Equal-time Kinetic Theory and Quark Spin in Electromagnetic Fields

Xingyu Guo, Anping Huang, Jinfeng Liao and Pengfei Zhuang
Tsinghua University

1. *Equal-time Transport*
2. *NJL Model in Electromagnetic Fields*
3. *Classical Wigner function*
4. *Spin Induced $U_A(1)$ symmetry Breaking*
5. *Summary and Outlook*

Covariant and Equal-time Wigner Operators

1. Classical transport

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \vec{\nabla} f = C$$

$$+ \text{ on-shell condition } \left(p^2 - m^2\right) f = 0 \;\rightarrow\; p_0 = E = \sqrt{\vec{p}^{\,2} + m^2} \quad in \;\; f$$

2. Covariant Wigner operator

$$\widehat{W}(x,p) = \int d^4 y e^{ipy} \widehat{\psi}(x + \frac{y}{2}) e^{iQ\int_{-1/2}^{1/2} ds \widehat{A}(x+sy)\cdot y} \widehat{\overline{\psi}}(x - \frac{y}{2})$$

The kinetic equation for $W(x,p)$ is obtained from the field equations.
Advantage: the kinetic equation is equivalent to the field equations.
Disadvantage: the initial Wigner function can not be obtained from the initial fields.

3. Equal-time Wigner operator

$$\widehat{W}(x,\vec{p}) = \int d^3 \vec{y} e^{-i\vec{p}\cdot\vec{y}} \widehat{\psi}(t, \vec{x} + \frac{\vec{y}}{2}) e^{-iQ\int_{-1/2}^{1/2} ds \widehat{\vec{A}}(\vec{x}+s\vec{y})\cdot\vec{y}} \widehat{\psi}^{+}(t, \vec{x} - \frac{\vec{y}}{2})$$

Advantage: a well defined initial problem
Question: how to derive a complete kinetic equation for $W(x,\vec{p})$?

Equal-time Kinetic Theory

Covariant kinetic theory for QED:

D.Vasak, M.Gyulassy and H.Elze, Ann.Phys.173, 462(1987)

recent application to CME: J.Chen, S.Pu, Q.Wang, X.Wang, PRL110,262301(2013)

Equal-time transport for QED:

I.Bialynicki-Birula, P.Gornicki and J.Rafelski, Phys.Rev.D44, 1825(1991)

recent application to CME: A.Huang, Y.Jiang, S.Shi, J.Liao, P.Zhuang, arXiV:1703.08856

A Complete Equal-time kinetic theory (transport + constraint):

P.Zhuang and U.Heinz, Ann.Phys.245, 311(1996)

P.Zhuang and U.Heinz, Phys.Rev.D53, 2096(1996)

P.Zhuang and U.Heinz, Phys.Rev.D57, 6525(1998)

Steps to derive a complete equal-time kinetic theory:

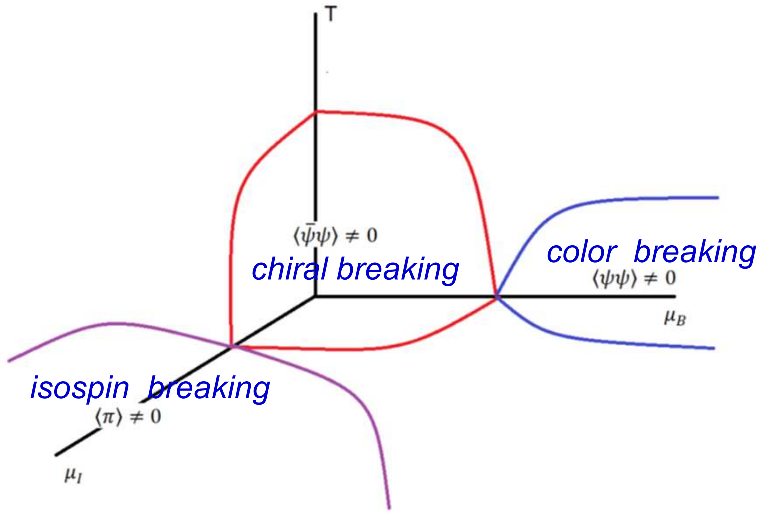
1) *Covariant kinetic equation for* $W(x,p)$, *obtained from field equations,*

2) $$W(x,\vec{p}) = \int dp_0 W(x,p)\gamma_0$$

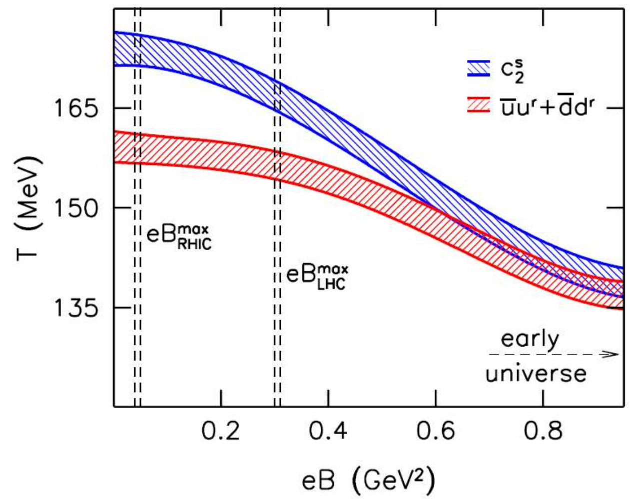
p_0*-integrating the covariant equation for* $W(x,p) \rightarrow$
a complete equal-time theory for $W(x,\vec{p})$*: transport and constraint (off-shell) equations for* $W(x,\vec{p})$*, the latter is often missed in many calculations.*

Spontaneous QCD Symmetry Breaking

QCD symmetry at finite T and μ:

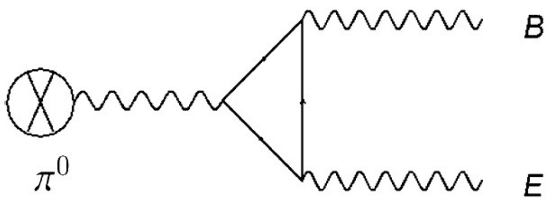


QCD symmetry in E and B fields:



IMC for chiral phase transition

$U_A(1)$ symmetry breaking due to electromagnetic triangle anomaly:



$\langle\pi\rangle=\langle\bar{\psi}i\gamma_5\tau_3\psi\rangle \neq 0$

Cao and Huang, Phys. Lett. B757, 1(2016)

What we want to do here: a kinetic description of chiral and $U_A(1)$ symmetry in equal-time transport frame.

Covariant Kinetic Equations in NJL

NJL model describes well the spontaneous symmetry breaking of QCD, since it is motivated from the successful BCS theory. The SU(2) NJL model:

$$\mathcal{L} = \bar{\psi}\left(i\gamma^\mu \mathcal{D}_\mu - m_0\right)\psi + G[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}_3\psi)^2]$$

$$\mathcal{D}_\mu = \partial_\mu + i(q + \Delta q\tau_3)A_\mu, \quad q = (q_u + q_d)/2, \quad \Delta q = (q_u - q_d)/2$$

2 order parameters for chiral and $U_A(1)$ symmetry breaking

$$\sigma(x) = 2G\langle\bar{\psi}\psi\rangle, \qquad \pi(x) = 2G\langle\bar{\psi}i\gamma_5\tau_3\psi\rangle$$

in mean field approximation

$$\mathcal{L}_{MF} = \bar{\psi}\left(i\gamma^\mu \mathcal{D}_\mu - (m_0 - \sigma) + i\gamma_5\tau_3\pi\right)\psi - \frac{1}{4G}[\sigma^2 + \pi^2]$$

Dirac equation

$$\left(i\gamma^\mu \mathcal{D}_\mu - (m_0 - \sigma) + i\gamma_5\tau_3\pi\right)\psi = 0$$

Kinetic equation for the Wigner function in external EM fields

$$W(x,p) = \int d^4y e^{ip\cdot y} e^{i(q+\Delta q\tau_3)\int_{-1/2}^{1/2} dsA(x+sy)\cdot y}\left\langle\psi(x+\frac{y}{2})\bar{\psi}(x-\frac{y}{2})\right\rangle$$

$$\left(\gamma^\mu D_\mu + \gamma^5\Pi_5 + \Sigma\right)W = 0 \quad (real\ and\ imaginary\ parts)$$

with electromagnetic operator D_μ, scalar operator Σ, and pseudoscalar operator Π_5.

Spin and Isospin Decomposition

Spin decomposition for $W(x,p)$

$$W(x,p) \;=\; \frac{1}{4}(F + i\gamma_5 P + \gamma_\mu V^\mu + \gamma_\mu\gamma_5 A^\mu + \frac{1}{2}\sigma_{\mu\nu}S^{\mu\nu})$$

→ 16 transport + 16 constraint (off-shell) equations.

Spin decomposition for $W(x,\vec{p})$

$$W(x,\vec{p}) = \int dp_0 W(x,p)\gamma_0 = \frac{1}{4}\left[f_0 + \gamma_5 f_1 - i\gamma_0\gamma_5 f_2 + \gamma_0 f_3 + \gamma_5\gamma_0\vec{\gamma}\cdot\vec{g}_0 + \gamma_0\vec{\gamma}\cdot\vec{g}_1 - i\vec{\gamma}\cdot\vec{g}_2 - \gamma_5\vec{\gamma}\cdot\vec{g}_3\right]$$

Isospin decomposition

$$f_i = \frac{1}{2}\left[f_{i0} + \tau_1 f_{i1} + \tau_2 f_{i2} + \tau_3 f_{i3}\right],$$

$$\vec{g}_i = \frac{1}{2}\left[\vec{g}_{i0} + \tau_1\vec{g}_{i1} + \tau_2\vec{g}_{i2} + \tau_3\vec{g}_{i3}\right]$$

Condensates in terms of the spin and isospin components

$$\sigma(x) = 2G\int\frac{d^3\vec{p}}{(2\pi)^3} f_{30}(x,\vec{p}), \qquad \pi(x) = 2G\int\frac{d^3\vec{p}}{(2\pi)^3} f_{23}(x,\vec{p})$$

Transport and Constraint Equations

p_0-integrating the equations for the covariant components gives a group of transport equations and a group of constraint equations.

For constant EM fields,

$$
\begin{aligned}
\hbar(\mathrm{D}_{1t}f_{0\pm}+\vec{\mathrm{D}}_1\cdot\vec{g}_{1\pm}+\mathrm{D}_{2t}f_{0\mp}+\vec{\mathrm{D}}_2\cdot\vec{g}_{1\mp}) &= -2\sigma_o f_{3\pm}-2\pi_o f_{2\mp}\\
\hbar(\mathrm{D}_{1t}f_{1\pm}+\vec{\mathrm{D}}_1\cdot\vec{g}_{0\pm}+\mathrm{D}_{2t}f_{1\mp}+\vec{\mathrm{D}}_2\cdot\vec{g}_{0\mp}) &= -2\sigma_e f_{2\pm}+2\pi_e f_{3\mp}\\
\hbar\mathrm{D}_{1t}f_{2\pm}+\hbar\mathrm{D}_{2t}f_{2\mp}+2\vec{p}\cdot\vec{g}_{3\pm} &= 2\sigma_e f_{1\pm}+2\pi_o f_{0\mp}\\
\hbar\mathrm{D}_{1t}f_{3\pm}+\hbar\mathrm{D}_{2t}f_{3\mp}-2\vec{p}\cdot\vec{g}_{2\pm} &= -2\sigma_o f_{0\pm}-2\pi_e f_{1\mp}\\
\hbar(\mathrm{D}_{1t}\vec{g}_{0\pm}+\vec{\mathrm{D}}_1 f_{1\pm}+\mathrm{D}_{2t}\vec{g}_{0\mp}+\vec{\mathrm{D}}_2 f_{1\mp})-2\vec{p}\times\vec{g}_{1\pm} &= -2\sigma_o\vec{g}_{3\pm}+2\pi_o\vec{g}_{2\mp}\\
\hbar(\mathrm{D}_{1t}\vec{g}_{1\pm}+\vec{\mathrm{D}}_1 f_{0\pm}+\mathrm{D}_{2t}\vec{g}_{1\mp}+\vec{\mathrm{D}}_2 f_{0\mp})-2\vec{p}\times\vec{g}_{0\pm} &= -2\sigma_e\vec{g}_{2\pm}+2\pi_e\vec{g}_{3\mp}\\
\hbar(\mathrm{D}_{1t}\vec{g}_{2\pm}+\vec{\mathrm{D}}_1\times\vec{g}_{3\pm}+\mathrm{D}_{2t}\vec{g}_{2\mp}+\vec{\mathrm{D}}_2\times\vec{g}_{3\mp})+2\vec{p}f_{3\pm} &= 2\sigma_e\vec{g}_{1\pm}+2\pi_o\vec{g}_{0\mp}\\
\hbar(\mathrm{D}_{1t}\vec{g}_{3\pm}+\vec{\mathrm{D}}_1\times\vec{g}_{2\pm}+\mathrm{D}_{2t}\vec{g}_{3\mp}+\vec{\mathrm{D}}_2\times\vec{g}_{2\mp})-2\vec{p}f_{2\pm} &= -2\sigma_o\vec{g}_{0\pm}-2\pi_e\vec{g}_{1\mp}
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{D}_t &= \partial_t+(\bar{q}+\Delta q\tau_3)\vec{E}\cdot\vec{\nabla}_p\\
\vec{\mathrm{D}} &= \vec{\nabla}+(\bar{q}+\Delta q\tau_3)\vec{B}\times\vec{\nabla}_p\\
\sigma_e &= m_0-\cos(\frac{\hbar}{2}\vec{\nabla}\cdot\vec{\nabla}_p)\sigma(x)\\
\sigma_o &= \sin(\frac{\hbar}{2}\vec{\nabla}\cdot\vec{\nabla}_p)\sigma(x)\\
\pi_e &= \cos(\frac{\hbar}{2}\vec{\nabla}\cdot\vec{\nabla}_p)\pi(x)\\
\pi_o &= \sin(\frac{\hbar}{2}\vec{\nabla}\cdot\vec{\nabla}_p)\pi(x)
\end{aligned}
$$

$$
\begin{aligned}
\int dp_0p_0F_\pm-\frac{\hbar}{2}(\vec{\mathrm{D}}_1\cdot\vec{g}_{2\pm}+\vec{\mathrm{D}}_2\cdot\vec{g}_{2\mp}) &= \pi_o f_{1\mp}+\sigma_e f_{0\pm}\\
\int dp_0p_0P_\pm-\frac{\hbar}{2}(\vec{\mathrm{D}}_1\cdot\vec{g}_{3\pm}+\vec{\mathrm{D}}_2\cdot\vec{g}_{3\mp}) &= \pi_e f_{0\mp}+\sigma_o f_{1\pm}\\
\int dp_0p_0V_{0\pm}-\vec{p}\cdot\vec{g}_{1\pm} &= \pi_e f_{2\mp}+\sigma_e f_{3\pm}\\
\int dp_0p_0A_{0\pm}+\vec{p}\cdot\vec{g}_{0\pm} &= \pi_o f_{3\mp}+\sigma_o f_{2\pm}\\
\int dp_0p_0\vec{V}_\pm-\frac{\hbar}{2}(\vec{\mathrm{D}}_1\times\vec{g}_{0\pm}+\vec{\mathrm{D}}_2\times\vec{g}_{0\mp})-\vec{p}f_{0\pm} &= -\pi_o\vec{g}_{3\mp}-\sigma_o\vec{g}_{2\pm}\\
\int dp_0p_0\vec{A}_\pm+\frac{\hbar}{2}(\vec{\mathrm{D}}_1\times\vec{g}_{1\pm}+\vec{\mathrm{D}}_2\times\vec{g}_{1\mp})+\vec{p}f_{1\pm} &= -\pi_e\vec{g}_{2\mp}-\sigma_e\vec{g}_{3\pm}\\
\int dp_0p_0S^{0i}_\pm\vec{e}_i-\frac{\hbar}{2}(\vec{\mathrm{D}}_1 f_{3\pm}+\vec{\mathrm{D}}_2 f_{3\mp})+\vec{p}\times\vec{g}_{3\pm} &= -\pi_e\vec{g}_{0\mp}-\sigma_o\vec{g}_{1\pm}\\
\int dp_0p_0S_{jk\pm}\epsilon^{ijk}\vec{e}_i-\hbar(\vec{\mathrm{D}}_1 f_{2\pm}+\vec{\mathrm{D}}_2 f_{2\mp})+2\vec{p}\times\vec{g}_{2\pm} &= 2\pi_o\vec{g}_{1\mp}+2\sigma_e\vec{g}_{0\pm}
\end{aligned}
$$

explicit $\hbar$ dependence !

Classical (Mean Field) Limit

$\hbar \to 0$ *in transport equations*

$$f_{1\pm} = \frac{\vec{p}\cdot\vec{g}_{3\pm}}{m_0-\sigma},\quad f_{2\pm} = \frac{\pi}{m_0-\sigma} f_{3\mp},\quad \vec{g}_{1\pm} = \frac{\vec{p}}{m_0-\sigma} f_{3\pm},\quad \vec{g}_{2\pm} = \frac{\vec{p}\times\vec{g}_{0\pm}+\pi\vec{g}_{3\mp}}{m_0-\sigma}$$

$f_0, f_3, \vec{g}_0, \vec{g}_3$ are independent components

$\hbar \to 0$ *in both transport and constraint equations*

$$W(x,p) = W(x,\vec{p})\delta(p_0 - E_p)$$

$$f_{1\pm} = \frac{\vec{p}\cdot\vec{g}_{0\pm}}{E_p},\quad f_{2\pm} = \frac{\pi}{E_p} f_{0\mp},\quad f_{3\pm} = \frac{m_0-\sigma}{E_p} f_{0\pm},\quad \vec{g}_{1\pm} = \frac{\vec{p}}{E_p} f_{0\pm},$$

$$\vec{g}_{2\pm} = \frac{\vec{p}\times\vec{g}_{0\pm}+\pi\vec{g}_{3\mp}}{m_0-\sigma},\quad \vec{g}_{3\pm} = \frac{E_p^2(m_0-\sigma)\vec{g}_{0\pm} - (m_0-\sigma)\left(\vec{p}\cdot\vec{g}_{0\pm}\right)\vec{p} - E_p\pi\vec{p}\times\vec{g}_{0\pm}}{E_p m^2},$$

$$m^2 = \left(m_0-\sigma\right)^2 + \pi^2,\quad E_p^2 = m^2 + \vec{p}^{\,2}\quad \text{(automatic on-shell condition !)}$$

only $f_0, \vec{g}_0$ are independent components

Physics of the classical components (from conservation rules):

f_0 : charge (number) density, $\quad \vec{g}_1 = \vec{v} f_0$: charge (number) current ,

$\vec{g}_0$: spin density $(\vec{J} = \vec{r}\times\vec{p} f_0 + \frac{\hbar}{2}\vec{g}_0)$, $\quad f_1 = \vec{v}\cdot\vec{g}_0$: helicity density

f_3 : mass density $(\sigma \sim \int d^3\vec{p} f_3)$, $\quad f_2$: pion condensate $(\pi \sim \int d^3\vec{p} f_2)$,

$\vec{g}_3$: magnetic moment density, $\quad \vec{g}_2$:

Classical Condensates

Momentum-integrating the components f_3 and f_2 , gap equations for the order parameters $\sigma(x)$ and $\pi(x)$,

$$(m_0-\sigma)\left(1+2G\int\frac{d^3\vec{p}}{(2\pi)^3}\frac{f_{0+}(x,\vec{p})}{E_p}\right)-m_0=0$$

$$\pi\left(1+2G\int\frac{d^3\vec{p}}{(2\pi)^3}\frac{f_{0+}(x,\vec{p})}{E_p}\right)=0$$

1) For $m_0 \neq 0$,

$\pi \equiv 0$, no $U_A(1)$ breaking in classical (mean field) limit,

$$(m_0-\sigma)\left(1+2G\int\frac{d^3\vec{p}}{(2\pi)^3}\frac{f_{0+}(x,\vec{p})}{E_p}\right)-m_0=0 \quad \rightarrow \quad \sigma$$

2) For $m_0=0$, σ and π cannot be separately determined,

$$1+2G\int\frac{d^3\vec{p}}{(2\pi)^3}\frac{f_{0+}(x,\vec{p})}{E_p}=0 \quad \rightarrow \quad m^2=(m_0-\sigma)^2+\pi^2$$

Independent Charge and Spin Density

The classical transport equations are from the transport equations to the 1st order in ℏ:

$$D_t f_{0\pm} + \vec{D}\cdot\vec{g}_{1\pm} = \left(\vec{\nabla} m\cdot\vec{\nabla}_p\right) f_{3\pm}$$

$$D_t f_{1\pm} + \vec{D}\cdot\vec{g}_{0\pm} = -2m f_{2\pm}^{(1)} + 2\pi^{(1)} f_{3\mp}$$

$$D_t f_{2\pm} + 2\vec{p}\cdot\vec{g}_{3\pm}^{(1)} = 2m f_{1\pm}^{(1)} - 2\sigma^{(1)} f_{1\pm}$$

$$D_t f_{3\pm} - 2\vec{p}\cdot\vec{g}_{2\pm}^{(1)} = \left(\vec{\nabla} m\cdot\vec{\nabla}_p\right) f_{0\pm} - 2\pi^{(1)} f_{1\mp}$$

$$D_t \vec{g}_{0\pm} + \vec{D} f_{1\pm} - 2\vec{p}\times\vec{g}_{1\pm}^{(1)} = \left(\vec{\nabla} m\cdot\vec{\nabla}_p\right)\vec{g}_{3\pm}$$

$$D_t \vec{g}_{1\pm} + \vec{D} f_{0\pm} - 2\vec{p}\times\vec{g}_{0\pm}^{(1)} = -2m\vec{g}_{2\pm}^{(1)} + 2\sigma^{(1)}\vec{g}_{2\pm} + 2\pi^{(1)}\vec{g}_{3\mp}$$

$$D_t \vec{g}_{2\pm} + \vec{D}\times\vec{g}_{3\div} + 2\vec{p} f_{3\pm}^{(1)} = 2m\vec{g}_{1\pm}^{(1)} - 2\sigma^{(1)}\vec{g}_{1\pm}$$

$$D_t \vec{g}_{3\pm} - \vec{D}\times\vec{g}_{2\pm} - 2\vec{p} f_{2\pm}^{(1)} = \left(\vec{\nabla} m\cdot\vec{\nabla}_p\right)\vec{g}_{0\pm} - 2\pi^{(1)}\vec{g}_{1\mp}$$

From the 1st, 2nd and 8th transport equations and considering the relations among the classical components,

$$\left(D_t + \vec{v}\cdot\vec{D}\right) f_{0\pm} - \frac{\vec{\nabla} m^2\cdot\vec{\nabla}_p}{2E_p} f_{0\pm} = 0, \qquad \vec{v} = \frac{\vec{p}}{E_p}$$

$$\left(D_t + \vec{v}\cdot\vec{D}\right)\vec{g}_{0\pm} - \frac{\vec{\nabla} m^2\cdot\vec{\nabla}_p}{2E_p}\vec{g}_{0\pm} = \frac{e}{E_p^2}\left[\vec{p}\times\left(\vec{E}\times\vec{g}_{0\pm}\right) - E_p\vec{B}\times\vec{g}_{0\pm}\right]$$

$$-\frac{1}{2E_p^4}\left[\partial_t m^2\vec{p} - E_p\vec{\nabla} m^2\right]\times\left(\vec{p}\times\vec{g}_0\right)$$

charge density is conserved, but spin density not, it changes in EM fields.

Homogeneous Solution

2 Transport equations in homogeneous case

$$\left(\vec{E}+\vec{v}\times\vec{B}\right)\cdot\vec{\nabla}_p f_0 = 0, \qquad \vec{v}=\frac{\vec{p}}{E_p}$$

$$\left(\vec{E}+\vec{v}\times\vec{B}\right)\cdot\vec{\nabla}_p \vec{g}_0 = \frac{e}{E_p^2}\left[\vec{p}\times\left(\vec{E}\times\vec{g}_0\right)-E_p\vec{B}\times\vec{g}_0\right]$$

with solution

$$\vec{g}_0 = \frac{q_i}{m^2}\left[\vec{v}\times\vec{E}+\vec{B}\right]$$

$$f_0 = \begin{cases} \dfrac{q_i}{m^6}\left(\vec{B}\times\vec{p}\right)^2 & \text{for } \vec{E}\parallel\vec{B} \\ \dfrac{q_i}{m^3}\vec{B}\cdot\vec{p} & \text{for } \vec{E}\perp\vec{B} \end{cases}$$

Quantum Correction $\pi^{(1)}$

Momentum-integrating the 2nd transport equation

$$D_t f_{1-} + \vec{D}\cdot\vec{g}_{0-} = -2mf_{2-}^{(1)} + 2\pi^{(1)} f_{3+}$$

and considering the relations among the classical components, the first order quantum correction to the pion condensate

$$\pi^{(1)}(x) = \frac{G}{m_0}\int\frac{d^3\vec{p}}{(2\pi)^3}\left[D_t f_{1-} + \vec{D}\cdot\vec{g}_{0-}\right]$$

In homogeneous case,

$$\pi^{(1)}(x,\vec{p}) = -\frac{G}{2m_0}\left(q_u^2 - q_d^2\right)\left(\frac{\left(\vec{p}\cdot\vec{E}\right)\left(\vec{p}\cdot\vec{B}\right)}{m^2E_p^3} + \frac{1}{E_p}\left(\frac{2}{m^2}+\frac{1}{E_p^2}\right)\vec{E}\cdot\vec{B}\right)$$

$$\pi^{(1)}(x) = \int\frac{d^3\vec{p}}{(2\pi)^3}\ \pi^{(1)}(x,\vec{p})$$

$$= -\frac{G\vec{E}\cdot\vec{B}}{8m_0\pi^2}(q_u^2-q_d^2)\left[\frac{\Lambda}{\sqrt{\Lambda^2+m^2}}\left(1+\frac{7\Lambda^2}{3m^2}\right)+\ln\frac{m}{\Lambda+\sqrt{\Lambda^2+m^2}}\right]$$

$$\sim\left(q_u^2-q_d^2\right)\vec{E}\cdot\vec{B}$$

- *$U_A(1)$ symmetry is broken due to quark spin interaction in EM field,*
- *only in the case $\vec{E}\cdot\vec{B}\neq 0$.*

Full Solution

Solving the full transport equations, including off-shell effect.

Considering heavy ion collisions, where the EM fields exist only in the very initial stage and can be neglected in the later evolution. Taking into account only time dependence,

$$\begin{aligned}
&\partial_t f_0 = 0\\
&\partial_t f_1 + 2(m_0-\sigma) f_2 - 2\pi f_3 = 0\\
&\partial_t f_2 + 2\vec{p}\cdot\vec{g}_3 - 2(m_0-\sigma) f_1 = 0\\
&\partial_t f_3 - 2\vec{p}\cdot\vec{g}_2 + 2\pi f_1 = 0\\
&\partial_t \vec{g}_0 - 2\vec{p}\times\vec{g}_1 = 0\\
&\partial_t \vec{g}_1 - 2\vec{p}\times\vec{g}_0 + 2(m_0-\sigma)\vec{g}_2 - 2\pi\vec{g}_3 = 0\\
&\partial_t \vec{g}_2 + 2\vec{p} f_3 - 2(m_0-\sigma)\vec{g}_1 = 0\\
&\partial_t \vec{g}_3 - 2\vec{p} f_2 + 2\pi\vec{g}_1 = 0
\end{aligned}$$

the only independent vector $\vec{p}$

$$\vec{g}_i = \frac{\vec{p}}{|\vec{p}|} g_i$$

→

$$\begin{aligned}
&\partial_t f_0 = 0\\
&\partial_t f_1 + 2(m_0-\sigma) f_2 - 2\pi f_3 = 0\\
&\partial_t f_2 + 2p g_3 - 2(m_0-\sigma) f_1 = 0\\
&\partial_t f_3 - 2p g_2 + 2\pi f_1 = 0\\
&\partial_t g_0 = 0\\
&\partial_t g_1 + 2(m_0-\sigma) g_2 - 2\pi g_3 = 0\\
&\partial_t g_2 + 2p f_3 - 2(m_0-\sigma) g_1 = 0\\
&\partial_t g_3 - 2p f_2 + 2\pi g_1 = 0
\end{aligned}$$

+ condensates

$$\sigma(t) = \frac{G}{\pi^2}\int dp\, p^2 f_3(t,p), \qquad \pi(t) = \frac{G}{\pi^2}\int dp\, p^2 f_2(t,p)$$

starting with the initial condition with EM fields

$$f_0(t_0) = \frac{q}{m^3} pB(t_0), \qquad g_0(t_0) = \frac{q}{m^2} B(t_0),$$

$$f_1(t_0) = \frac{p}{E_p} g_0(t_0), \qquad f_2(t_0) = \frac{\pi(t_0)}{E_p} f_0(t_0), \qquad f_3(t_0) = \frac{m_0-\sigma(t_0)}{E_p} f_0(t_0),$$

$$g_1(t_0) = \frac{p}{E_p} f_0(t_0), \qquad g_2(t_0) = \frac{\pi(t_0)}{m_0-\sigma(t_0)} g_3(t_0), \qquad g_3(t_0) = \frac{m_0-\sigma(t_0)}{E_p} g_0(t_0)$$

- *We want to see how long the QCD chirality effect can survive.*
- *Waiting for the numerical result.*

Summary and Outlook

1) *Quark transport is controlled by 16 components in general case, due to the off-shell effect. Even in classical limit, it is controlled by the charge density and spin density.*

2) *QCD chirality is closely related to the quark spin interaction with the external electromagnetic fields. We considered here the $U_A(1)$ symmetry breaking as an example. Other calculations, for instance CME, CVE and pair production, are under progress.*

3) *Equal-time kinetic theory for QCD chirality can be solved as an initial problem and then applied to high energy nuclear collisions at RHIC and LHC.*