
Squeezed BBC of $\phi\phi$ in Au+Au and d+Au at RHIC energies

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(arXiv:1611.05770v3)

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1. Motivations

- In HEHIC QGP environment, ϕ meson can produce readily, bypassing the OZI rules [5].
- Small interaction expected between ϕ meson and hadronic medium makes it as a sensitive probe of the QGP properties [10-12,14-20].
- The results of ϕ elliptic flow in the HIC at the RHIC indicate that the flow reflects dominantly the anisotropy of the QGP and the hadronic scattering effect is unimportant [10-12,15-20].

1. Motivations

- However, φ meson is also argued to be with a larger hadronic-interaction cross section than the estimations by current theories, based on the recent measurements of the elliptic flow of identified hadrons in the Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ GeV at the LHC [21].
- It is still an open issue to determine the interaction between φ and the hadronic medium.

1. Motivations

- In the particle-emitting sources in HEHIC, the interaction between particle and source medium leads to a modification of the particle mass and a squeezed boson-antiboson correlation, known as back-to-back correlation (BBC) [22–24].
- The measurements of the BBC of $\varphi\varphi$ may give knowledge of the interaction between φ meson and source medium, and provide a new way to probe the thermal and evolve properties of the hadronic sources [22–26].

2. Bases of squeezed BBC

Two-particle correlation function:

$$C_2(\mathbf{k}_1, \mathbf{k}_2) = \frac{N_2(\mathbf{k}_1, \mathbf{k}_2)}{N_1(\mathbf{k}_1)N_1(\mathbf{k}_2)}$$

where

$$N_1(\mathbf{k}_1) = \omega_{\mathbf{k}_1} \frac{d^3 N}{d\mathbf{k}_1} = \omega_{\mathbf{k}_1} \langle a_{\mathbf{k}_1}^\dagger a_{\mathbf{k}_1} \rangle$$

$$\begin{aligned} N_2(\mathbf{k}_1, \mathbf{k}_2) &= \omega_{\mathbf{k}_1} \omega_{\mathbf{k}_2} \langle a_{\mathbf{k}_1}^\dagger a_{\mathbf{k}_2}^\dagger a_{\mathbf{k}_2} a_{\mathbf{k}_1} \rangle \\ &\stackrel{\text{Wick Theorem}}{=} \omega_{\mathbf{k}_1} \omega_{\mathbf{k}_2} \left[\underbrace{\langle a_{\mathbf{k}_1}^\dagger a_{\mathbf{k}_1} \rangle \langle a_{\mathbf{k}_2}^\dagger a_{\mathbf{k}_2} \rangle}_{\text{Noninterference term}} + \underbrace{\langle a_{\mathbf{k}_1}^\dagger a_{\mathbf{k}_2} \rangle \langle a_{\mathbf{k}_2}^\dagger a_{\mathbf{k}_1} \rangle}_{\text{Interference Term}} \right. \\ &\quad \left. + \underbrace{\langle a_{\mathbf{k}_1}^\dagger a_{\mathbf{k}_2}^\dagger \rangle \langle a_{\mathbf{k}_2} a_{\mathbf{k}_1} \rangle}_{\text{HBT-Correlation}} \right], \end{aligned}$$

**Wick
Theorem**

— **Noninterference term**

— **Interference Term**

Squeezed Correlation

HBT-Correlation



2. Bases of squeezed BBC

- 1996, M.Asakawa and T.Csorog put forward the squeezed correlation of boson-antiboson due to their mass modification in medium, back-to-back correlation.

Heavy Ion Physics, 1996, 4: 233. Uniform system, without time evolution.

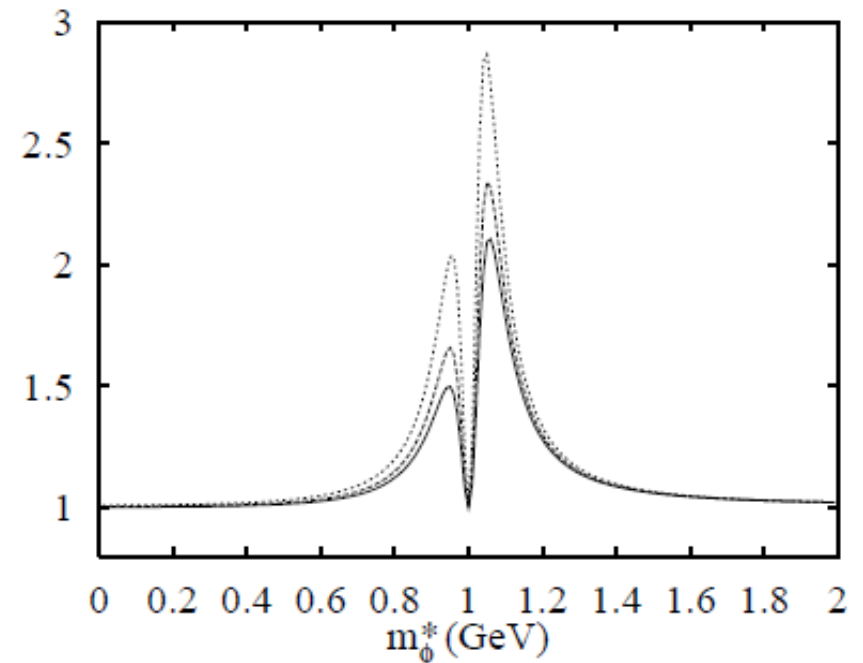
- 1999, M.Asakawa, T.Csorgo, and M.Gyulassy, back-to-back correlation (BBC) of $\phi\phi$, Phys. Rev. Lett., 1999, 83: 4013. Uniform system, with

Emission time factor:

$$F(\tau) = \frac{\theta(\tau - \tau_0)}{\Delta t} e^{-(\tau - \tau_0)/\Delta t}$$

The solid, dashed, and dotted lines stand for $|\mathbf{k}|=0, 300, \text{ and } 500 \text{ MeV}$.

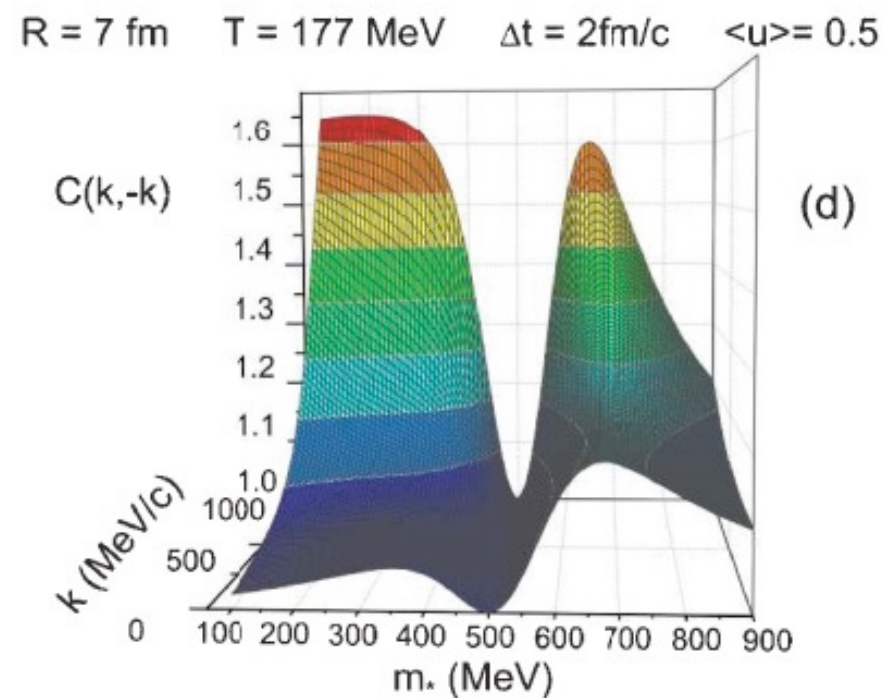
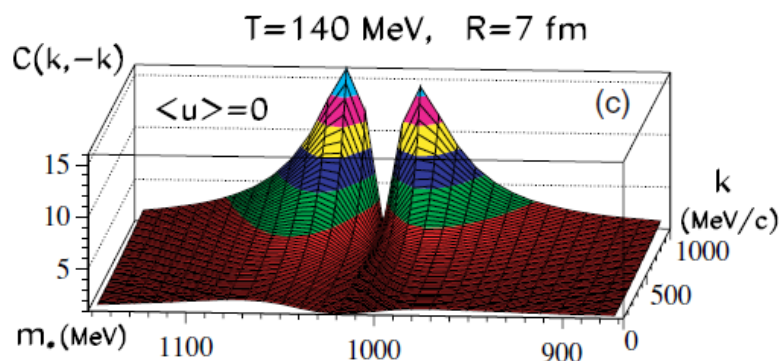
$C_2(k,-k)$



2. Bases of squeezed BBC

2006, S.Padula et al., Phys. Rev. C, 2006, 73: 044906.

2010, S.Padula et al., BBC of K^+K^- Phys. Rev. C, 2010, 82 : 034905.



2. Bases of squeezed BBC

hadronic gas

●
quasi-particle
(in medium)

Freeze out

$$m_* \neq m$$

$$m_*^2 = m^2 - \delta M^2$$

● Free Particle
(observed)

a_k and b_k are related by Bogoliubov transformation

$$a_k = c_k b_k + s_{-k}^* b_{-k}^\dagger, \quad c_k = \cosh f_k, \quad s_k = \sinh f_k, \quad f_k = \frac{1}{2} \log(\omega_k / \Omega_k)$$

$$\omega_k = \sqrt{k^2 + m^2}, \quad \Omega_k = \sqrt{k^2 + m_*^2}, \quad n_k = \frac{1}{\exp(\Omega_k/T) - 1}, \quad n_1(\mathbf{k}) = |c_k|^2 n_k + |s_{-k}|^2 (n_{-k} + 1).$$

BBC Function

$$C(\mathbf{k}_1, \mathbf{k}_2) = 1 +$$

$$\frac{|G_s(\mathbf{k}_1, \mathbf{k}_2)|^2}{G_c(\mathbf{k}_1, \mathbf{k}_1) G_c(\mathbf{k}_2, \mathbf{k}_2)}$$

$$C(\mathbf{k}, -\mathbf{k}) = 1 + \frac{V |c_k s_k^* n_k + c_{-k} s_{-k}^* (n_{-k} + 1)|^2}{V [n_1(\mathbf{k}) n_1(-\mathbf{k})]}$$

The BBC function $C(\mathbf{k}, -\mathbf{k})$ will be 1 if there is no mass modification. However, for a finite mass modification, $\delta m^2 = m^2 - m_*^2$, $f_k \sim \delta m^2 / (4k^2)$ as $|\mathbf{k}| \rightarrow \infty$, and the BBC function will increase with increasing particle momentum [2], $C(\mathbf{k}, -\mathbf{k}) \sim 1 + 1/|s_{-k}|^2 \sim 1 + k^4 / (\delta m^2 / 4)^2$.



3. Method & results

We use the VISH2+1 code with event-by-event MC-Glb initial condition to simulate the particle-emitting sources for the Au+Au & d+Au collisions at RHIC energies.

We extract modified mass in medium m^* from experimental data.

The BBC function of the two ϕ mesons with momenta \mathbf{k}_1 and \mathbf{k}_2 is defined as [23, 24]

$$C(\mathbf{k}_1, \mathbf{k}_2) = 1 + \frac{|G_s(\mathbf{k}_1, \mathbf{k}_2)|^2}{G_c(\mathbf{k}_1, \mathbf{k}_1)G_c(\mathbf{k}_2, \mathbf{k}_2)}, \quad (1)$$

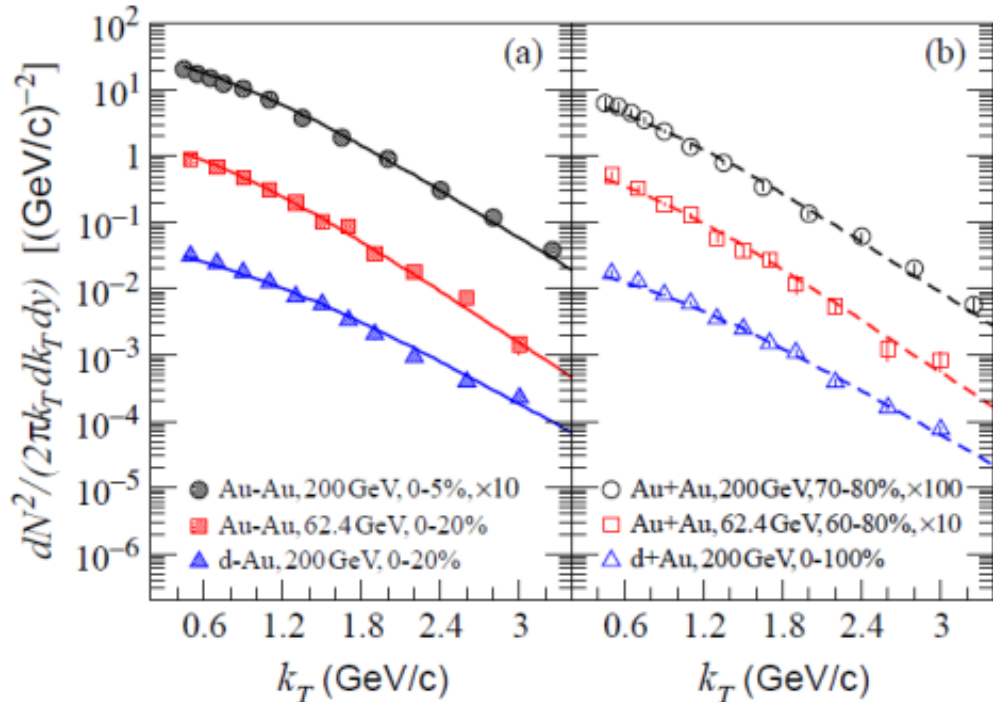
where $G_c(\mathbf{k}_1, \mathbf{k}_2)$ and $G_s(\mathbf{k}_1, \mathbf{k}_2)$ are the chaotic and squeezed amplitudes, respectively. For evolution particle-emitting sources, they can be expressed as [23–26, 29]

$$G_c(\mathbf{k}_1, \mathbf{k}_2) = \int \frac{d^4\sigma_\mu(r)}{(2\pi)^3} K_{1,2}^\mu e^{i q_{1,2} \cdot r} \left\{ |c'_{\mathbf{k}'_1, \mathbf{k}'_2}|^2 n'_{\mathbf{k}'_1, \mathbf{k}'_2} + |s'_{-\mathbf{k}'_1, -\mathbf{k}'_2}|^2 [n'_{-\mathbf{k}'_1, -\mathbf{k}'_2} + 1] \right\}, \quad (2)$$

$$G_s(\mathbf{k}_1, \mathbf{k}_2) = \int \frac{d^4\sigma_\mu(r)}{(2\pi)^3} K_{1,2}^\mu e^{2i K_{1,2} \cdot r} \left\{ s'^*_{-\mathbf{k}'_1, \mathbf{k}'_2} c'_{\mathbf{k}'_2, -\mathbf{k}'_1} \times n'_{-\mathbf{k}'_1, \mathbf{k}'_2} + c'_{\mathbf{k}'_1, -\mathbf{k}'_2} s'^*_{-\mathbf{k}'_2, \mathbf{k}'_1} [n'_{\mathbf{k}'_1, -\mathbf{k}'_2} + 1] \right\}, \quad (3)$$

where $d^4\sigma_\mu(r)$ is the four-dimension element of freeze-out hypersurface, $q_{1,2}^\mu = k_1^\mu - k_2^\mu$, $K_{1,2}^\mu = (k_1^\mu + k_2^\mu)/2$, and \mathbf{k}'_i is the local-frame momentum corresponding to \mathbf{k}_i ($i =$

3. Method & results

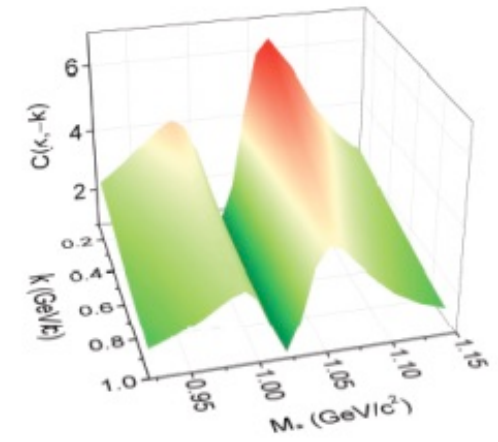
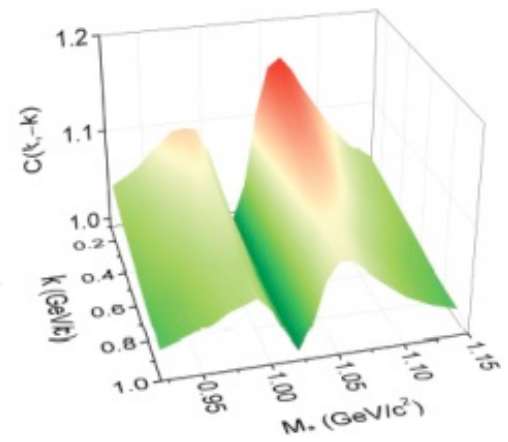


The transverse-momentum spectra of the ϕ meson calculated with the viscous hydrodynamic code at the freeze-out temperature $T_f=140\text{MeV}$. The experimental data measured by STAR collaboration [10] are also plotted. **The calculated spectra suit the experimental data well.**

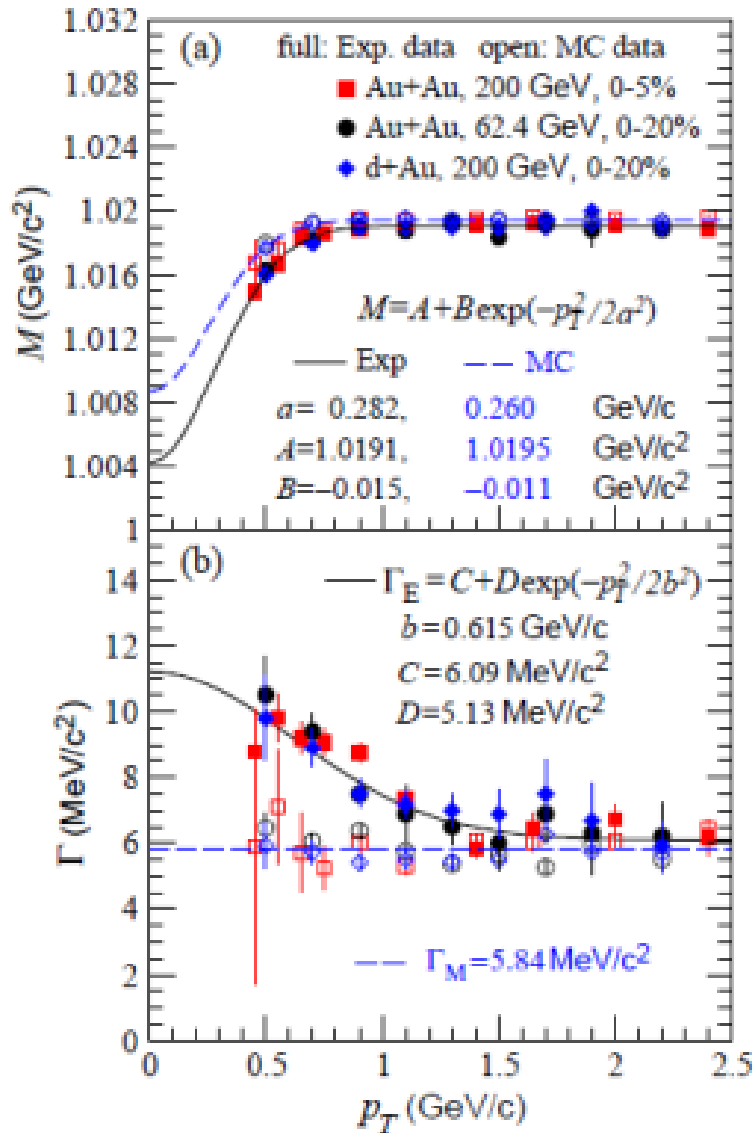
$$G_s(\mathbf{k}_1, \mathbf{k}_2) = \int \frac{d^4\sigma_\mu(r)}{(2\pi)^3} K_{1,2}^\mu e^{2iK_{1,2}\cdot r} \left\{ s_{-\mathbf{k}'_1, \mathbf{k}'_2}^{1/4} c'_{\mathbf{k}'_2, -\mathbf{k}'_1} \right. \\ \left. \times n'_{-\mathbf{k}'_1, \mathbf{k}'_2} + c'_{\mathbf{k}'_1, -\mathbf{k}'_2} s_{-\mathbf{k}'_2, \mathbf{k}'_1}^{1/4} [n'_{\mathbf{k}'_1, -\mathbf{k}'_2} + 1] \right\}.$$

(a) Au+Au, 200 GeV, 0-5%

(b) Au+Au, 200 GeV, 70-80%



3. Method & results



On the other hand, the ϕ mesons with larger p_T (v_T) escape the source more easily and with more possibility decaying outside of the source. ($\phi \rightarrow K+K^-$)

There are larger differences between the experimental and MC data in the low p_T region.

Considering the ϕ mesons with small p_T , thus with small average transverse velocity v_T , have more possibility decaying inside the source medium which with a transverse expanding velocity comparable to v_T , the differences between the EXP and MC data at small p_T reflect the medium effects on the measured mass and mass-width.



3. Method & results

PRL 98, 042501 (2007)

PHYSICAL REVIEW LETTERS

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26 JANUARY 2007

Evidence for In-Medium Modification of the ϕ Meson at Normal Nuclear Density

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Invariant mass spectra of e^+e^- pairs have been measured in 12 GeV $p + A$ reactions to detect possible in-medium modification of vector mesons. Copper and carbon targets are used to study the nuclear-size dependence of e^+e^- invariant mass distributions. A significant excess on the low-mass side of the ϕ meson peak is observed in the low $\beta\gamma (= \beta/\sqrt{1-\beta^2})$ region of ϕ mesons ($\beta\gamma < 1.25$) with copper targets. However, in the high $\beta\gamma$ region ($\beta\gamma > 1.25$), spectral shapes of ϕ mesons are well described by the Breit-Wigner shape when experimental effects are considered. Thus, in addition to our earlier publications on ρ/ω modification, this study has experimentally verified vector meson mass modification at normal nuclear density.

The measurements of the electron-positron decay of ϕ by the KEK-PS E325 Collaboration are consistent with our analyses.



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3. Method & results

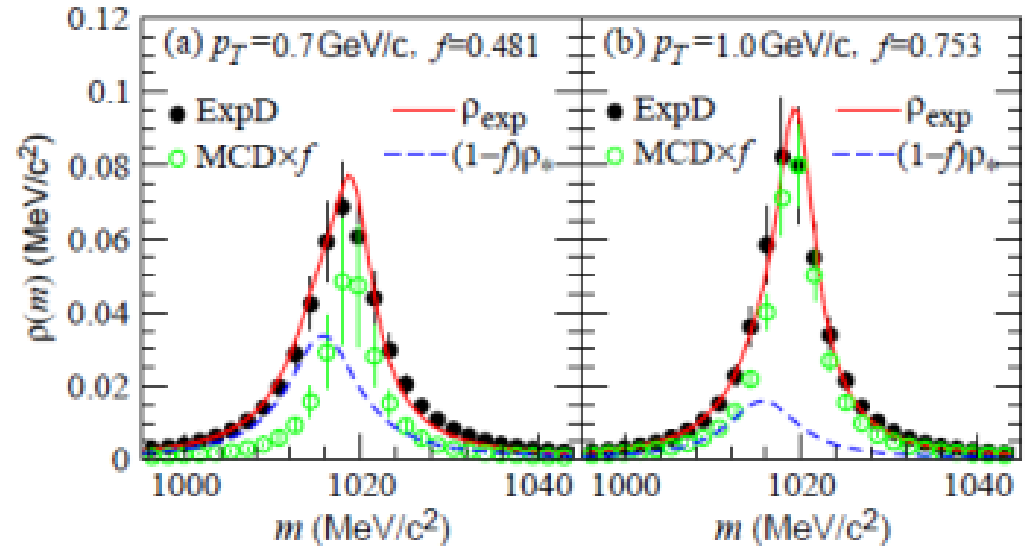
Assuming the measured mass distribution of $K+K^-$ in the experiments [10] consists of the two parts, one from the contribution of the mesons decaying inside the source medium and another from the contribution of the ϕ decaying outside of the source, we have the normalized density distribution of the mass as,

$$\rho_{\text{exp}}(m; M_{\text{exp}}, \Gamma_{\text{exp}}) = \frac{\Gamma_{\text{exp}}/2\pi}{(m - M_{\text{exp}})^2 + (\Gamma_{\text{exp}}/2)^2}$$

$$= f(p_T)\rho_0(m; M_0, \Gamma_0) + [1 - f(p_T)]\rho_*(m; M_*, \Gamma_*)$$

TABLE I: Results of fitting $\rho_{\text{exp}}(M)$ with Eq. (4).

$p_T(\text{GeV}/c)$	$m_*(\text{GeV}/c^2)$	$\Gamma_*(\text{MeV}/c^2)$	f
0.5	1.0157 ± 0.2110	9.785 ± 0.271	0.000 ± 0.008
0.7	1.0157(fixed)	9.785(fixed)	0.481 ± 0.031
1.0	1.0157(fixed)	9.785(fixed)	0.753 ± 0.014
1.4	1.0157(fixed)	9.785(fixed)	0.853 ± 0.013



3. Method & results

TABLE I: Results of fitting $\rho_{\text{exp}}(m)$ with Eq. (4).

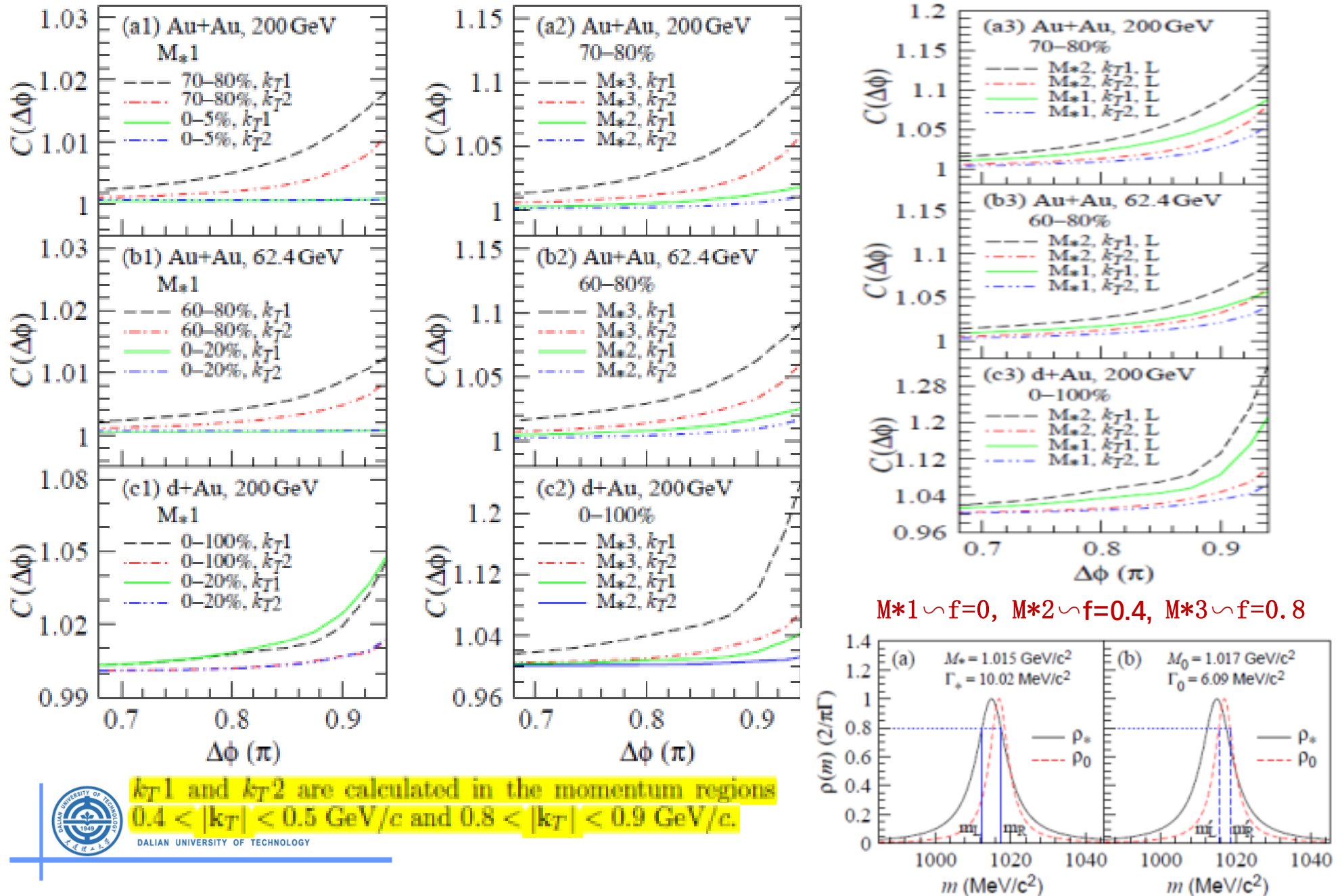
	$p_T(\text{GeV}/c)$	$M_*(\text{GeV}/c^2)$	$\Gamma_*(\text{MeV}/c^2)$	f	χ^2/NBF
Fit 1	0.5	1.0157 ± 0.0003	9.785 ± 0.373	0.000 ± 0.008	0.04/30
	0.7	1.0157(fixed)	9.785(fixed)	0.481 ± 0.031	65.05/30
	1.0	1.0157(fixed)	9.785(fixed)	0.753 ± 0.014	68.43/30
Fit 2	0.5	1.0148 ± 0.0005	12.069 ± 0.921	$0.40(\text{fixed})$	0.56/30
	0.7	1.0148(fixed)	12.069(fixed)	0.640 ± 0.021	51.98/30
	1.0	1.0148(fixed)	12.069(fixed)	0.829 ± 0.009	65.82/30
Fit 3	0.5	1.0107 ± 0.0016	20.197 ± 5.773	$0.80(\text{fixed})$	5.95/30
	0.7	1.0107(fixed)	20.197(fixed)	0.846 ± 0.009	71.80/30
	1.0	1.0107(fixed)	20.197(fixed)	0.926 ± 0.004	111.79/30

$$\rho_{\text{exp}}(m; M_{\text{exp}}, \Gamma_{\text{exp}}) = \frac{\Gamma_{\text{exp}}/2\pi}{(m - M_{\text{exp}})^2 + (\Gamma_{\text{exp}}/2)^2}$$

$$= f(p_T)\rho_0(m; M_0, \Gamma_0) + [1 - f(p_T)]\rho_*(m; M_*, \Gamma_*)$$



3. Method & results



k_{T1} and k_{T2} are calculated in the momentum regions $0.4 < |k_T| < 0.5 \text{ GeV}/c$ and $0.8 < |k_T| < 0.9 \text{ GeV}/c$.



4. Summary & conclusion

We investigate the squeezed BBC of ϕ meson caused by the mass modification in the source medium for the Au+Au and d+Au collisions at the RHIC energies.

The BBC functions are calculated using the modified masses extracted from experimental data and the source space-time distributions provided by the viscous hydrodynamic code VISH2+1.

It is found that the BBC of ϕ may perhaps be observed in the collisions of d+Au and the peripheral collisions of Au+Au at the RHIC energies.

We suggest to measure the BBC experimentally for understanding the mass modifications of the ϕ meson in the collisions.



Thanks !



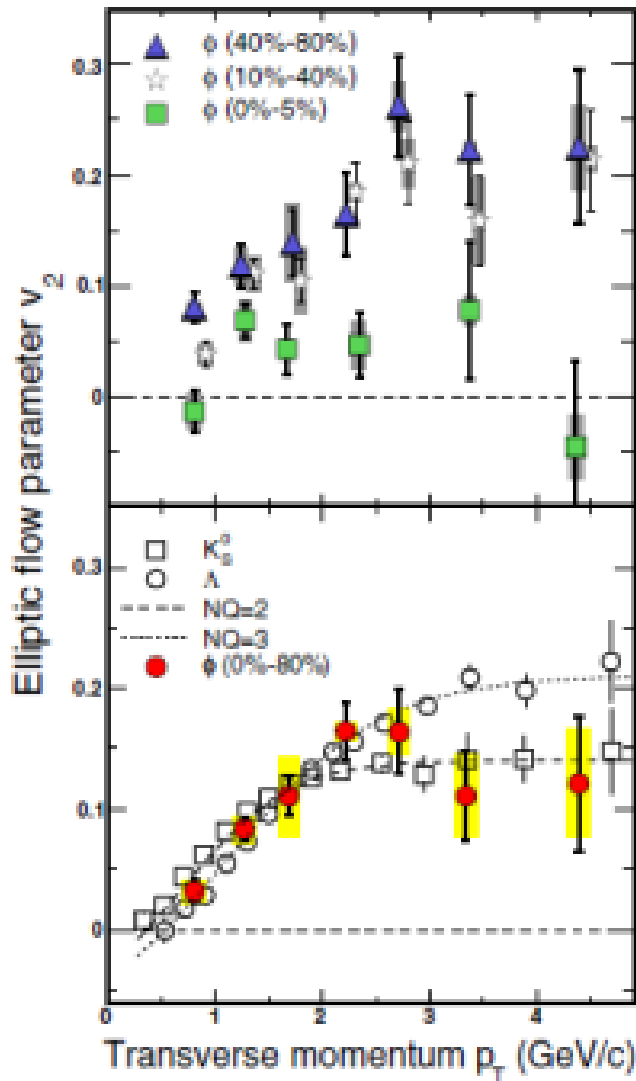
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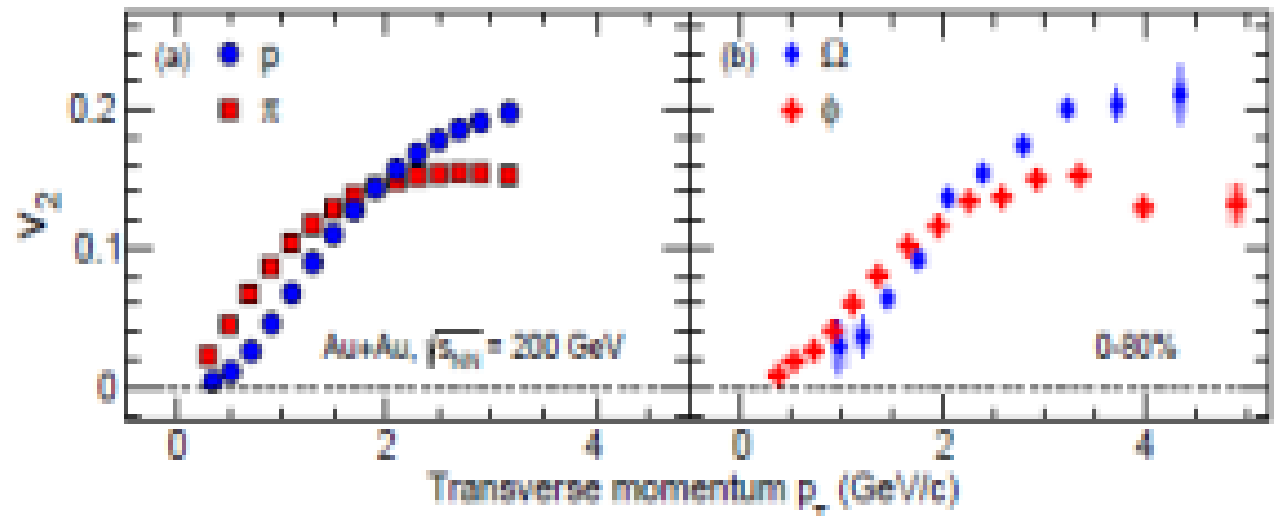


1. Motivations



[11] STAR Collab, PRL99,112301, 2007:

... for $p_T < 2 \text{ GeV}/c$, the v_2 follows a mass-ordered hierarchy where the values of v_2 , within errors, fall between those of the heavier Λ (open circles) and lighter K (open squares). However, at intermediate p_T , between 2–5 GeV/c, the v_2 appears to follow the same trend as K .



[20] STAR Collaboration, PRL116,062301, 2016.

1. Motivations

[16] PHENIX Collab, PRL99,052301, 2007.

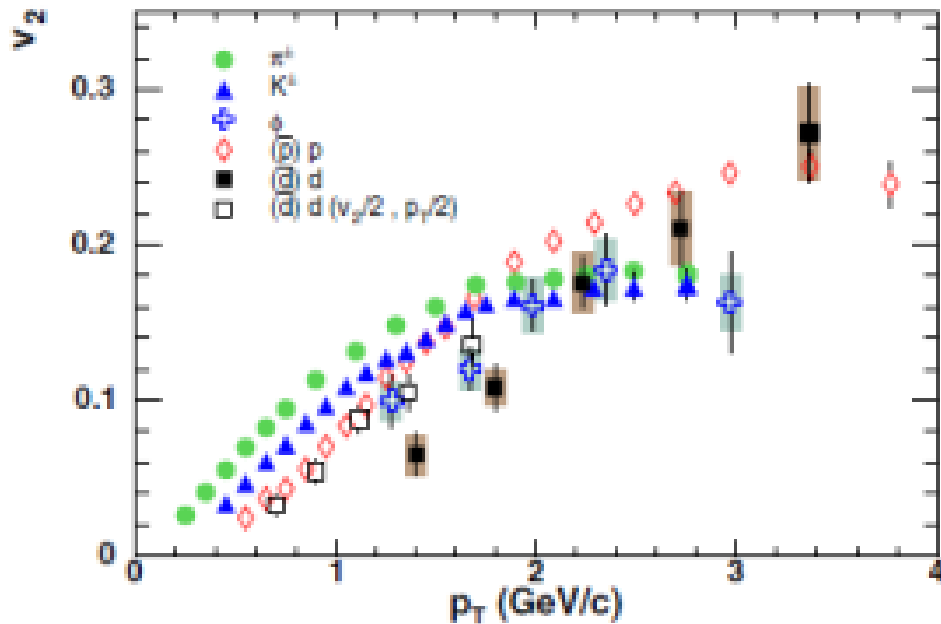


FIG. 2 (color online). Comparison of differential $v_2(p_T)$ for mesons, $(\bar{d})d$, π^\pm , K^\pm , and $(\bar{p})p$ (as indicated). Results are shown for 20%–60% central Au + Au collisions.

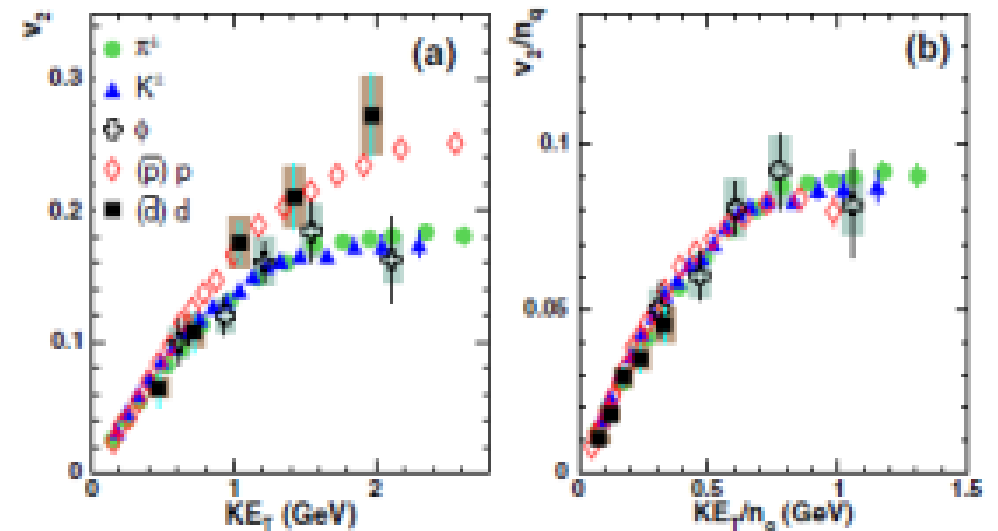
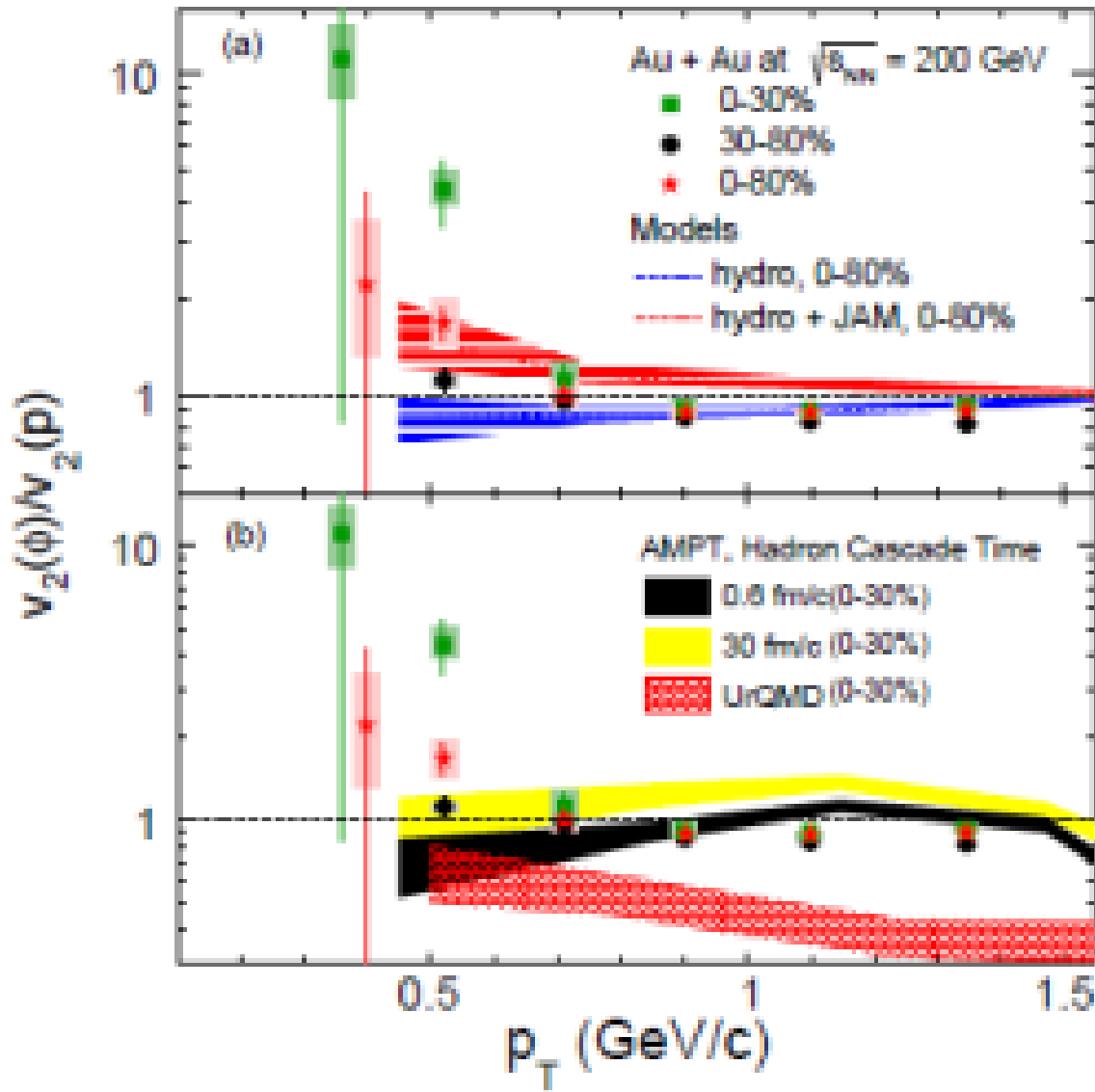


FIG. 3 (color online). (a) v_2 vs KE_T for several identified particle species obtained in midcentral (20%–60%) Au + Au collisions. (b) v_2/n_q vs KE_T/n_q for the same particle species shown in (a). The shaded bands indicate systematic error estimates for $(\bar{d})d$ and ϕ mesons (see text).

$$KE_T = mT - m$$



1. Motivations



[20] STAR Collaboration, PRL 116, 062301, 2016:

There is an indication of the breakdown of previous observed mass ordering between ϕ and proton v_2 at low transverse momentum in the 0-30% centrality range, possibly indicating late hadronic interactions affecting the proton v_2 .

1. Motivations

[21] ALICE Collab,
JHEP06,190, 2015:

The v_2 values of
the ϕ -meson in
figure 5 indicate
that for $p_T < 3$ GeV/c
it follows the mass-
ordered hierarchy.

However, for higher
 p_T values the ϕ
data points appear
to follow the band
of baryons for
central events
within uncertainties.

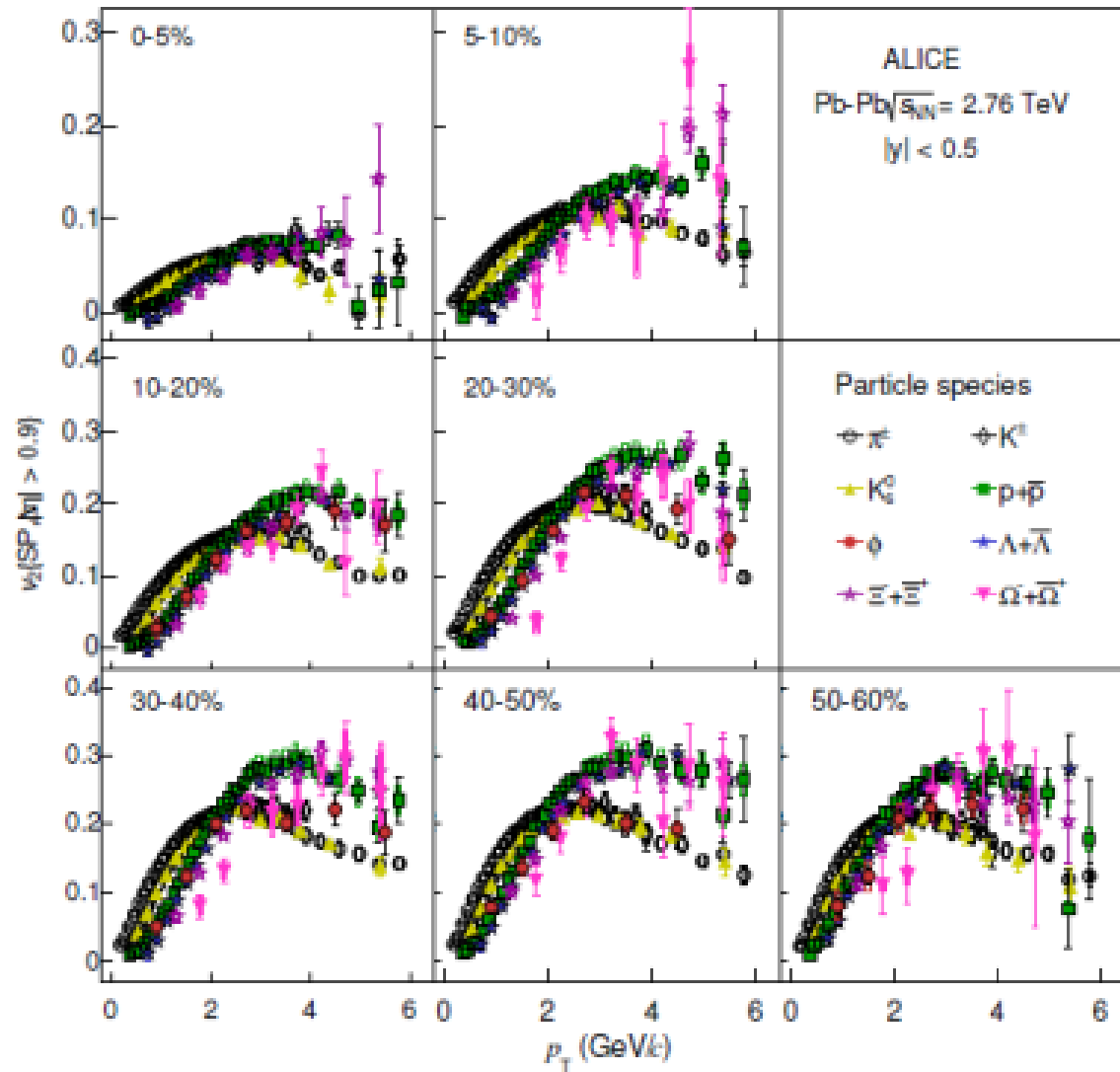


Figure 5. The p_T -differential v_2 for different particle species grouped by centrality class of Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV.



1. Motivations

[21] ALICE Collab,
JHEP06,190, 2015:

A distinct mass ordering was found for all centralities in the low transverse momentum region i.e. for $p_T < 3$ GeV/c, which is attributed to the interplay between elliptic and radial flow that modifies the $v_2(p_T)$ according to particle mass.

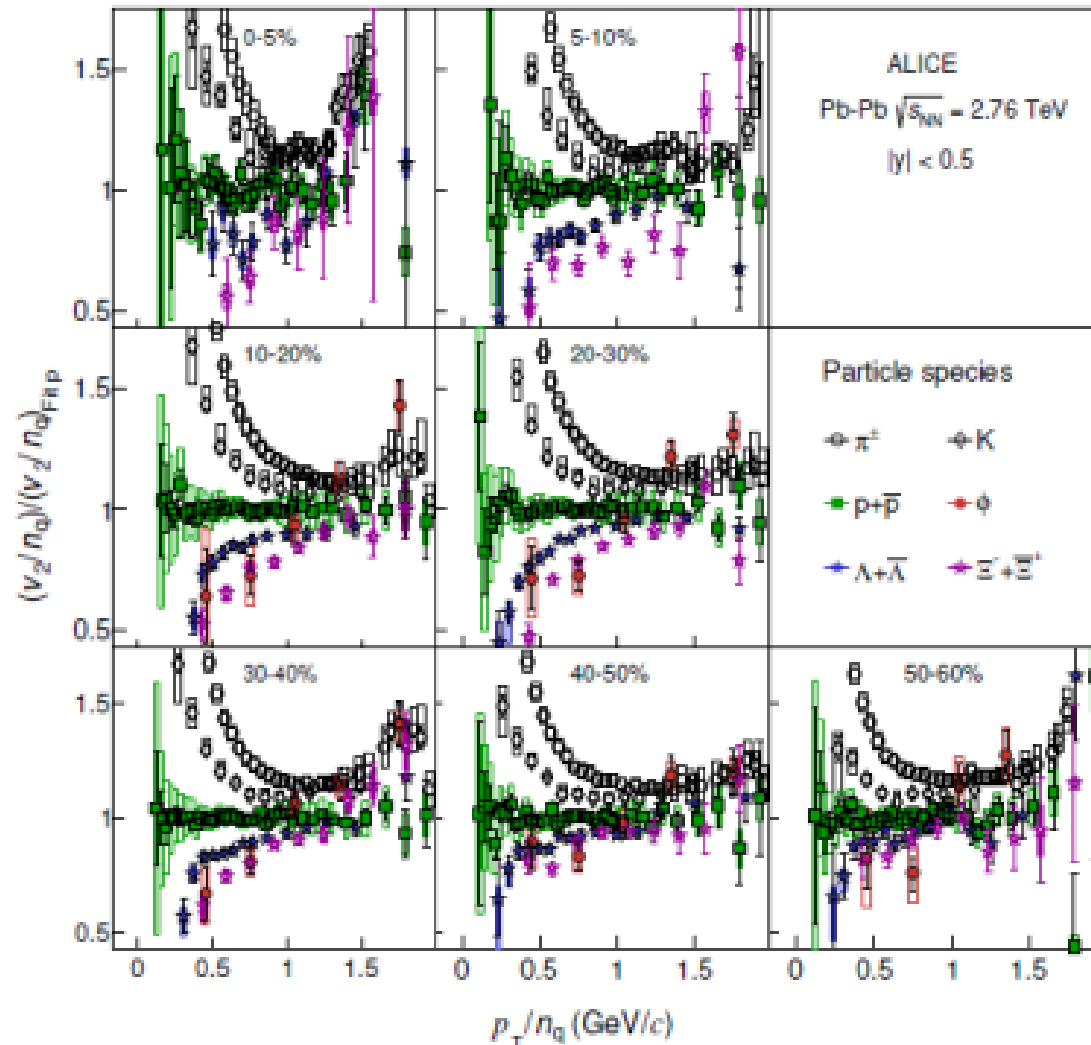


Figure 9. The p_T/n_q dependence of the double ratio of v_2/n_q for every particle species relative to a fit to v_2/n_q of p and \bar{p} (see text for details) for Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV.



2. Bases of BBC

1996年, M. Asakawa和T. Csorog指出当粒子在源内与介质相互作用发生质量改变时, 会产生一种正反玻色子对的压缩关联, 即背对背关联。

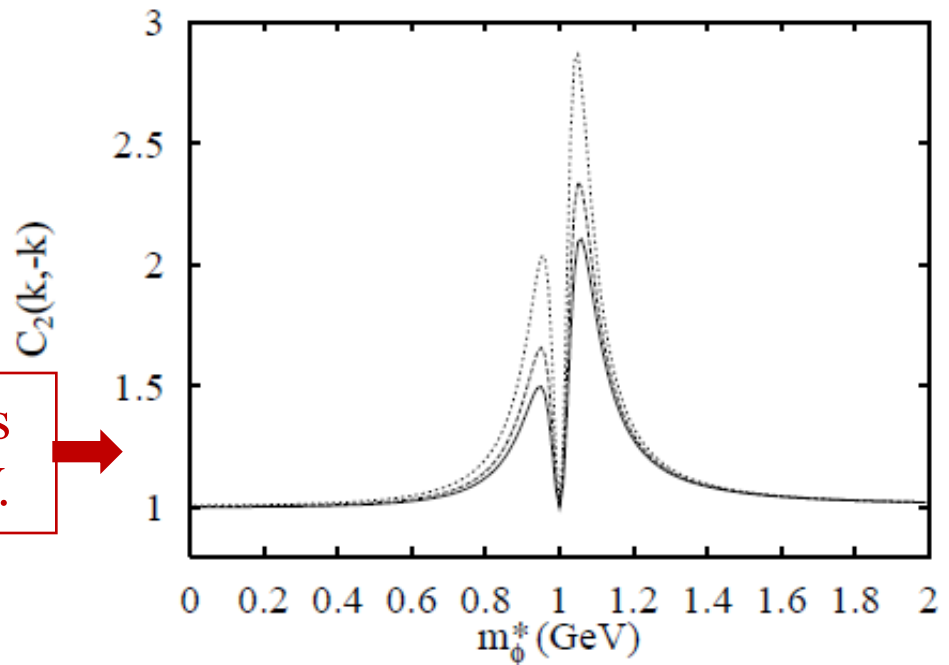
Heavy Ion Physics, 1996, 4: 233. 未考虑源发射时间分布对背对背关联的影响。

1999年, M. Asakawa, T. Csorgo和M. Gyulassy进一步在瞬时冻出假设下, 得出了均匀源的 $\phi\phi$ 背对背关联函数。 **Phys. Rev. Lett., 1999, 83: 4013.**

源发射时间分布为:

$$F(\tau) = \frac{\theta(\tau - \tau_0)}{\Delta t} e^{-(\tau - \tau_0)/\Delta t}$$

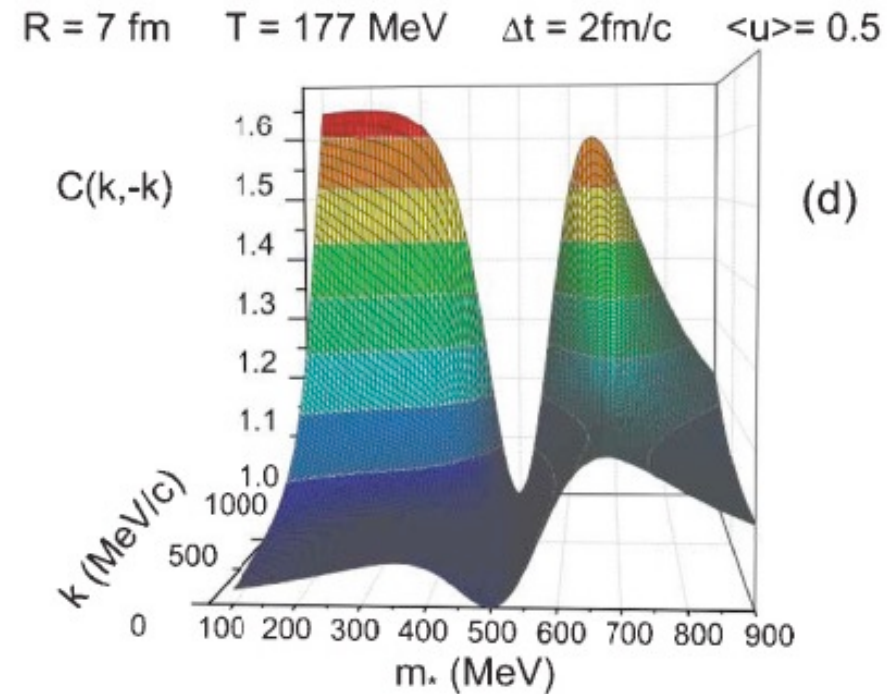
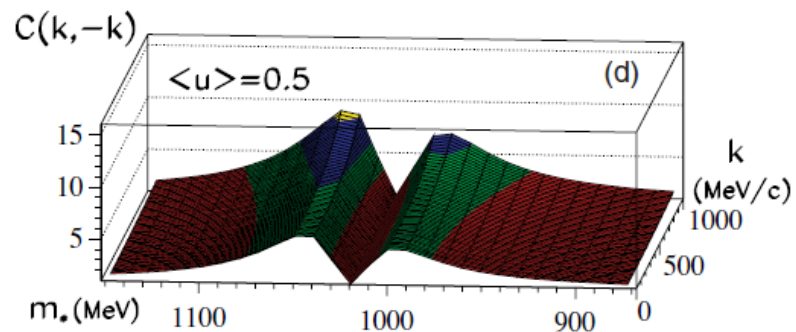
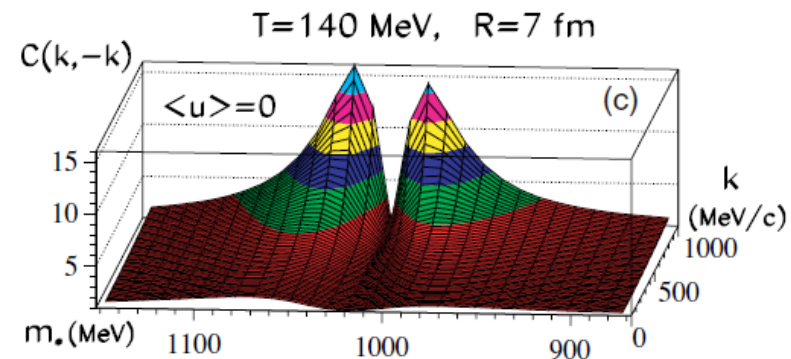
The solid, dashed, and dotted lines stand for $|\mathbf{k}|=0, 300, \text{ and } 500 \text{ MeV}$.



2. Bases of BBC

2006年, S. Padula等人进一步研究了有限大并且膨胀的系统在**非相对论情况**下的背对背关联。**Phys. Rev. C, 2006, 73: 044906.**

之后, S. Padula等人还进一步研究了同样情况下 K^+K^- 的背对背关联。**Phys. Rev. C, 2010, 82 : 034905.**



2. Bases of BBC

强子气体媒介

准粒子
(处于媒介中)

热力学冻出

自由粒子
(实验探测到)

$$m_* \neq m$$

$$m_*^2 = m^2 - \delta M^2$$

$(b_{\mathbf{k}}^\dagger, b_{\mathbf{k}})$

m_*

$$H = H_0 - \frac{1}{2} \int d\mathbf{x} d\mathbf{y} \phi(\mathbf{x}) \delta M^2(\mathbf{x} - \mathbf{y}) \phi(\mathbf{y})$$

$$H = \int \Omega_{\mathbf{k}} b_{\mathbf{k}}^\dagger b_{\mathbf{k}} d^3 k$$

$$\Omega_{\mathbf{k}} = \sqrt{m_*^2 + k^2}$$

$(a_{\mathbf{k}}^\dagger, a_{\mathbf{k}})$

m

$$H_0 = \frac{1}{2} \int d\mathbf{x} (\dot{\phi}^2 + |\nabla \phi|^2 + m_0^2 \phi^2)$$

$$H_0 = \int \omega_{\mathbf{k}} a_{\mathbf{k}}^\dagger a_{\mathbf{k}} d^3 k$$

$$\omega_{\mathbf{k}} = \sqrt{m^2 + k^2}$$

$a_{\mathbf{k}}^\dagger$ 与 $b_{\mathbf{k}}^\dagger$ 由波戈留波夫变换联系起来

$$a_{\mathbf{k}} = c_{\mathbf{k}} b_{\mathbf{k}} + s_{-\mathbf{k}}^* b_{-\mathbf{k}}^\dagger, \quad c_{\mathbf{k}} = \cosh f_{\mathbf{k}}, \quad s_{\mathbf{k}} = \sinh f_{\mathbf{k}}, \quad f_{\mathbf{k}} = \frac{1}{2} \log(\omega_{\mathbf{k}}/\Omega_{\mathbf{k}})$$



2. Bases of BBC

$$\begin{aligned}
 N_2(\mathbf{k}_1, \mathbf{k}_2) &= \omega_{\mathbf{k}_1} \omega_{\mathbf{k}_2} \langle a_{\mathbf{k}_1}^\dagger a_{\mathbf{k}_2}^\dagger a_{\mathbf{k}_2} a_{\mathbf{k}_1} \rangle \\
 &= \omega_{\mathbf{k}_1} \omega_{\mathbf{k}_2} \left[\langle a_{\mathbf{k}_1}^\dagger a_{\mathbf{k}_1} \rangle \langle a_{\mathbf{k}_2}^\dagger a_{\mathbf{k}_2} \rangle + \langle a_{\mathbf{k}_1}^\dagger a_{\mathbf{k}_2} \rangle \langle a_{\mathbf{k}_2}^\dagger a_{\mathbf{k}_1} \rangle \right. \\
 &\quad \left. + \langle a_{\mathbf{k}_1}^\dagger a_{\mathbf{k}_2}^\dagger \rangle \langle a_{\mathbf{k}_2} a_{\mathbf{k}_1} \rangle \right],
 \end{aligned}$$

$$a_{\mathbf{k}} = c_{\mathbf{k}} b_{\mathbf{k}} + s_{-\mathbf{k}}^* b_{-\mathbf{k}}^\dagger$$

$$\begin{aligned}
 \langle \underline{a_{\mathbf{k}} a_{-\mathbf{k}}} \rangle &= \langle c_{\mathbf{k}}^2 b_{\mathbf{k}} b_{-\mathbf{k}} + c_{\mathbf{k}} s_{\mathbf{k}} b_{\mathbf{k}} b_{\mathbf{k}}^+ \\
 &\quad + c_{\mathbf{k}} s_{\mathbf{k}} b_{-\mathbf{k}}^+ b_{-\mathbf{k}} + s_{\mathbf{k}}^2 b_{-\mathbf{k}}^+ b_{\mathbf{k}}^+ \rangle \\
 &= \langle c_{\mathbf{k}} s_{\mathbf{k}} (1 + b_{\mathbf{k}}^+ b_{\mathbf{k}}) + c_{\mathbf{k}} s_{\mathbf{k}} b_{-\mathbf{k}}^+ b_{-\mathbf{k}} \rangle \\
 &= 2c_{\mathbf{k}} s_{\mathbf{k}} n_{\mathbf{k}} + c_{\mathbf{k}} s_{\mathbf{k}} \\
 n_{\mathbf{k}} &= \langle b_{\mathbf{k}}^+ b_{\mathbf{k}} \rangle
 \end{aligned}$$



2. Bases of BBC

混沌振幅: $G_c(\mathbf{k}_1, \mathbf{k}_2) = \sqrt{\omega_{\mathbf{k}_1} \omega_{\mathbf{k}_2}} \langle a_{\mathbf{k}_1}^\dagger a_{\mathbf{k}_2} \rangle,$

压缩振幅: $G_s(\mathbf{k}_1, \mathbf{k}_2) = \sqrt{\omega_{\mathbf{k}_1} \omega_{\mathbf{k}_2}} \langle a_{\mathbf{k}_1} a_{\mathbf{k}_2} \rangle,$

背对背关联函数:

$$C(\mathbf{k}, -\mathbf{k}) = 1 + \frac{|G_s(\mathbf{k}, -\mathbf{k})|^2}{G_c(\mathbf{k}, \mathbf{k})G_c(-\mathbf{k}, -\mathbf{k})}$$

对于流体力学演化源, **Makhlín** 和**Sinyukov**推导得出:

$$G_c(k_1, k_2) = \frac{1}{(2\pi)^3} \int d^4 \sigma_\mu K_{1,2}^\mu e^{iq_{1,2} \cdot x} n_c(x, K_{1,2}^\mu u_\mu)$$

$$G_s(k_1, k_2) = \frac{1}{(2\pi)^3} \int d^4 \sigma_\mu K_{1,2}^\mu e^{2iK_{1,2} \cdot x} n_s(x, K_{1,2}^\mu u_\mu)$$

Sov. J. Nucl. Phys. 46 (1987) 354;
Nucl.Phys. A566 (1994) 589c



2. Bases of BBC

$$G_s(k_1, k_2) = \frac{1}{(2\pi)^3} \int d^4 \sigma_\mu K_{1,2}^\mu e^{2iK_{1,2} \cdot x} n_s(x, K_{1,2}^\mu u_\mu)$$

空间部分：

$$d^4 \sigma_\mu = (dx dy dz, -dt dy dz, -dt dx dz, -dt dx dy)$$

采用瞬时冻出假设：（ $dt = 0$ ）

$$d^4 \sigma_\mu K_{1,2}^\mu = K^0 d^3 x$$

均匀源

$$C(\mathbf{k}, -\mathbf{k}) = 1 + \frac{V |c_{\mathbf{k}} s_{\mathbf{k}}^* n_{\mathbf{k}} + c_{-\mathbf{k}} s_{-\mathbf{k}}^* (n_{-\mathbf{k}} + 1)|^2}{V [n_1(\mathbf{k}) n_1(-\mathbf{k})]}$$

Phys. Rev. Lett., 1999, 83: 4013.



大连理工大学
DALIAN UNIVERSITY OF TECHNOLOGY

$$e^{i(\omega_1 + \omega_2)t}$$

时间部分：

采用指数衰减发射时间分布：

$$F(\tau) = \frac{\theta(\tau - \tau_0)}{\Delta t} e^{-(\tau - \tau_0)/\Delta t}$$

$$\tilde{F}(\omega) = \int dt F(t) \exp(-i\omega t)$$

$$|\tilde{F}(\omega_{\mathbf{k}} + \omega_{-\mathbf{k}}, \Delta t)|^2 = [1 + (\omega_{\mathbf{k}} + \omega_{-\mathbf{k}})^2 \Delta t^2]^{-1}$$

$$C_-(\mathbf{k}, -\mathbf{k}) - 1 = \frac{|c_{\mathbf{k}}^* s_{\mathbf{k}} n_{\mathbf{k}} + c_{-\mathbf{k}}^* s_{-\mathbf{k}} (n_{-\mathbf{k}} + 1)|^2}{n_1(\mathbf{k}) n_1(-\mathbf{k})} \times |\tilde{F}(\omega_{\mathbf{k}} + \omega_{-\mathbf{k}})|^2,$$

2. Bases of BBC

$$G_s(k_1, k_2) = \frac{1}{(2\pi)^3} \int d^4 \sigma_\mu K_{1,2}^\mu e^{2iK_{1,2} \cdot x} n_s(x, K_{1,2}^\mu u_\mu)$$

空间部分：

$$d^4 \sigma_\mu = (dx dy dz, -dt dy dz, -dt dx dz, -dt dx dy)$$

采用瞬时冻出假设： ($dt = 0$)

$$d^4 \sigma_\mu K_{1,2}^\mu = K^0 d^3 x$$

高斯球形膨胀源
+ 非相对论近似



S.Padula et al. Phys. Rev. C, 2006, 73: 044906

$$e^{i(\omega_1 + \omega_2)t}$$

时间部分：

采用指数衰减发射时间分布：

$$F(\tau) = \frac{\theta(\tau - \tau_0)}{\Delta t} e^{-(\tau - \tau_0)/\Delta t}$$

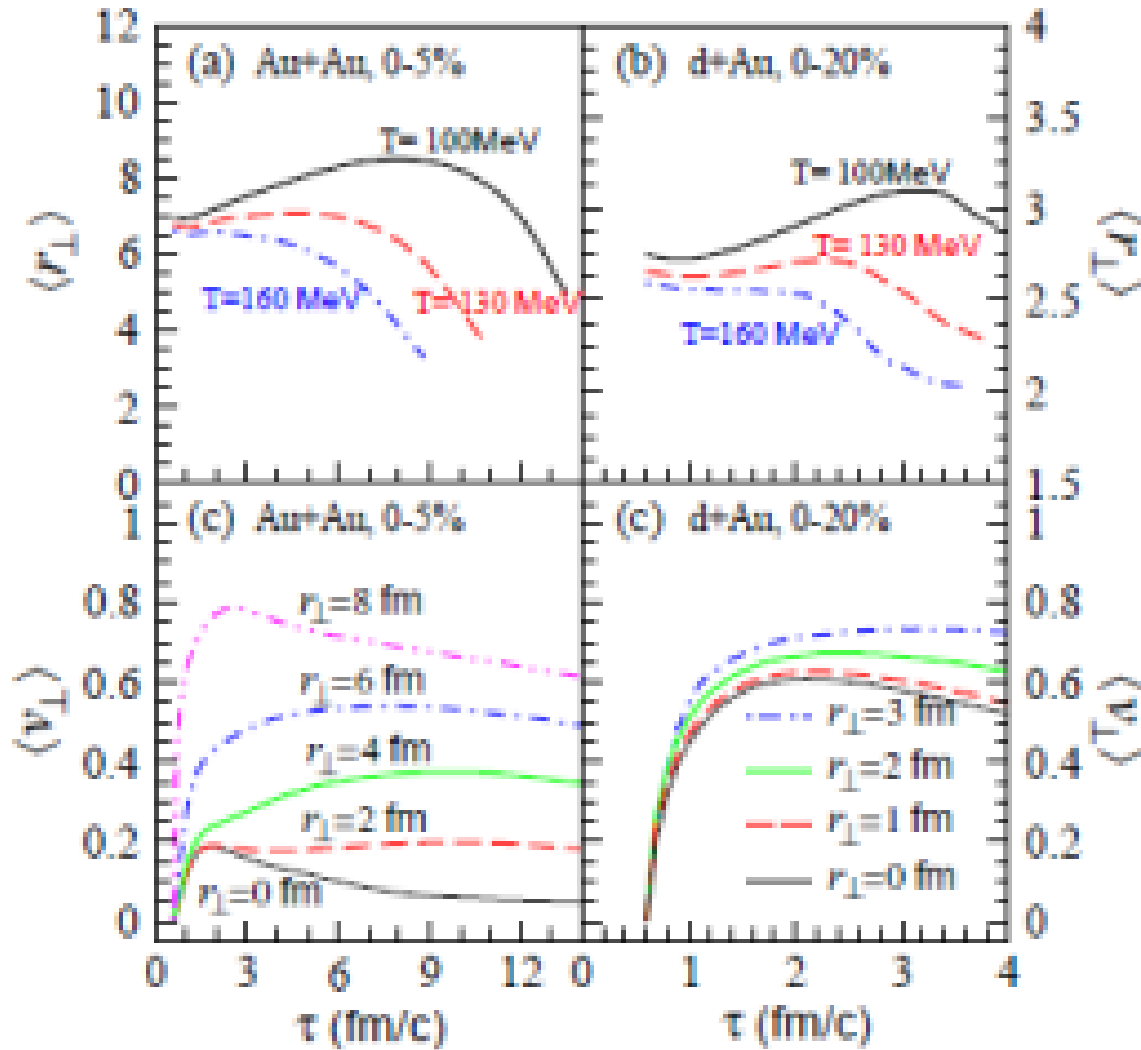
$$F(\omega) = \int dt F(t) \exp(-i\omega t)$$



$$|\tilde{F}(\omega_k + \omega_{-k}, \Delta t)|^2 = [1 + (\omega_k + \omega_{-k})^2 \Delta t^2]^{-1}$$



3. Method & results



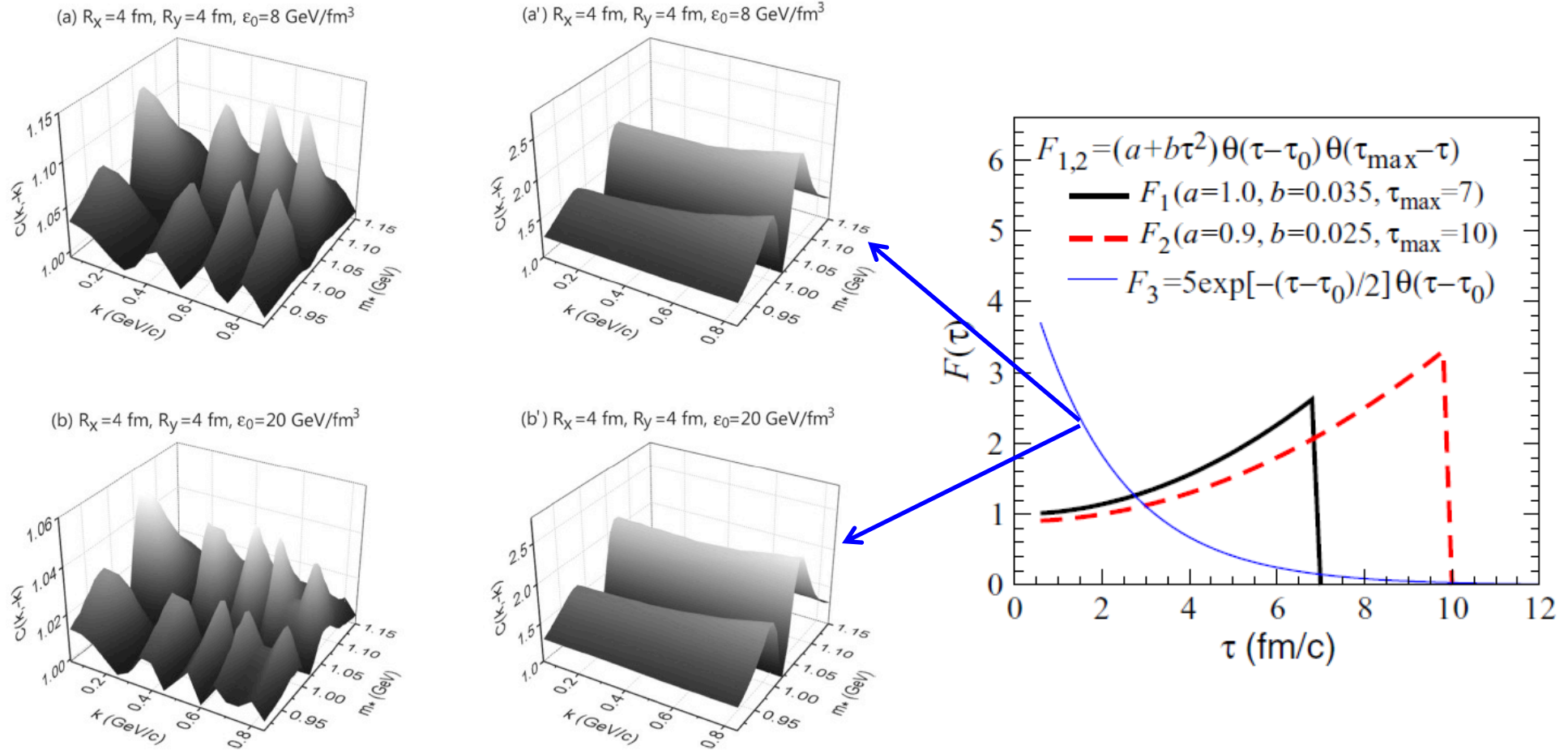
The source isothermals and average transverse velocities $\langle v_{\perp} \rangle$ calculated with the 1000 events for the collisions of the Au+Au with 0-5% centrality and the d+Au with 0-20% centrality at $\sqrt{s_{NN}} = 200$ GeV.

The sources for the d+Au collisions have smaller space-time distribution and evolve faster than that for the Au+Au collisions.

The average velocity $\langle v_{\perp} \rangle$ at $r_{\perp} = 0$ is small for the collisions of Au+Au because the center of the fireball for each the event is near to $r_{\perp} = 0$ for the symmetric collisions.

Previous works

流体力学演化源的正反玻色子对的背对背关联 (高斯初始条件)



Previous works

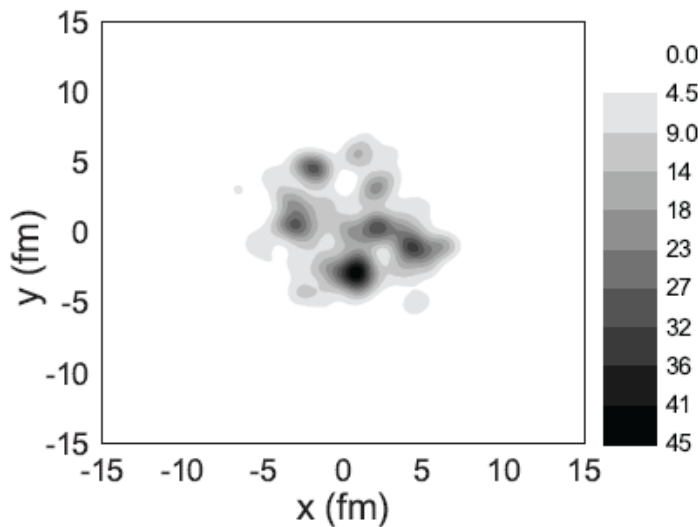
流体力学演化源的正反玻色子对的背对背关联 (涨落初始条件)

涨落初始条件 (AMPT中的HIJING) :

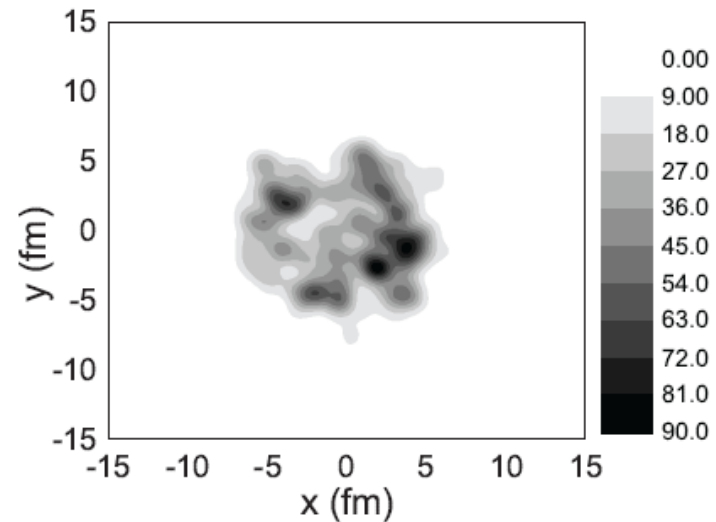
[Ying Hu *et al.* J. Phys. G: Nucl. Part. Phys. 42, 045105 (2015)]

$$\epsilon(x, y) = K \sum_i \frac{p_{\perp i}}{\tau_0} \frac{1}{2\pi\sigma^2} \times \exp\left\{-\frac{[x - x_i(\tau_0)]^2 + [y - y_i(\tau_0)]^2}{2\pi\sigma^2}\right\}$$

$\sqrt{s_{NN}} = 200$ GeV Au+Au $b = 0$

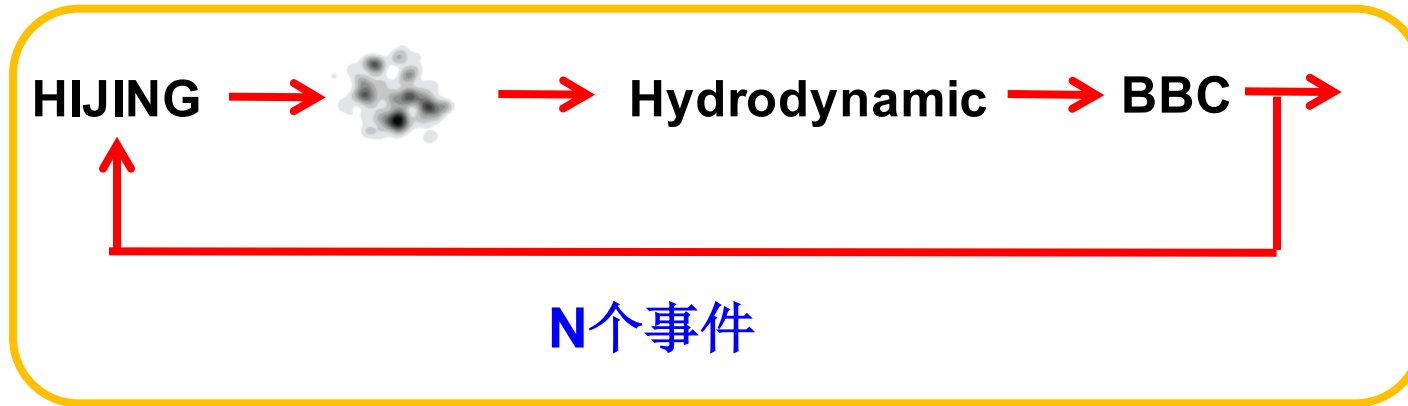


$\sqrt{s_{NN}} = 2.76$ TeV Pb+Pb $b = 0$



Previous works

流体力学演化源的正反玻色子对的
背对背关联（涨落初始条件）



背对背关联函数:

$$C(\mathbf{k}, -\mathbf{k}) = \frac{\frac{1}{N_E} \sum_{i=1}^{N_E} [G_{ci}(\mathbf{k}, \mathbf{k})G_{ci}(-\mathbf{k}, -\mathbf{k}) + |G_{si}(\mathbf{k}, -\mathbf{k})|^2]}{\frac{1}{N_E} \sum_{i=1}^{N_E} G_{ci}(\mathbf{k}, \mathbf{k})G_{ci}(-\mathbf{k}, -\mathbf{k})}$$

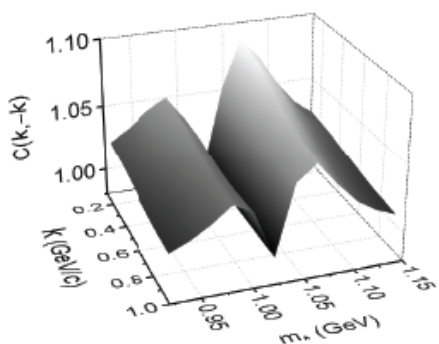


Previous works

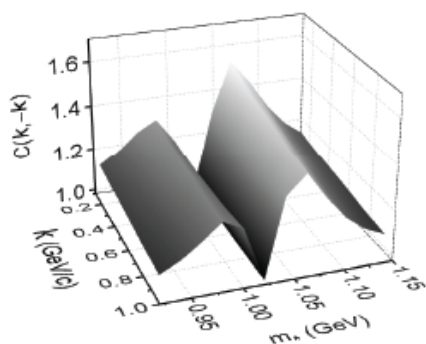
流体力学演化源的正反玻色子对的背对背关联 (涨落初始条件)

ϕ 介子多事件混合后的背对背关联(2000个事件)

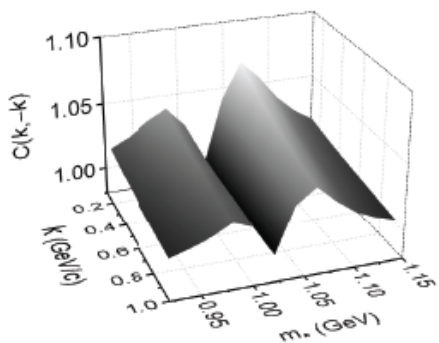
(a) Au+Au, 200 GeV, 0-10%



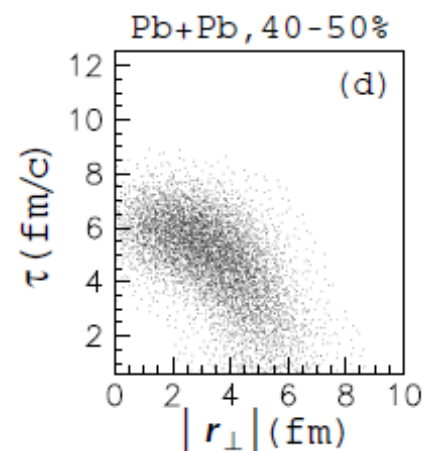
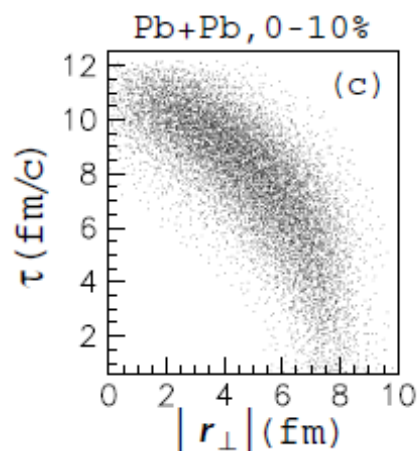
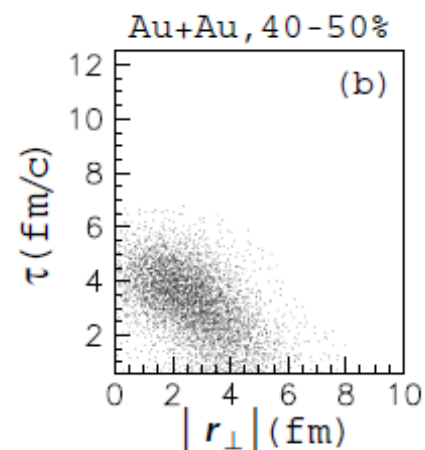
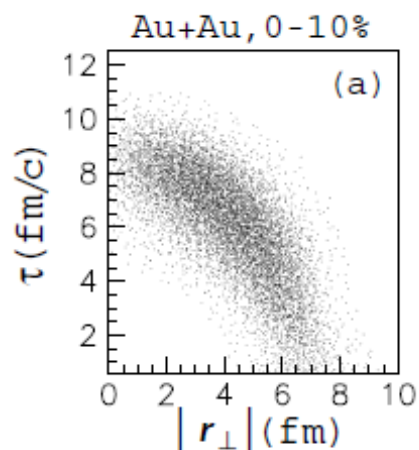
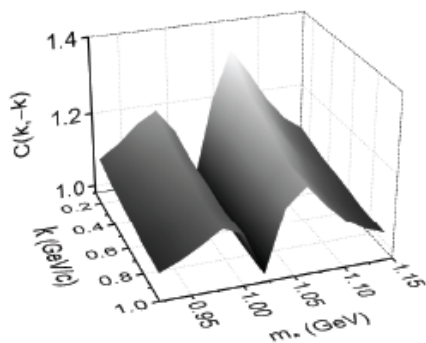
(b) Au+Au, 200 GeV, 40-50%



(c) Pb+Pb, 2.76 TeV, 0-10%



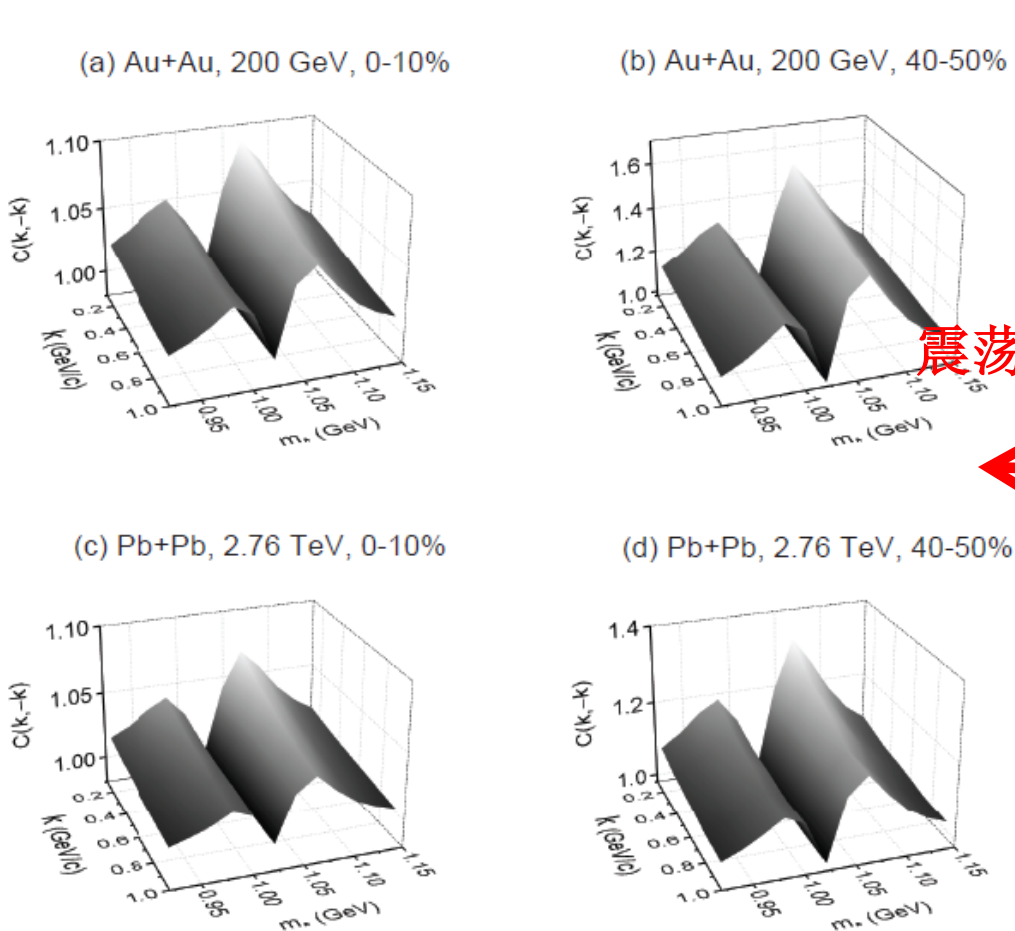
(d) Pb+Pb, 2.76 TeV, 40-50%



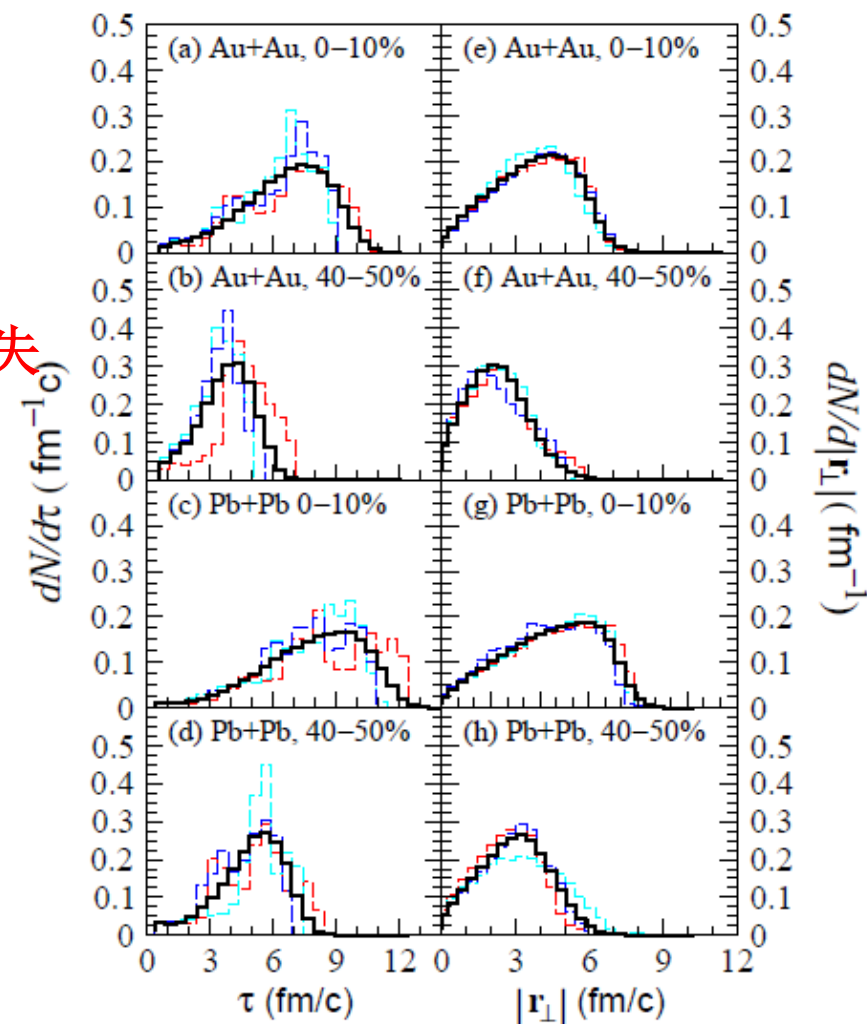
Previous works

流体力学演化源的正反玻色子对的背对背关联 (涨落初始条件)

ϕ 介子多事件混合后的背对背关联(2000个事件)



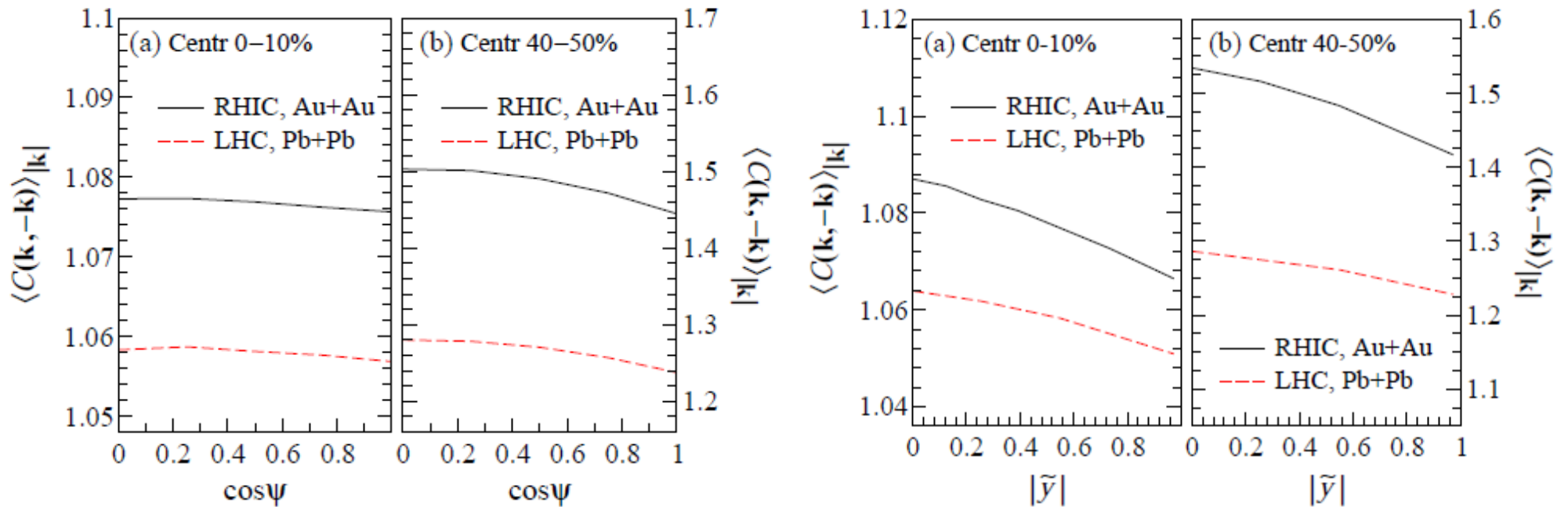
震荡消失



Previous works

流体力学演化源的正反玻色子对的背对背关联 (涨落初始条件)

$\phi\phi$ 背对背关联对粒子动量方向的依赖



Previous works

流体力学演化源的正反玻色子对的背对背关联 (涨落初始条件)

Cu+Cu碰撞 ϕ 介子多事件混合后的背对背关联(2000个事件)

(a) Cu+Cu, 200 GeV, $b=0-4$ fm

(b) Cu+Cu, 200 GeV, $b=4-8$ fm

