2}$, the result follows immediately from Lemma 3.4. If $\alpha > \frac{1}{2}$, observe that it follows easily from the Langlands classification that

$$\pi_0 \leq \mathcal{L}(a) \times \mathfrak{s}([\nu^{-\alpha-m}\rho, \nu^{-\alpha}\rho]) \rtimes L(\nu^{\alpha-1}\rho, \dots \nu^{\frac{1}{2}}\rho; \delta([\nu^{\alpha}\rho, \nu^{\alpha+m}\rho];\sigma)).$$

Next, we claim that

$$L(\nu^{\alpha-1}\rho, \dots \nu^{\frac{1}{2}}\rho; \delta([\nu^{\alpha}\rho, \nu^{\alpha+m}\rho];\sigma)) \leq \mathfrak{s}([\nu^{-\alpha+2}\rho, \nu^{-\frac{1}{2}}\rho]) \rtimes L(\nu^{\alpha-1}\rho; \delta([\nu^{\alpha}\rho, \nu^{\alpha+m}\rho];\sigma)).$$

To see this, observe that $\mathfrak{s}([\nu^{-\alpha+2}\rho, \nu^{-\frac{1}{2}}\rho]) \rtimes L(\nu^{\alpha-1}\rho; \delta([\nu^{\alpha}\rho, \nu^{\alpha+m}\rho];\sigma))$ contains $\nu^{-\alpha+1}\rho \otimes \cdots \otimes \nu^{-\frac{1}{2}}\rho \otimes \delta([\nu^{\alpha}\rho, \nu^{\alpha+m}\rho];\sigma)$ in its Jacquet module, and it is the unique term in the Jacquet module having this central character. The irreducible subquotient containing this term then embeds in

$$\nu^{-\alpha+1}\rho\otimes\cdots\otimes\nu^{-\frac{1}{2}}\rho\rtimes\delta([\nu^{\alpha}\rho,\nu^{\alpha+m}\rho];\sigma)$$

by central character considerations, hence must be

$$L(\nu^{\alpha-1}\rho,\ldots\nu^{\frac{1}{2}}\rho;\delta([\nu^{\alpha}\rho,\nu^{\alpha+m}\rho];\sigma)),$$

as claimed. Combining these observations,

$$\pi_0\leq\mathcal{L}(a)\times\mathfrak{s}([\nu^{-\alpha-m}\rho,\nu^{-\alpha}\rho])\times\mathfrak{s}([\nu^{-\alpha+2}\rho,\nu^{-\frac{1}{2}}\rho])\rtimes L(\nu^{\alpha-1}\rho;\delta([\nu^{\alpha}\rho,\nu^{\alpha+m}\rho];\sigma)).$$

By supercuspidal support considerations, in order for

$$\mu^*(\pi_0)\geq\mathfrak{s}([\nu^{\frac{1}{2}}\rho,\nu^{\alpha+m}\rho])\otimes\theta$$

to hold we would have to have

$$\nu^{\alpha-1}\rho\otimes\theta'\leq\mu^*(L(\nu^{\alpha-1}\rho;\delta([\nu^{\alpha}\rho,\nu^{\alpha+m}\rho];\sigma))$$

for some θ'. Now, it follows from [Mu] that

$$L(\nu^{\alpha-1}\rho;\delta([\nu^{\alpha}\rho,\nu^{\alpha+m}\rho;\sigma))\leq\delta([\nu^{\alpha-1}\rho,\nu^{\alpha+m}\rho])\rtimes\sigma$$

(Proposition 3.1(i) if $\alpha>3/2$; Theorem 5.1(ii) if $\alpha=3/2$). From this, it follows immediately that $\mu^*(L(\nu^{\alpha-1}\rho;\delta([\nu^{\alpha}\rho,\nu^{\alpha+m}\rho];\sigma)))$ contains no terms of the form $\nu^{\alpha-1}\rho\otimes\theta'$.

We now consider the case $\alpha\equiv 0 \bmod 1$. By Lemma 3.5, it suffices to show

$$\mu^*(\mathcal{L}(a)\times\rho\rtimes L(\nu^{\alpha+m}\rho,\ldots,\nu\rho;\delta([\nu^{\alpha}\rho,\nu^{\alpha+m}\rho];\sigma))$$

contains no such terms. For $\mu^*(\mathcal{L}(a)\times\rho\rtimes L(\nu^{\alpha+m}\rho,\ldots,\nu\rho;\delta([\nu^{\alpha}\rho,\nu^{\alpha+m}\rho];\sigma))$ to contain such a term, we would have to have either

$$\mu^*(L(\nu^{\alpha+m}\rho,\ldots,\nu\rho;\delta([\nu^{\alpha}\rho,\nu^{\alpha+m}\rho];\sigma))\geq\mathfrak{s}([\rho,\nu^{\alpha+m}\rho])\otimes\theta'$$

for some θ'–which cannot happen by supercuspidal support considerations–or

$$\mu^*(L(\nu^{\alpha+m}\rho,\ldots,\nu\rho;\delta([\nu^{\alpha}\rho,\nu^{\alpha+m}\rho];\sigma))\geq\mathfrak{s}([\nu\rho,\nu^{\alpha+m}\rho])\otimes\theta'$$

–which cannot happen by Lemma 3.4. The result follows. ■

Theorem 3.7. *Suppose* $|\Delta_{k+1}|>1$. *Then* $\pi=L(\delta(\Delta_1),\ldots,\delta(\Delta_k);\delta(\Delta_{k+1};\sigma))$ *is not unitary.*

Proof. By Lemma 3.1,

$$\pi_1\leq\mathfrak{s}([\nu^{-\alpha-m}\rho,\nu^{\alpha+m}\rho])\rtimes\pi.$$

Were π unitary, we would have to have π_1 appearing as a subrepresentation. It therefore suffices to show π_1 is not a subrepresentation of $\mathfrak{s}([\nu^{-\alpha-m}\rho,\nu^{\alpha+m}\rho])\rtimes\pi$, or by Frobenius reciprocity, that $\mu^*(\pi_1)\not\geq\mathfrak{s}([\nu^{-\alpha-m}\rho,\nu^{\alpha+m}\rho])\otimes\pi$. Now, by Lemma 3.3,

$$\pi_1\leq\mathfrak{s}([\nu^{-\alpha-m}\rho,\nu^{-\alpha_0}\rho])\rtimes\pi_0.$$

It therefore suffices to show $\mu^*(\mathfrak{s}([\nu^{-\alpha-m}\rho,\nu^{-\alpha_0}\rho])\rtimes\pi_0)\not\geq\mathfrak{s}([\nu^{-\alpha-m}\rho,\nu^{\alpha+m}\rho])\otimes\pi$. Write $\mu^*(\pi_0)=\sum\tau_i\otimes\theta_i$. A straightforward calculation using Theorem 2.1 gives

$$\mu^*(\mathfrak{s}([\nu^{-\alpha-m}\rho,\nu^{-\alpha_0}\rho])\rtimes\pi_0)=\sum_i\sum_{j=-\alpha-m-1}^{-\alpha_0}\sum_{\ell=-\alpha-m-1}^{j}\mathfrak{s}([\nu^{\alpha_0}\rho,\nu^{-j-1}\rho])\times \mathfrak{s}([\nu^{-\alpha-m}\rho,\nu^{\ell}\rho])\times\tau_i\otimes\mathfrak{s}([\nu^{\ell+1}\rho,\nu^{j}\rho])\rtimes\theta_i.$$

First, suppose $\alpha\equiv\frac{1}{2}\bmod 1$. If $\ell=-\frac{1}{2}$, then $j=-\frac{1}{2}$; in order to pick up $\mathfrak{s}([\nu^{-\alpha-m}\rho,\nu^{\alpha+m}\rho])\otimes\pi$ we would then need $\tau_i\otimes\theta_i=\mathfrak{s}([\nu^{\frac{1}{2}}\rho,\nu^{\alpha+m}\rho])\otimes\pi$. However, by Lemma 3.6 this is not the case. If $\ell<-\frac{1}{2}$, then $s_{(1)}\tau_i$–hence $s_{(1)}\pi_0$–must contain a term of the form $\nu^{\ell+1}\rho\otimes\lambda$. However, by Lemma 3.4 this is not the case. Thus $\mu^*(\mathfrak{s}([\nu^{-\alpha-m}\rho,\nu^{-\frac{1}{2}}\rho])\rtimes\pi_0)\not\geq\mathfrak{s}([\nu^{-\alpha-m}\rho,\nu^{\alpha+m}\rho])\otimes\pi$, and non-unitarizability follows. Now, suppose $\alpha\equiv 0\bmod 1$. If $\ell=-1$, then $j=-1$ and we need $\tau_i\otimes\theta_i=\mathfrak{s}([\rho,\nu^{\alpha+m}\rho])\otimes\pi'$ for a suitable π'. However, by Lemma 3.6, this does not happen. If $\ell<-1$, then $s_{(1)}\tau_i$–hence $s_{(1)}\pi_0$–must contain a term of the form $\nu^{\ell+1}\rho\otimes\lambda$. Again, by Lemma 3.4 this is not the case. Thus $\mu^*(\mathfrak{s}([\nu^{-\alpha-m}\rho,\nu^{-1}\rho])\rtimes\pi_0)\not\geq\mathfrak{s}([\nu^{-\alpha-m}\rho,\nu^{\alpha+m}\rho])\otimes\pi$, and non-unitarizability follows. ■

4. The case $\boldsymbol{\Delta_{k+1}=\emptyset}$ and $\boldsymbol{|\Delta_k|=1}$

In this section, we address the case $\Delta_{k+1}=\emptyset$ and $|\Delta_k|=1$. In particular, we show that this case is dual to that covered in section 3; the results needed to prove non-unitarity are then obtained by duality from those in section 3.

Proposition 4.1. *Suppose π has the form $\pi=L(\Delta_1,\dots,\Delta_k;\sigma)$ with $|\Delta_k|=\cdots=|\Delta_{k-\ell+1}|=1$ and $|\Delta_{k-\ell}|>1$ for some $\ell\geq 1$. Then, $\hat{\pi}$ has the form*

$$\hat{\pi}=L(\Delta'_1,\dots,\Delta'_{k'};\delta(\Delta'_{k'+1};\sigma))$$

with $|\Delta_{k'+1}|=\ell+1$. Conversely, if π has the form $\pi=L(\Delta'_1,\dots,\Delta'_{k'};\delta(\Delta'_{k'+1};\sigma))$ with $|\Delta'_{k'+1}|>1$, then $\hat{\pi}$ has the form $\hat{\pi}=L(\Delta_1,\dots,\Delta_k;\sigma)$ with $|\Delta_k|=\cdots=|\Delta_{k-\ell+1}|=1$ and $|\Delta_{k-\ell}|>1$ for $\ell=|\Delta'_{k'+1}|-1$.

Proof. Write

$$\pi=L(\delta([\nu^{b_s}\rho,\nu^{a_s}\rho]),\dots,\delta([\nu^{b_{m+1}}\rho,\nu^{a_{m+1}}\rho]),\nu^{\alpha+m}\rho,\nu^{\alpha+m-1}\rho,\dots,\nu^{\alpha}\rho;\sigma)$$

with $m\geq 0$. Note that $b_{m+1}=\alpha+m+1$ and for $\ell>m+1$, $b_\ell=a_{\ell-1}+1$. Observe that for the appropriate standard Levi factor M, we have

$$\delta([\nu^{-a_s}\rho,\nu^{-b_s}\rho])\otimes\cdots\otimes\nu^{-a_{m+1}}\rho,\nu^{-b_{m+1}}\rho])\otimes\nu^{-\alpha-m}\rho\otimes\nu^{-\alpha-m+1}\rho\otimes\cdots\otimes\nu^{-\alpha}\rho\otimes\sigma$$

in $r_{M,G}(\pi)$, and this is the unique irreducible subquotient of $r_{M,G}(\pi)$ having its central character (see Proposition 5.3 [B-J]). By duality,

$$\mathfrak{s}([\nu^{b_s}\rho,\nu^{a_s}\rho])\otimes\cdots\otimes\mathfrak{s}([\nu^{b_{m+1}}\rho,\nu^{a_{m+1}}\rho])\otimes\nu^{\alpha+m}\rho\otimes\cdots\otimes\nu^{\alpha}\rho\otimes\sigma$$

is the unique irreducible subquotient of $r_{M,G}(\hat{\pi})$ with its central character. Therefore, by central character considerations,

$$\begin{aligned}\hat{\pi} &\hookrightarrow \mathfrak{s}([\nu^{b_s}\rho,\nu^{a_s}\rho])\times\cdots\times\mathfrak{s}([\nu^{b_{m+1}}\rho,\nu^{a_{m+1}}\rho])\times\nu^{\alpha+m}\rho\times\cdots\times\nu^{\alpha}\rho\rtimes\sigma\\ &\hookrightarrow \mathfrak{s}([\nu^{b_s}\rho,\nu^{a_s}\rho])\times\cdots\times(\nu^{\alpha+m+1}\rho\times\mathfrak{s}([\nu^{b_{m+1}+1}\rho,\nu^{a_{m+1}}\rho]))\times\nu^{\alpha+m}\rho\times\cdots\times\nu^{\alpha}\rho\rtimes\sigma\\ &\cong \nu^{\alpha+m+1}\rho\times\nu^{\alpha+m}\rho\times\cdots\times\nu^{\alpha}\rho\times\mathfrak{s}([\nu^{b_s}\rho,\nu^{a_s}\rho])\times\cdots\times\mathfrak{s}([\nu^{b_{m+2}}\rho,\nu^{a_{m+2}}\rho])\times\\ &\qquad \mathfrak{s}([\nu^{b_{m+1}+1}\rho,\nu^{a_{m+1}}\rho])\rtimes\sigma\end{aligned}$$

using the observation that $\delta([\nu^c\rho,\nu^d\rho])\times\nu^x\rho\cong\nu^x\rho\times\delta([\nu^c\rho,\nu^d\rho])$ (irreducible if $x<c-1$) to "commute" $\nu^{\alpha+m-1}\rho,\ldots,\nu^{\alpha}\rho$ to the front. Next, using the fact that

$$\mathfrak{s}([\nu^c\rho,\nu^d\rho])\rtimes\sigma\cong\mathfrak{s}([\nu^{-d}\rho,\nu^{-c}\rho])\rtimes\sigma$$

for $c>\alpha$ (again irreducible), we get $\hat{\pi}\hookrightarrow$

$$\begin{aligned}&\nu^{\alpha+m+1}\rho\times\nu^{\alpha+m}\rho\times\cdots\times\nu^{\alpha}\rho\times\mathfrak{s}([\nu^{-a_{m+1}}\rho,\nu^{-b_{m+1}-1}\rho])\times\mathfrak{s}([\nu^{-a_{m+2}}\rho,\nu^{-b_{m+2}}\rho])\\ &\qquad\times\cdots\times\mathfrak{s}([\nu^{-a_s}\rho,\nu^{-b_s}\rho])\rtimes\sigma\\ &\cong\mathfrak{s}([\nu^{-a_{m+1}}\rho,\nu^{-b_{m+1}-1}\rho])\times\mathfrak{s}([\nu^{-a_{m+2}}\rho,\nu^{-b_{m+2}}\rho])\times\cdots\times\mathfrak{s}([\nu^{-a_s}\rho,\nu^{-b_s}\rho])\times\\ &\qquad\nu^{\alpha+m+1}\rho\times\nu^{\alpha+m}\rho\times\cdots\times\nu^{\alpha}\rho\rtimes\sigma\end{aligned}$$

so, by Lemma 5.5 of [J2], $\hat{\pi}\hookrightarrow\mathcal{L}(b)\rtimes\theta$ for some irreducible

$$\mathcal{L}(b)\le\mathfrak{s}([\nu^{-a_{m+1}}\rho,\nu^{-b_{m+1}-1}\rho])\times\mathfrak{s}([\nu^{-a_{m+2}}\rho,\nu^{-b_{m+2}}\rho])\times\cdots\times\mathfrak{s}([\nu^{-a_s}\rho,\nu^{-b_s}\rho])$$

and $\theta\le\nu^{\alpha+m+1}\rho\times\nu^{\alpha+m}\rho\times\cdots\times\nu^{\alpha}\rho\rtimes\sigma$. Since $s_{min}(\hat{\pi})$ contains terms of the form $\cdots\otimes\nu^{\alpha+m+1}\rho\otimes\nu^{\alpha+m}\rho\otimes\cdots\otimes\nu^{\alpha}\rho\otimes\sigma$, it follows that $\mu^*(\hat{\pi})\ge\mathcal{L}'\otimes\delta([\nu^{\alpha}\rho,\nu^{\alpha+m+1}\rho];\sigma)$ for some $\mathcal{L}'$. Since $\mu^*(\hat{\pi})\le M^*(\mathcal{L}(b))\rtimes\mu^*(\theta)$, supercuspidal support considerations tell us $\theta\cong\delta([\nu^{\alpha}\rho,\nu^{\alpha+m+1}\rho];\sigma)$. It then follows from the Langlands classification that $\hat{\pi}=L_{sub}(b;\delta([\nu^{\alpha}\rho,\nu^{\alpha+m+1}\rho];\sigma)$. Note that this corresponds to $|\Delta'_{k+1}|=m+2=\ell+1$, as needed.

In the other direction, suppose π' has the form

$$\pi'=L(\delta([\nu^{b_s}\rho,\nu^{a_s}\rho]),\ldots,\delta([\nu^{b_{m+1}}\rho,\nu^{a_{m+1}}\rho]);\delta([\nu^{\alpha}\rho,\nu^{\alpha+m}\rho];\sigma)).$$

Again, for a suitable Levi factor M, we have

$$\delta([\nu^{-a_s}\rho,\nu^{-b_s}\rho])\otimes\cdots\otimes\delta([\nu^{-a_{m+1}}\rho,\nu^{-b_{m+1}}\rho])\otimes\delta([\nu^{\alpha}\rho,\nu^{\alpha+m}\rho];\sigma)$$

unique in $r_{M,G}(\pi')$ having its central character. Again, by duality we have

$$\begin{aligned}\hat{\pi'}&\hookrightarrow\mathfrak{s}([\nu^{b_s}\rho,\nu^{a_s}\rho])\times\cdots\times\mathfrak{s}([\nu^{b_{m+1}}\rho,\nu^{a_{m+1}}\rho])\rtimes L(\nu^{\alpha+m}\rho,\nu^{\alpha+m-1}\rho,\ldots,\nu^{\alpha}\rho;\sigma)\\ &\hookrightarrow\mathfrak{s}([\nu^{b_s}\rho,\nu^{a_s}\rho])\times\cdots\times\mathfrak{s}([\nu^{b_{m+1}}\rho,\nu^{a_{m+1}}\rho])\times\nu^{-\alpha-m}\rho\rtimes L(\nu^{\alpha+m-1}\rho,\ldots,\nu^{\alpha}\rho;\sigma)\\ &\cong\nu^{-\alpha-m}\rho\times\mathfrak{s}([\nu^{b_s}\rho,\nu^{a_s}\rho])\times\cdots\times\mathfrak{s}([\nu^{b_{m+1}}\rho,\nu^{a_{m+1}}\rho])\rtimes L(\nu^{\alpha+m-1}\rho,\ldots,\nu^{\alpha}\rho;\sigma)\\ &\cong\nu^{-\alpha-m}\rho\times\mathfrak{s}([\nu^{-a_{m+1}}\rho,\nu^{-b_{m+1}}\rho])\times\ldots\times\mathfrak{s}([\nu^{-a_s}\rho,\nu^{-b_s}\rho])\rtimes L(\nu^{\alpha+m-1}\rho,\ldots,\nu^{\alpha}\rho;\sigma)\end{aligned}$$

using

$$\mathfrak{s}([\nu^c\rho,\nu^d\rho])\rtimes L(\nu^{\alpha+m-1}\rho,\ldots,\nu^{\alpha}\rho;\sigma)\cong\mathfrak{s}([\nu^{-d}\rho,\nu^{-c}\rho])\rtimes L(\nu^{\alpha+m-1}\rho,\ldots,\nu^{\alpha}\rho;\sigma)$$

for $c > \alpha + m$ by irreducibility (cf. [Mu]) and a "commuting" argument. It now follows from Lemma 5.5 [J2] that

$$\hat{\pi}' \hookrightarrow \mathcal{L}(b) \rtimes L(\nu^{\alpha+m-1}\rho, \ldots, \nu^{\alpha}\rho; \sigma)$$

for some irreducible $\mathcal{L}(b) \leq \nu^{-\alpha-m}\rho \times \mathfrak{s}([\nu^{-a_{m+1}}\rho, \nu^{-b_{m+1}}\rho]) \times \cdots \times \mathfrak{s}([\nu^{-a_s}\rho, \nu^{-b_s}\rho])$. Further, since $r_{M_{(r)},G}(\widehat{\pi'})$ contains a term of the form $\nu^{-\alpha-m}\rho \otimes \ldots$, we must have

$$r_{M_{(r)},G}(\mathcal{L}(b)) = r_{M_{(r)},G}(\mathcal{L}_{sub}(\delta([\nu^{-c_1}\rho, \nu^{-d_1}\rho]), \ldots, \delta([\nu^{-c_u}\rho, \nu^{-d_u}\rho])) \geq \nu^{-\alpha-m}\rho \otimes \ldots$$

By Proposition 2.4.3 [J3], we have $\delta([\nu^{-c_u}\rho, \nu^{-d_u}\rho]) \ncong \nu^{-\alpha-m}\rho$. In particular, by the Langlands classification,

$$\hat{\pi} \cong L(\delta([\nu^{d_1}\rho, \nu^{c_1}\rho]), \ldots, \delta([\nu^{d_u}\rho, \nu^{c_u}\rho]), \nu^{\alpha+m-1}\rho, \nu^{\alpha+m-2}\rho, \ldots, \nu^{\alpha}\rho; \sigma)$$

with $\delta([\nu^{d_u}\rho, \nu^{c_u}\rho]) \ncong \nu^{\alpha+m}\rho$. The result follows. ∎

Theorem 4.2. *Let $\pi' = L(\Delta_1, \ldots, \Delta_k; \sigma)$ with $|\Delta_k| = \cdots = |\Delta_{k-\ell+1}| = 1$ and $|\Delta_{k-\ell}| > 1$ for some $\ell \geq 1$. Then π' is non-unitary.*

Proof. We argue using duality and the results of section 3. By Proposition 4.1, let

$$\pi = \hat{\pi'} \cong L(a, \delta([\nu^{\alpha}\rho, \nu^{\alpha+m}\rho]; \sigma))$$

as in section 3. We then let π_0 and π_1 be as in section 3.

First, observe that by duality and Lemma 3.1, we have

$$\hat{\pi}_1 \leq \delta([\nu^{-\alpha-m}\rho, \nu^{\alpha+m}\rho]) \rtimes \hat{\pi}.$$

Further, by duality and Lemma 3.3,

$$\hat{\pi}_1 \leq \delta([\nu^{\alpha_0}\rho, \nu^{\alpha+m}\rho]) \rtimes \hat{\pi}_0.$$

Now, by duality and Lemma 3.6, we have $\mu^*(\hat{\pi}_0)$ contains no terms of the form

$$\delta([\nu^{-\alpha-m}\rho, \nu^{-\frac{1}{2}}\rho]) \otimes \theta$$

if $\alpha \equiv \frac{1}{2} \bmod 1$, and no terms of the form $\delta([\nu^{-\alpha-m}\rho, \rho]) \otimes \theta$ if $\alpha \equiv 0 \bmod 1$.

At this point, the same basic argument as in the proof of Theorem 3.7 may be used to show that $\hat{\pi} = \pi'$ is nonunitary, as needed. ∎

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Received March 27, 2011
and in final form April 9, 2012