Squeezed BBC of $\phi\phi$ in Au+Au and d+Au at RHIC energies

Wei-Ning Zhang

Co-author: Yong Zhang

(arXiv:1611.05770v3)
Contents:

- Motivations
- Bases of squeezed BBC
- Method & results
- Summary & conclusion
1. Motivations

- In HEHIC QGP environment, $\phi$ meson can produce readily, bypassing the OZI rules [5].

- Small interaction expected between $\phi$ meson and hadronic medium makes it as a sensitive probe of the QGP properties [10-12, 14-20].

- The results of $\phi$ elliptic flow in the HIC at the RHIC indicate that the flow reflects dominantly the anisotropy of the QGP and the hadronic scattering effect is unimportant [10-12, 15-20].
1. Motivations

- However, φ meson is also argued to be with a larger hadronic-interaction cross section than the estimations by current theories, based on the recent measurements of the elliptic flow of identified hadrons in the Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ GeV at the LHC [21].

- It is still an open issue to determine the interaction between φ and the hadronic medium.
1. Motivations

- In the particle-emitting sources in HEHIC, the interaction between particle and source medium leads to a modification of the particle mass and a squeezed boson-antiboson correlation, known as back-to-back correlation (BBC) [22–24].

- The measurements of the BBC of $\phi\phi$ may give knowledge of the interaction between $\phi$ meson and source medium, and provide a new way to probe the thermal and evolve properties of the hadronic sources [22–26].
2. Bases of squeezed BBC

Two-particle correlation function:

\[
C_2(k_1, k_2) = \frac{N_2(k_1, k_2)}{N_1(k_1)N_1(k_2)}
\]

where

\[
N_1(k_1) = \omega_{k_1} \frac{d^3 N}{dk_1} = \omega_{k_1} \langle a_{k_1}^{\dagger} a_{k_1} \rangle
\]

\[
N_2(k_1, k_2) = \omega_{k_1} \omega_{k_2} \langle a_{k_1}^{\dagger} a_{k_2}^{\dagger} a_{k_2} a_{k_1} \rangle
\]

\[
= \omega_{k_1} \omega_{k_2} \left[ \langle a_{k_1}^{\dagger} a_{k_1} \rangle \langle a_{k_2}^{\dagger} a_{k_2} \rangle + \langle a_{k_1}^{\dagger} a_{k_2} \rangle \langle a_{k_2}^{\dagger} a_{k_1} \rangle \right]
\]

Wick Theorem

Noninterference term

Interference Term

HBT-Correlation

Squeezed Correlation
2. Bases of squeezed BBC


**Emission time factor:**

$$F(\tau) = \frac{\theta(\tau - \tau_0)}{\Delta t} e^{-(\tau - \tau_0)/\Delta t}$$

The solid, dashed, and dotted lines stand for $|k_1|=0, 300, \text{ and } 500 \text{ MeV}$. 

![Graph showing $C_2(k-k)$ vs $m_\phi$ (GeV)]
2. Bases of squeezed BBC


2. Bases of squeezed BBC

hadronic gas

Freeze out

\[ m_* \neq m \]
\[ m_*^2 = m^2 - \delta M^2 \]

Free Particle (observed)

\[ a_k \text{ and } b_k \text{ are related by Bogoliubov transformation} \]

\[ a_k = c_k b_k + s^*_{-k} b^+_k, \quad c_k = \cosh f_k, \quad s_k = \sinh f_k, \quad f_k = \frac{1}{2} \log(\omega_k/\Omega_k) \]

\[ \omega_k = \sqrt{k^2 + m^2}, \quad \Omega_k = \sqrt{k^2 + m_*^2}, \quad n_k = \frac{1}{\exp(\Omega_k/T) - 1}, \quad n_1(k) = |c_k|^2 n_k + |s_{-k}|^2 (n_{-k} + 1). \]

BBC Function

\[ C(k_1, k_2) = \frac{|G_s(k_1, k_2)|^2}{G_c(k_1, k_1)G_c(k_2, k_2)} \]

The BBC function \( C(k, -k) \) will be 1 if there is no mass modification. However, for a finite mass modification, \( \delta m^2 = m^2 - m_*^2, \quad f_k \sim \delta m^2/(4k^2) \) as \( |k| \to \infty \), and the BBC function will increase with increasing particle momentum [2], \( C(k, -k) \sim 1 + 1/|s_{-k}|^2 \sim 1 + k^4/(\delta m^2/4)^2. \)
3. Method & results

We use the VISH2+1 code with event-by-event MC-Glb initial condition to simulate the particle-emitting sources for the Au+Au & d+Au collisions at RHIC energies.

We extract modified mass in medium m* from experimental data.

\[
C(k_1, k_2) = 1 + \frac{|G_s(k_1, k_2)|^2}{G_c(k_1, k_1)G_c(k_2, k_2)},
\]

where \(G_c(k_1, k_2)\) and \(G_s(k_1, k_2)\) are the chaotic and squeezed amplitudes, respectively. For evolution particle-emitting sources, they can be expressed as [23–26, 29]

\[
G_c(k_1, k_2) = \int \frac{d^4 \sigma_\mu(r)}{(2\pi)^3} K_{1,2}^\mu e^{i q_1,2 \cdot r} \left\{ |c_{k_1}',k_2'|^2 n_{k_1}',k_2' \\
+ |s'_{-k_1}',-k_2'|^2 [n'_{-k_1}',-k_2'+1] \right\},
\]

\[
G_s(k_1, k_2) = \int \frac{d^4 \sigma_\mu(r)}{(2\pi)^3} K_{1,2}^\mu e^{2i q_1,2 \cdot r} \left\{ s'_{-k_1}',k_2' c'_{k_2',-k_1}' \\
\times n'_{-k_1}',k_2' + c'_{k_1}',-k_2' s'_{-k_2}',k_1' [n'_{k_1}',-k_2'+1] \right\},
\]

where \(d^4 \sigma_\mu(r)\) is the four-dimension element of freeze-out hypersurface, \(q_1,2 = k_1' - k_2'\), \(K_{1,2}^\mu = (k_1'^\mu + k_2'^\mu)/2\), and \(k_i'\) is the local-frame momentum corresponding to \(k_i\) (i =
3. Method & results

The transverse-momentum spectra of the $\phi$ meson calculated with the viscous hydrodynamic code at the freeze-out temperature $T_f=140\text{MeV}$. The experimental data measured by STAR collaboration [10] are also plotted. The calculated spectra suit the experimental data well.
3. Method & results

On the other hand, the φ mesons with larger \( p_T \) (\( v_T \)) escape the source more easily and with more possibility decaying outside of the source. (\( \phi \rightarrow K^+K^- \))

There are larger differences between the experimental and MC data in the low \( p_T \) region.

Considering the φ mesons with small \( p_T \), thus with small average transverse velocity \( v_T \), have more possibility decaying inside the source medium which with a transverse expanding velocity comparable to \( v_T \), the differences between the EXP and MC data at small \( p_T \) reflect the medium effects on the measured mass and mass-width.
3. Method & results

The measurements of the electron-positron decay of φ by the KEK-PS E325 Collaboration are consistent with our analyses.
3. Method & results

Assuming the measured mass distribution of K+K− in the experiments [10] consists of the two parts, one from the contribution of the mesons decaying inside the source medium and another from the contribution of the φ decaying outside of the source, we have the normalized density distribution of the mass as,

\[
\rho_{\text{exp}}(m; M_{\text{exp}}, \Gamma_{\text{exp}}) = \frac{\Gamma_{\text{exp}}/2\pi}{(m - M_{\text{exp}})^2 + (\Gamma_{\text{exp}}/2)^2}
\]

\[
= f(p_T)\rho_0(m; M_0, \Gamma_0) + [1 - f(p_T)]\rho_*(m; M_*, \Gamma_*)
\]
3. Method & results

<table>
<thead>
<tr>
<th>$p_T$(GeV/c)</th>
<th>$M_*$ (GeV/c$^2$)</th>
<th>$\Gamma_*$ (MeV/c$^2$)</th>
<th>$f$</th>
<th>$\chi^2$/NBF</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>$1.0157 \pm 0.0003$</td>
<td>$9.785 \pm 0.373$</td>
<td>$0.000 \pm 0.008$</td>
<td>0.04/30</td>
</tr>
<tr>
<td>Fit 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.7</td>
<td>$1.0157$(fixed)</td>
<td>$9.785$(fixed)</td>
<td>$0.481 \pm 0.031$</td>
<td>65.05/30</td>
</tr>
<tr>
<td>1.0</td>
<td>$1.0157$(fixed)</td>
<td>$9.785$(fixed)</td>
<td>$0.753 \pm 0.014$</td>
<td>68.43/30</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>$1.0148 \pm 0.0005$</td>
<td>$12.069 \pm 0.921$</td>
<td>$0.40$(fixed)</td>
<td>0.56/30</td>
</tr>
<tr>
<td>Fit 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.7</td>
<td>$1.0148$(fixed)</td>
<td>$12.069$(fixed)</td>
<td>$0.640 \pm 0.021$</td>
<td>51.98/30</td>
</tr>
<tr>
<td>1.0</td>
<td>$1.0148$(fixed)</td>
<td>$12.069$(fixed)</td>
<td>$0.829 \pm 0.009$</td>
<td>65.82/30</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>$1.0107 \pm 0.0016$</td>
<td>$20.197 \pm 5.773$</td>
<td>$0.80$(fixed)</td>
<td>5.95/30</td>
</tr>
<tr>
<td>Fit 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.7</td>
<td>$1.0107$(fixed)</td>
<td>$20.197$(fixed)</td>
<td>$0.846 \pm 0.009$</td>
<td>71.80/30</td>
</tr>
<tr>
<td>1.0</td>
<td>$1.0107$(fixed)</td>
<td>$20.197$(fixed)</td>
<td>$0.926 \pm 0.004$</td>
<td>111.79/30</td>
</tr>
</tbody>
</table>

$$\rho_{\exp}(m; M_{\exp}, \Gamma_{\exp}) = \frac{\Gamma_{\exp}/2\pi}{(m - M_{\exp})^2 + (\Gamma_{\exp}/2)^2}$$

$$= f(p_T)\rho_0(m; M_0, \Gamma_0) + [1 - f(p_T)]\rho_*(m; M_*, \Gamma_*)$$
3. Method & results

$k_{T1}$ and $k_{T2}$ are calculated in the momentum regions $0.4 < |k_T| < 0.5$ GeV/c and $0.8 < |k_T| < 0.9$ GeV/c.
We investigate the squeezed BBC of $\phi$ meson caused by the mass modification in the source medium for the Au+Au and d+Au collisions at the RHIC energies.

The BBC functions are calculated using the modified masses extracted from experimental data and the source space-time distributions provided by the viscous hydrodynamic code VISH2+1.

It is found that the BBC of $\phi$ may perhaps be observed in the collisions of d+Au and the peripheral collisions of Au+Au at the RHIC energies.

We suggest to measure the BBC experimentally for understanding the mass modifications of the $\phi$ meson in the collisions.
Thanks!
1. Motivations

… for $p_T < 2\text{GeV/c}$, the $v_2$ follows a mass-ordered hierarchy where the values of $v_2$, within errors, fall between those of the heavier $\Lambda$ (open circles) and lighter $K$ (open squares). However, at intermediate $p_T$, between 2–5GeV/c, the $v_2$ appears to follow the same trend as $K$.

1. Motivations


FIG. 2 (color online). Comparison of differential $v_2(p_T)$ for mesons, $(\bar{d})d$, $\pi^\pm$, $K^\pm$, and $(\bar{p})p$ (as indicated). Results are shown for 20%–60% central Au + Au collisions.

FIG. 3 (color online). (a) $v_2$ vs KE$_T$ for several identified particle species obtained in midcentral (20%–60%) Au + Au collisions. (b) $v_2/n_q$ vs KE$_T/n_q$ for the same particle species shown in (a). The shaded bands indicate systematic error estimates for $(\bar{d})d$ and $\phi$ mesons (see text).

KE$_T$ = m$_T$ – m
1. Motivations

[20] STAR Collaboration, PRL 116, 062301, 2016:
There is a indication of the breakdown of previous observed mass ordering between $\varphi$ and proton v2 at low transverse momentum in the 0-30% centrality range, possibly indicating late hadronic interactions affecting the proton v2.
1. Motivations

[21] ALICE Collab, JHEP06,190, 2015: The v2 values of the $\phi$-meson in figure 5 indicate that for $p_T<3$ GeV/c it follows the mass-ordered hierarchy. However, for higher $p_T$ values the $\phi$ data points appear to follow the band of baryons for central events within uncertainties.

Figure 5. The $p_T$-differential $v_2$ for different particle species grouped by centrality class of Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV.
1. Motivations

[21] ALICE Collab, JHEP06,190, 2015:
A distinct mass ordering was found for all centralities in the low transverse momentum region i.e. for pT<3 GeV/c, which is attributed to the interplay between elliptic and radial flow that modifies the v2(pT) according to particle mass.
2. Bases of BBC

1996年，M. Asakawa和T. Csorog指出当粒子在源内与介质相互作用发生质量改变时，会产生一种正反玻色子对的压缩关联，即背对背关联。

Heavy Ion Physics, 1996, 4: 233. 未考虑源发射时间分布对背对背关联的影响。


源发射时间分布为：

$$F(\tau) = \frac{\theta(\tau - \tau_0)}{\Delta t} e^{-(\tau - \tau_0)/\Delta t}$$

The solid, dashed, and dotted lines stand for $|k| = 0, 300, \text{ and } 500 \text{ MeV}$. 
2. Bases of BBC


2. Bases of BBC

强子气体媒介
准粒子（处于媒介中）

热力学冻出

自由粒子（实验探测到）

\[ m_* \neq m \]
\[ m_*^2 = m^2 - \delta M^2 \]

\[(b_{k}^+, b_{k})\]

\[ H = H_0 - \frac{1}{2} \int dxdy \phi(x) \delta M^2(x - y) \phi(y) \]
\[ H = \int \Omega_k b_k^+ b_k d^3k \]
\[ \Omega_k = \sqrt{m_*^2 + k^2} \]

\[(a_{k}^+, a_{k})\]

\[ H_0 = \frac{1}{2} \int dx (\dot{\phi}^2 + |\nabla \phi|^2 + m_0^2 \phi^2) \]
\[ H_0 = \int \omega_k a_k^+ a_k d^3k \]
\[ \omega_k = \sqrt{m^2 + k^2} \]

\[ a_k = c_k b_k + s_{-k}^* b_{-k}^+ , \quad c_k = \cosh f_k , \quad s_k = \sinh f_k , \quad f_k = \frac{1}{2} \log(\omega_k/\Omega_k) \]
2. Bases of BBC

\[ N_2(k_1, k_2) = \omega_{k_1} \omega_{k_2} \langle a_{k_1}^\dagger a_{k_2}^\dagger a_{k_2} a_{k_1} \rangle \]

\[ = \omega_{k_1} \omega_{k_2} \left[ \langle a_{k_1}^\dagger a_{k_1} \rangle \langle a_{k_2}^\dagger a_{k_2} \rangle + \langle a_{k_1}^\dagger a_{k_2} \rangle \langle a_{k_2}^\dagger a_{k_1} \rangle \right. \]

\[ \left. + \langle a_{k_1}^\dagger a_{k_2}^\dagger \rangle \langle a_{k_2} a_{k_1} \rangle \right] , \]

\[ a_k = c_k b_k + s_{-k}^* b_{-k}^\dagger \]

\[ \langle a_k a_{-k} \rangle = \langle c_k^2 b_k b_{-k} + c_k s_k b_k b_k^+ + c_k s_k b_{-k}^+ b_{-k} + s_k^2 b_{-k}^+ b_{-k}^+ \rangle \]

\[ = \langle c_k s_k (1 + b_k^+ b_k) + c_k s_k b_{-k}^+ b_{-k} \rangle \]

\[ = 2c_k s_k n_k + c_k s_k \]

\[ n_k = \langle b_k^+ b_k \rangle \]
2. Bases of BBC

混沌振幅：\[ G_c(k_1, k_2) = \sqrt{\omega_{k_1} \omega_{k_2}} \langle a_{k_1}^{\dagger} a_{k_2} \rangle, \]

压缩振幅：\[ G_s(k_1, k_2) = \sqrt{\omega_{k_1} \omega_{k_2}} \langle a_{k_1} a_{k_2} \rangle, \]

背对背关联函数：\[ C(k, -k) = 1 + \frac{|G_s(k, -k)|^2}{G_c(k, k)G_c(-k, -k)} \]

对于流体力学演化源，Makhlin 和 Sinyukov推导得出：

\[ G_c(k_1, k_2) = \frac{1}{(2\pi)^3} \int d^4 \sigma \mu K_{1,2}^\mu e^{i\sigma_{1,2} x} n_c(x, K_{1,2}^\mu u_\mu) \]

\[ G_s(k_1, k_2) = \frac{1}{(2\pi)^3} \int d^4 \sigma \mu K_{1,2}^\mu e^{2i\sigma_{1,2} x} n_s(x, K_{1,2}^\mu u_\mu) \]

2. Bases of BBC

\[ G_s(k_1, k_2) = \frac{1}{(2\pi)^3} \int d^4\sigma_\mu K_{1,2}^\mu e^{2iK_{1,2} \cdot x} n_s(x, K_{1,2}^\mu u_\mu) \]

空间部分:

\[ d^4\sigma_\mu = (dx dy dz, -dt dy dz, -dt dx dz, -dt dx dy) \]

采用瞬时冻出假设: (\( dt = 0 \))

\[ d^4\sigma_\mu K_{1,2}^\mu = K^0 d^3x \]

均匀源

\[ C(k, -k) = 1 + \frac{V |c_k s_k^* n_k + c_{-k} s_{-k}^* (n_{-k} + 1)|^2}{V [n_1(k) n_1(-k)]} \]

\[ |\tilde{F}(\omega_k + \omega_{-k}, \Delta t)|^2 = [1 + (\omega_k + \omega_{-k})^2 \Delta t^2]^{-1} \]


\[ C(k, -k) - 1 = \frac{|c_k s_k n_k + c_{-k} s_{-k} (n_{-k} + 1)|^2}{n_1(k) n_1(-k)} \times |\tilde{F}(\omega_k + \omega_{-k})|^2, \]
2. Bases of BBC

\[ G_s(k_1, k_2) = \frac{1}{(2\pi)^3} \int d^4 \sigma \mu K_{1,2}^{\mu} e^{2iK_{1,2} \cdot x} n_s(x, K_{1,2}^{\mu} u_{\mu}) \]

空间部分：
\[ d^4 \sigma_{\mu} = (dx dy dz, -dt dy dz, -dt dx dz, -dt dx dy) \]
采用瞬时冻出假设：（\( dt = 0 \)）
\[ d^4 \sigma_{\mu} K_{1,2}^{\mu} = K^0 d^3 x \]

采用指数衰减发射时间分布：
\[ F(\tau) = \frac{\theta(\tau - \tau_0)}{\Delta t} e^{-(\tau - \tau_0)/\Delta t} \]
\[ F(\omega) = \int dt F(t) \exp(-i\omega t) \]
\[ |\tilde{F}(\omega_k + \omega_{-k}, \Delta t)|^2 = [1 + (\omega_k + \omega_{-k})^2 \Delta t^2]^{-1} \]

3. Method & results

The source isothermals and average transverse velocities \( \langle v_\perp \rangle \) calculated with the 1000 events for the collisions of the Au+Au with 0-5% centrality and the d+Au with 0-20% centrality at \( \sqrt{s_{NN}} = 200 \text{ GeV} \).

The sources for the d+Au collisions have smaller space-time distribution and evolve faster than that for the Au+Au collisions.

The average velocity \( \langle v_\perp \rangle \) at \( r_\perp = 0 \) is small for the collisions of Au+Au because the center of the fireball for each the event is near to \( r_\perp = 0 \) for the symmetric collisions.
Previous works

流体力学演化源的正反玻色子对的背对背关联（高斯初始条件）

(a) $R_X = 4 \text{ fm}, R_Y = 4 \text{ fm}, \varepsilon_0 = 8 \text{ GeV/fm}^3$

(a') $R_X = 4 \text{ fm}, R_Y = 4 \text{ fm}, \varepsilon_0 = 8 \text{ GeV/fm}^3$

(b) $R_X = 4 \text{ fm}, R_Y = 4 \text{ fm}, \varepsilon_0 = 20 \text{ GeV/fm}^3$

(b') $R_X = 4 \text{ fm}, R_Y = 4 \text{ fm}, \varepsilon_0 = 20 \text{ GeV/fm}^3$

$$F_{1,2} = (a + b \tau^2) \theta(\tau - \tau_0) \theta(\tau_{\text{max}} - \tau)$$

- $F_1 (a = 1.0, b = 0.035, \tau_{\text{max}} = 7)$
- $F_2 (a = 0.9, b = 0.025, \tau_{\text{max}} = 10)$
- $F_3 = 5 \exp[-(\tau - \tau_0)/2] \theta(\tau - \tau_0)$

$F(\tau)$

$\tau \text{ (fm/c)}$
Previous works

流体力学演化源的正反玻色子对的背对背关联 (涨落初始条件):


\[ \epsilon(x, y) = K \sum_i \frac{p_{\perp i}}{\tau_0} \frac{1}{2\pi\sigma^2} \times \exp\left\{ -\frac{[x-x_i(\tau_0)]^2 + [y-y_i(\tau_0)]^2}{2\pi\sigma^2} \right\} \]

\[ \sqrt{s_{NN}} = 200 \text{ GeV \ Au+Au \ b = 0} \]

\[ \sqrt{s_{NN}} = 2.76 \text{ TeV \ Pb+Pb \ b = 0} \]
Previous works

背对背关联函数:

$$C'(k, -k) = \frac{1}{N_E} \sum_{i=1}^{N_E} \left[ G_{ci}(k, k)G_{ci}(-k, -k) + |G_{si}(k, -k)|^2 \right]$$

$$\frac{1}{N_E} \sum_{i=1}^{N_E} G_{ci}(k, k)G_{ci}(-k, -k)$$
Previous works

流体力学演化源的正反玻色子对的背对背关联（涨落初始条件）

流体力学演化源的正反玻色子对的背对背关联（涨落初始条件）

φ介子多事件混合后的背对背关联（2000个事件）

(a) Au+Au, 200 GeV, 0-10%
(b) Au+Au, 200 GeV, 40-50%
(c) Pb+Pb, 2.76 TeV, 0-10%
(d) Pb+Pb, 2.76 TeV, 40-50%
Previous works

流体力学演化源的正反玻色子对的背对背关联（涨落初始条件）

$\phi$介子多事件混合后的背对背关联（2000个事件）

(a) Au+Au, 200 GeV, 0-10%
(b) Au+Au, 200 GeV, 40-50%
(c) Pb+Pb, 2.76 TeV, 0-10%
(d) Pb+Pb, 2.76 TeV, 40-50%

震荡消失
Previous works

ϕϕ背对背关联对粒子动量方向的依赖

图示了不同碰撞中心的能量下，RHIC和LHC在Au+Au和Pb+Pb碰撞中的背对背关联对的依赖关系。
Previous works

Cu+Cu collision ϕ meson multi-event mixing-back-to-back correlation (2000 events)

(a) Cu+Cu, 200 GeV, b=0-4 fm
(b) Cu+Cu, 200 GeV, b=4-8 fm

流体力学演化源的正反玻色子对的背对背关联（涨落初始条件）