An extended AMPT model with mean-field potentials

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Search for signals of critical point at finite $\mu_B$!

RHIC-BES: $\sqrt{s} \sim 7.7-39$ GeV

FAIR-CBM: $\sqrt{s} < 12$ GeV
Highlights in RHIC-BES I

$v_2$ splitting

$\sqrt{s_{NN}}$ (GeV)

$0.06$

$0.04$

$0.02$

$0.00$

$0.00$

$0.02$

$0.04$

$0.06$

Au+Au, 0%-80%  Ξ^*-Ξ^+
η-sub EP

STAR, PRL (2013)

Baryon-Antibaryon polarization

Effects of mean-field potentials on these observables?

$U_i \sim \sum_{i \neq j} V_{ij} + \ldots$

HBT correlation

STAR, PRL (2013)

STAR, PRC (2015)

STAR, arXiv:1701.06657
Transport model simulations of intermediate-energy heavy-ion collisions

Main approaches:
Boltzmann transport and Quantum Molecular Dynamics

Heavy-ion experiments ↔ Transport simulations ↔ Mean-field potential

Initialization ↔ Mean-field potential ↔ NN scatterings

Nuclear EOS, $E_{\text{sym}}$
A multiphase transport (AMPT) model with string melting

Structure of AMPT model with string melting

A+B

HIJING: energy in excited strings and minijet partons
fragment into partons

ZPC (Zhang's Parton Cascade):

till parton freezeout

Quark Coalescence

ART (A Relativistic Transport model for hadrons)

Lund string fragmentation function

\[ f(z) \approx z^{-1}(1 - z)^{a} \exp \left[ -\frac{b(m^2 + p_t^2)}{z} \right] \]

z : light-cone momentum fraction

Parton scattering cross section

\[ \frac{d\sigma}{dt} \approx \frac{9\pi\alpha^2}{2s^2} \left(1 + \frac{\mu^2}{s}\right) \left(\frac{1}{t - \mu^2}\right)^2, \quad \sigma \approx \frac{9\pi\alpha^2}{2\mu^2} \]

\( \alpha \): strong coupling constant
\( \mu \): screening mass

a, b: particle multiplicity
\( \alpha, \mu \): partonic interaction
A multiphase transport (AMPT) model with string melting

Structure of AMPT model with string melting

Hijing
energy in excited strings and minijet partons nucleon spectators

fragment into partons

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Turn on hadronic mean-field potentials
hadronic potentials for particles and antiparticles

Nucleon and antinucleon potential
\[ L = \bar{\psi} [i\gamma_\mu \partial^\mu - m - g_\sigma \sigma - g_\omega \gamma_\mu \omega^\mu] \psi + \frac{1}{2} (\partial^\mu \sigma)^2 - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} b_\sigma^3 - \frac{1}{4} c_\sigma^4 - \frac{1}{4} (\partial_\mu \omega^\nu - \partial_\nu \omega^\mu)^2 + \frac{1}{2} m_\omega^2 \omega^\mu, \]

\[ \Sigma_s = g_\sigma \langle \sigma \rangle, \quad \Sigma_{s\mu} = g_\omega \langle \omega_\mu \rangle \]

\[ U_{N,\overline{N}} = \Sigma_s (\rho_B, \rho_{\overline{B}}) \pm \Sigma^0_{s\mu} (\rho_B, \rho_{\overline{B}}) \]

\[ U_{\Lambda,\overline{\Lambda}} \sim \frac{2}{3} U_{N,\overline{N}}, U_{\Xi,\overline{\Xi}} \sim \frac{1}{3} U_{N,\overline{N}} \]

Vector potential changes sign for antiparticles! (e^+e^- exchange γ)

Kaon and antikaon potential
\[ \omega_{K,\overline{K}} = \sqrt{m_K^2 + p^2 - a_K \rho_s + (b_K \rho_B^{\text{net}})^2} \pm b_K \rho_B^{\text{net}} \]
\[ U_K(\overline{K}) = \omega_{K(\overline{K})} - \omega_0 \quad \omega_0 = \sqrt{m_K^2 + p^2} \]

Pion s-wave potential
\[ \Pi_s^-(\rho_p, \rho_n) = \rho_n [T_{\pi N}^- - T_{\pi N}^+] - \rho_p [T_{\pi N}^- + T_{\pi N}^+] + \Pi_{\text{rel}}^-(\rho_p, \rho_n) + \Pi_{\text{cor}}^-(\rho_p, \rho_n) \]
\[ \Pi_s^+(\rho_p, \rho_n) = \Pi_s^-(\rho_n, \rho_p) \]
\[ \Pi_s^0(\rho_p, \rho_n) = - (\rho_p + \rho_n) T^+_{\pi N} + \Pi_{\text{cor}}^0(\rho_p, \rho_n) \]


N. Kaiser and W. Weise, PLB (2001)
In baryon-rich and neutron-rich matter:

- **Baryon potential**: weakly attractive
- **Antibaryon potential**: deeply attractive
- **K⁺ potential**: weakly repulsive
- **K⁻ potential**: deeply attractive
- **π⁺ potential**: weakly attractive
- **π⁻ potential**: weakly repulsive

Sub threshold particle production

Chiral perturbation theory

Introduced with test-particle method
Effects of mean-field potentials on elliptic flow

Particles with attractive potentials are more likely to be trapped in the system

\[ v_2 = \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle} \]

Particles with repulsive potentials are more likely to leave the system

\[ v_2 \text{ decrease} \]

\[ v_2 \text{ increase} \]

Fit AMPT parameters at RHIC-BES energies

Lund string fragmentation parameters, Parton scattering cross section, parton life time

Pseudorapidity distribution, elliptic flow, energy density at chemical freeze-out
Qualitatively consistent
proton and antiproton: underestimate
$K^+$ and $K^-$: overestimate
$\pi^+$ and $\pi^-$: underestimate

JX, L. W. Chen, C. M. Ko, and Z. W. Lin, PRC 85, 041901(R) (2012)
Effects of mean-field potentials on HBT correlation


a probe of neutron-proton U difference based on IBUU

\[ E_{\text{sym}}(\rho) = E_{\text{sym}}(\rho_0) \cdot u^\gamma \]

(a) nn \quad (d) nn

\(^{52}\text{Ca} + ^{48}\text{Ca}\)

\[ P < 300 \text{ MeV}/c \]

\[ P > 500 \text{ MeV}/c \]

E=80 AMeV, b=0 fm

(b) pp \quad (e) pp

(c) np \quad (f) np

\[ C(q) = (1 - \lambda) + \lambda K_{\text{coul}}(q_{\text{inv}}) \times \left( 1 + e^{-q_o^2 R_o^2 - q_s^2 R_s^2 - q_l^2 R_l^2 - 2q_o q_s R_o^2 - 2q_o q_l R_o^2} \right) \]

Q.F. Li, M. Bleicher, and H. Stocker, PLB (2008)

Q.F. Li, M. Bleicher, and H. Stocker, PLB (2008)

Effects of hadronic mean-field potentials on HBT correlation

Later emission and broader emission time distribution

Impact on \( R_{\text{out}} \) and \( R_{\text{side}} \)

Lead to a smaller freeze-out eccentricity

\( Au+Au@7.7\text{GeV}, 0-5\%, |\eta|<0.5 \)

\[ k_\tau=0.15-0.25\text{GeV/c} \]

\( \pi^+\pi^- \) correlation

\( C(q_{\text{inv}}) \)


\( dN/dt \) (c/fm)

\( q_{\text{inv}} \) (GeV/c)

\( \sqrt{s_{\text{NN}}} \) (GeV)

\( \sqrt{s_{\text{NN}}} \) (GeV)
Effects of hadronic mean-field potentials on HBT correlation

Affect correlation for identified particles

- Attractive potential $\Rightarrow$ correlation $\uparrow$
- Annihilation $\Rightarrow$ correlation $\downarrow$

Affect global system evolution +
Affect emission of individual particles

Interplay with $p$-$p\bar{p}$ annihilation

A multiphase transport (AMPT) model with string melting

**Structure of AMPT model with string melting**

- **A + B**
  - HIJING: energy in excited strings and minijet partons 
    - nucleon spectators
  - Fragment into partons
  - ZPC (Zhang's Parton Cascade): till parton freezeout
  - Turn on partonic mean-field potentials
  - Quark Coalescence
  - ART (A Relativistic Transport model for hadrons)

**Lund string fragmentation function**

\[ f(z) = z^{-1} (1 - z)^a \exp \left[ -\frac{b(m^2 + p_t^2)}{z} \right] \]

- \( z \): light-cone momentum fraction

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- \( \alpha \): strong coupling constant
- \( \mu \): screening mass
- \( a, b \): particle multiplicity
- \( \alpha, \mu \): partonic interaction
3-flavor Nambu-Jona-Lasinio transport model

**Lagrangian:**

\[ \mathcal{L} = \bar{\psi} (i \gamma \cdot \mathbf{D} - M) \psi + \frac{G}{2} \sum_{a=0}^{8} \left[ (\bar{\psi} \lambda^a \psi)^2 + (\bar{\psi} i \gamma_5 \lambda^a \psi)^2 \right] - \sum_{a=0}^{8} \left[ \frac{G_V}{2} (\bar{\psi} \gamma^\mu \lambda^a \psi)^2 + \frac{G_A}{2} (\bar{\psi} \gamma^\mu \gamma_5 \lambda^a \psi)^2 \right] \]

**Kobayashi-Maskawa-t’Hooft interaction**

\[ - K[\det_f(\bar{\psi}(1 + \gamma_5)\psi) + \det_f(\bar{\psi}(1 - \gamma_5)\psi)] \]

**Parameters taken from**

M. Lutz, S. Klimt, and W. Weise, NPA (1992) to reproduce meson properties

**Boltzmann equation:**

\[ \frac{\partial}{\partial t} f + \bar{\nu} \cdot \nabla_x f - \nabla_x H \cdot \nabla_p f = C \]

**Single-quark Hamiltonian:**

\[ H = \sqrt{M^*^2 + p^*^2} \pm g_V \rho \]

\[ M_u = m_u - 2G\langle \bar{u}u \rangle + 2K\langle \bar{d}d \rangle \langle s \rangle \]

\[ M_d = m_d - 2G\langle \bar{d}d \rangle + 2K\langle s \rangle \langle \bar{u}u \rangle \]

\[ M_s = m_s - 2G\langle \bar{s}s \rangle + 2K\langle \bar{u}u \rangle \langle \bar{d}d \rangle \]

\[ p^* = p \mp g_V \rho \]

\[ g_V \equiv (2/3)G_V \]

**Equations of motion:**

\[ \frac{dx_i}{dt} = \frac{\partial H}{\partial p_i} = \frac{p_i^*}{E^*} \]

\[ \frac{dp_i}{dt} = - \frac{\partial H}{\partial x_i} \]

\[ \frac{M^*}{E^*} \frac{\partial M^*}{\partial x_i} \pm g_V \left( v_j \frac{\partial \rho_j}{\partial x_i} - \frac{\partial \rho_0}{\partial x_i} \right) \]

**Solve with test particle method**

Yoichiro Nambu

2008年

Nobel Prize

For Physics

Yoichiro Nambu

2008年

Nobel Prize

For Physics
Phase diagram from NJL model

\[ R_V = \frac{G_V}{G} \]
Fierz transformation: $R_V = 0.5$

Vector meson-mass spectrum: $R_V = 1.1$

Fit the parton scattering cross section with charged-particle $v_2$

Total $v_2$ is less sensitive to $R_V$

Hadronization happens when chiral symmetry is broken, i.e., $M^*>M_{\text{vac}}/2$

global hadronization

Density evolution in the partonic and hadronic phase

![Graphs showing density evolution and $v_2$ as functions of $p_T$ and $t$ for different energies and cross sections.](attachment:image.png)
**v₂ right after hadronization**

\[ v_2(N) > v_2(\bar{N}) \quad v_2(K^+) < v_2(K^-) \]

\[ \text{Au+Au@7.7 GeV} \quad 0-80\% \quad |\eta|<1 \]

\[ R_v = \frac{G_v}{G} \]

\[ R_v = 0.5 \]

\[ R_v = 1.1 \]

\[ \text{JX, T. Song, C. M. Ko, and F. Li,} \]

\[ \text{Phys. Rev. Lett. 110, 012301 (2014)} \]

**Final v₂**

\[ v_2(N) > v_2(\bar{N}) \quad v_2(K^+) > v_2(K^-) \]

\[ \text{Au+Au@7.7 GeV} \quad 0-80\% \quad |\eta|<1 \]

\[ R_v = \frac{G_v}{G} \]

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\[ \text{Phys. Rev. Lett. 110, 012301 (2014)} \]

\[ 0.5 < R_v < 1.1? \]
Collision energy dependence of $v_2$ splitting

Difficult to reproduce quantitatively $v_2$ splitting at all collision energies at RHIC-BES

Effect of vector potential still holds

Energy dependence of $v_2$ splitting is qualitatively consistent with experimental observation

JX and C.M. Ko, PRC 94, 054909 (2016)
EOS effects on the directed flow

\[ v_1 = \frac{\langle p_x \rangle}{p_T} \]

\[ T \times p \times p \times v = 1 \]

D.H. Rischke et al., APH N.S. Heavy Ion Physics (1995)

EOS of quark phase from NJL

\[ \Omega_{NJL} = -2N_c \sum_{i=u,d,s} \int_0^{\Lambda} \frac{d^3p}{(2\pi)^3} \left[ E_i + T \ln(1 + e^{-\beta(E_i-\mu_i)}) \right] 
+ T \ln(1 + e^{-\beta(E_i+\mu_i)}) \right] + G_S(\sigma_u^2 + \sigma_d^2 + \sigma_s^2) 
- 4K\sigma_u\sigma_d\sigma_s - \frac{1}{3}G_V(\rho_u + \rho_d + \rho_s)^2 \]

\[ P = -\Omega_{NJL} \]

Pressure in the temperature-density (T-n) plane

- \( G_V = 1.1G \)
- \( G_V = 0 \)
- \( G_V = -1.1G \)?
Directed flow from AMPT+NJL
preliminary

Fitting problem?

large splitting

overestimate splitting?
Spin effects on high-energy nuclear reactions - polarization

$\Lambda$ polarization

$$\bar{P}_H = \frac{8}{\pi \alpha} \frac{\langle \sin(\phi_P^* - \Psi_{EP}^{(1)}) \rangle}{R_{EP}^{(1)}}$$

$$\frac{d\sigma}{d\Omega} \sim 1 + \alpha_H P_H \cos \theta \quad P_H = \frac{\Lambda^\uparrow - \Lambda^\downarrow}{\Lambda^\uparrow + \Lambda^\downarrow}$$

Impact parameter $|b|$

$$n_{re} = \frac{p_{in} \times b}{|p_{in} \times b|}$$

perpendicular to the reaction plane


STAR, arXiv:1701.06657

Larger $\bar{\Lambda}$ spin polarization than $\Lambda$?

Vorticity leads to same $\Lambda(\bar{\Lambda})$ polarization


H. Li, L.G. Pang, Q. Wang, and X.L. Xia, arXiv: 1704.01507

Y.F. Sun and C.M. Ko, arXiv: 1706.09467
Spin dynamics in intermediate- and low-energy heavy-ion collisions

Frontiers of Physics

Density evolution in HIC

Spin density

Density gradient

Flux density

invited review, selected as cover story


SIBUU12

Boltzmann-Uehling-Uhlenbeck equation

\[
\frac{df}{dt} + v \cdot \nabla f - \nabla U \cdot \nabla f = - \int \frac{d^3p_1 d^3p_2 d^3\tilde{p}}{(2\pi)^9} \sigma_{12} f f (1 - f_1)(1 - f_2) \\
- f_1 f_2 (1 - f) (1 - f_1)(2\pi)^3 \delta^3 (p_1 + p_2 - p_1 - p_2)
\]

Test-particle method

C.Y. Wong, PRC 25, 1460 (1982)

Equation of motion

\[
\frac{dr}{dt} = \frac{\tilde{p}}{m}, \quad \frac{dp}{dt} = -\nabla U
\]

Spin-dependent Boltzmann-Uehling-Uhlenbeck eq

\[
\dot{\varepsilon}(\vec{r}, \tilde{p}) = \varepsilon(\vec{r}, \tilde{p}) \hat{I} + \tilde{h}(\vec{r}, \tilde{p}) \cdot \hat{\sigma}, \\
\dot{f}(\vec{r}, \tilde{p}) = f_0(\vec{r}, \tilde{p}) \hat{I} + \tilde{g}(\vec{r}, \tilde{p}) \cdot \hat{\sigma}.
\]

Spin dynamics in intermediate- and low-energy heavy-ion collisions


Spin-dependent equation of motion

\[
\frac{dr}{dt} = \frac{\tilde{p}}{m} + \nabla \rho (\varepsilon + \tilde{h} \cdot \tilde{n}) \quad \frac{dp}{dt} = -\nabla (\varepsilon + \tilde{h} \cdot \tilde{n})
\]

\[
\frac{d\tilde{n}}{dt} = 2 \tilde{h} \times \tilde{n} \quad \tilde{N} \quad \text{spin expectation direction}
\]
Spin polarization from vector interactions

Consider quark spin in NJL Hamiltonian

\[ H \approx \sqrt{M_i^2 + (\vec{p} - \vec{A})^2} + A_0 - \frac{\vec{\sigma} \cdot (\nabla \times \vec{A})}{2\sqrt{M_i^2 + (\vec{p} - \vec{A})^2}} \]

Spin-orbit coupling

Comparing the real magnetic field from two calculation methods

\[ e\vec{A}_m(t, \vec{r}) = \frac{e^2}{4\pi} \sum_n Z_n \frac{\vec{v}_n \times \vec{R}_n}{R_n - \vec{v}_n \cdot \vec{R}_n} \quad e\vec{B} = \nabla \times e\vec{A}_m \]

\[ \vec{B}(t, \vec{r}) = \frac{e}{4\pi} \sum_n Z_n \frac{\vec{v}_n \times \vec{R}_n}{(R_n - \vec{v}_n \cdot \vec{R}_n)^3} (1 - v_n^2) \]

B\(_i\): baryon number; Q\(_i\): charge number

\[ A_0 = B_i g_V \rho_0 + Q_i e\varphi, \]
\[ \vec{A} = B_i g_V \vec{\rho} + Q_i e\vec{A}_m, \]
Effects on the different baryon and antibaryon polarizations

\[
P = \frac{N^\uparrow - N^\downarrow}{N^\uparrow + N^\downarrow}
\]

Further incorporating vorticity effects, polarizations of all particles are expected to increase.

\[
\Lambda^\uparrow(\downarrow) \sim uds^\uparrow(\downarrow)
\]

\[
\Lambda^\downarrow(\uparrow) \sim \bar{uds}^\downarrow(\uparrow)
\]

\[
P_{\Lambda} > P_{\overline{\Lambda}}
\]

\[
B^\uparrow(\downarrow) \sim qqq^\uparrow(\downarrow)
\]

\[
\overline{B}^\uparrow(\downarrow) \sim \overline{qqq}^\downarrow(\downarrow)
\]

\[
P_B < P_{\overline{B}}
\]

Baryon spin: quark spin, gluon spin, quark angular momentum, ...

Zhang-Zhu Han and JX, arXiv:1707.07262 [nucl-th]
Rapidity dependence of polarization

It is of great interest to confirm experimentally whether baryons have a stronger polarization than antibaryons at large rapidities.

Zhang-Zhu Han and JX, arXiv:1707.07262 [nucl-th]
Summary

Effects of mean-field potentials on

- Elliptic flow splitting between particles and antiparticles
- Directed flow splitting between particles and antiparticles
- HBT correlation, radii, and chaoticity parameter
- Different polarizations of quarks (baryons) and antiquarks (antibaryons)
对多相输运模型（AMPT）的改进

原始AMPT模型的结构

改进建AMPT模型的结构

适用于LHC能区、RHIC能区

适用于RHIC束流能量扫描能区、FAIR能区

Needs further improvements on

- Coalescence to reproduce better baryon/antibaryon ratio
- Finite thickness for initial partons and spectator nucleons
- Initial energy conservation
- and others ...
AMPT coalescence

Histogram of coalescence distance in coordinate space (dr) and in momentum space (dp)

Single event coalescence, dr criterion

Mix event coalescence, dr criterion

Single event coalescence, dr*dp criterion

Mix event coalescence, dr*dp criterion
$v_2$ from different coalescence treatments

Elliptic flow of initial hadrons from different coalescence treatments:

Au+Au@7.7GeV
20-30%
all the initial hadrons $|y|<1$
global hadronization

V2 from blast wave + Wigner function coalescence with dr+dp (Gaussian in r and p space), dr (Gaussian only in r space), dp (Gaussian only in p space)
Correct ART/AMPT charge violation

1) Correct inelastic channels where charges are not conserved

Subroutine for inelastic scatterings A+B->C+D

Z1 = charge of A+B

Sample branches for Inelastic scatterings ...

Z2 = C+D

If(Z1.ne.Z2)

2) Correct charge violation by turning off K^0 and K^0-bar
<table>
<thead>
<tr>
<th>$\sqrt{s}_{NN}$ (GeV)</th>
<th>$\lambda$</th>
<th>$R_o$ (fm)</th>
<th>$R_s$ (fm)</th>
<th>$R_l$ (fm)</th>
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<tr>
<td>7.7</td>
<td>cascade 0.673 ± 0.007</td>
<td>5.58 ± 0.03</td>
<td>4.75 ± 0.03</td>
<td>4.39 ± 0.03</td>
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<td></td>
<td>mean-field 0.719 ± 0.007</td>
<td>6.16 ± 0.04</td>
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<td>5.51 ± 0.04</td>
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<td></td>
<td>expt. 0.532 ± 0.007</td>
<td>5.57 ± 0.13</td>
<td>4.93 ± 0.10</td>
<td>5.01 ± 0.11</td>
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<td>19.6</td>
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<td></td>
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<td>4.84 ± 0.03</td>
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<td>5.99 ± 0.04</td>
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<td>39</td>
<td>cascade 0.655 ± 0.007</td>
<td>5.63 ± 0.03</td>
<td>4.79 ± 0.03</td>
<td>4.55 ± 0.03</td>
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<td>mean-field 0.678 ± 0.008</td>
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<tr>
<td></td>
<td>expt. 0.491 ± 0.004</td>
<td>5.86 ± 0.07</td>
<td>4.97 ± 0.05</td>
<td>6.18 ± 0.08</td>
</tr>
</tbody>
</table>
Equations of motion for solving Boltzmann equation

Substitute

\[ f(rp, t) = \int \frac{dr_0 \, dp_0 \, ds}{(2\pi \hbar)^3} \exp\{is \cdot [p - P(r_0p_0s, t)]/\hbar\} \delta[r - R(r_0p_0s, t)]f(r_0p_0, t_0) \]

into the Boltzmann-Vlasov equation

\[ \frac{\partial f(rp, t)}{\partial t} + \frac{p}{m} \cdot \nabla_r f(rp, t) - \frac{2}{\hbar} \sin \left\{ \frac{\hbar}{2} \nabla_r V \cdot \nabla^f \right\} V(r, t) f(rp, t) = 0 \]

First term:

\[ \frac{\partial f(rp, t)}{\partial t} = \int \frac{dr_0 \, dp_0 \, ds}{(2\pi \hbar)^3} f(r_0p_0, t_0) \left[ \frac{(-is)}{\hbar} \cdot \frac{\partial P}{\partial t} \right] \]

\[ \times \exp\{is \cdot [p - P(r_0p_0s, t)]/\hbar\} \delta[r - R(r_0p_0s, t)] \]

\[ + \exp\{is \cdot [p - P(r_0p_0s, t)]/\hbar\} \]

\[ \times (\nabla_R \delta[r - R(r_0p_0s, t)]) \cdot \partial R(r_0p_0s, t)/\partial t \). \]
Noting that
\[ \nabla_R \delta[r - R(r_0 p_0 s, t)] = -\nabla_r \delta[r - R(r_0 p_0 s, t)] \]

So
\[ \int \frac{dr_0 \, dp_0 \, ds}{(2\pi\hbar)^3} f(r_0 p_0, t_0) \exp\{is \cdot [p - P(r_0 p_0 s, t)]/\hbar\} \times (-\nabla_r \delta[r - R(r_0 p_0 s, t)]) \cdot \partial R(r_0 p_0 s, t)/\partial t \]
\[ = -\frac{\partial R(r_0 p_0 s, t)}{\partial t} \cdot \nabla_r f(rp, t), \]

The potential term:
\[ \frac{2}{\hbar} \sin \left\{ \frac{\hbar}{2} \nabla_r V \cdot \nabla_f p \right\} V(r, t) f(rp, t) \]
\[ = \frac{1}{\hbar} \int \frac{dr_0 \, dp_0 \, ds}{(2\pi\hbar)^3} f(r_0 p_0, t_0) \times \exp\{is \cdot [p - P(r_0 p_0 s, t)]/\hbar\} \delta[r - R(r_0 p_0 s, t)] \times \frac{[V(r - \frac{s}{2}, t) - V(r + \frac{s}{2}, t)]}{i}. \]
Put everything together:

\[
\left[ -\frac{\partial R(r_0 p_0 s, t)}{\partial t} + \frac{p}{m} \right] \cdot \nabla_r f(r p, t) + \int \frac{d r_0 \, d p_0 \, d s}{(2\pi \hbar)^3} \, f(r_0 p_0, t_0) \left[ \frac{-i s}{\hbar} \cdot \frac{\partial P}{\partial t} \right. \\
- \frac{[V(r - \frac{s}{2}, t) - V(r + \frac{s}{2}, t)]}{i\hbar} \left. \right] \times \exp\{i s \cdot [p - P(r_0 p_0 s, t)]/\hbar\} \times \delta[r - R(r_0 p_0 s, t)] = 0.
\]

Equations of motion:

\[
\frac{\partial R}{\partial t} = \frac{p}{m},
\]

\[
s \cdot \frac{\partial P}{\partial t} = V \left( R - \frac{s}{2}, t \right) - V \left( R + \frac{s}{2}, t \right).
\]

Momentum-dependent potential:

One more term in BV equation

\[
\frac{\partial R}{\partial t} = \frac{p}{m} + \nabla_p V
\]

\[
\frac{\partial P}{\partial t} \approx -\nabla_R V(R, t)
\]

Calculate phase-space distribution function

\[
f(\tilde{r}, \tilde{p}; t) = \frac{1}{N_{TP}} \sum_{i=1}^{N_{TP} A} g[\tilde{r} - \tilde{r}_i(t)] \tilde{g}[\tilde{p} - \tilde{p}_i(t)]
\]
Euler-Lagrange equation,

\[ [\gamma^\mu (i \partial_\mu - A_\mu) - M_i] \psi_i = 0. \]  \hfill (8)

Space and time components of the vector potential,

\[ A_0 = B_i g_V \rho_0 + Q_i e \varphi, \]  \hfill (9)

\[ \vec{A} = B_i g_V \vec{\rho} + Q_i e \vec{A}_m, \]  \hfill (10)

with \( g_V = \frac{2}{3} G_V \), \( \rho_0 = \langle \bar{\psi} \gamma^0 \psi \rangle \) and

\( \vec{\rho} \equiv \langle \bar{\psi} \vec{\gamma} \psi \rangle \)

The scalar and vector potential of the real electromagnetic field.

\[ e \varphi(t, \vec{r}) = \frac{e^2}{4\pi} \sum_n Z_n \frac{1}{R_n - \vec{v}_n \cdot \vec{R}_n}, \]  \hfill (11)

\[ e \vec{A}_m(t, \vec{r}) = \frac{e^2}{4\pi} \sum_n Z_n \frac{\vec{v}_n}{R_n - \vec{v}_n \cdot \vec{R}_n}, \]  \hfill (12)

Calculated from Eq. (8),

\[ i \partial_t \psi_i = [\gamma^0 \gamma^k (-i \partial_k + A_k) + \gamma^0 M_i + A_0] \psi_i. \]  \hfill (13)

Hamiltonian operator,

\[ \hat{H} = \gamma^0 \gamma^k (-\hat{p}_k + A_k) + \gamma^0 M_i + A_0, \]  \hfill (14)

Single-particle Hamiltonian,

\[ H = \sqrt{(\vec{p} - \vec{A})^2 + M_i^2 - \vec{\sigma} \cdot (\nabla \times \vec{A}) + A_0}. \]  \hfill (15)

\[ \vec{\sigma} \cdot (\nabla \times \vec{A}) \ll (\vec{p} - \vec{A})^2 + M_i^2 \]

Single-particle Hamiltonian can be further expressed as,

\[ H \approx \sqrt{M_i^2 + (\vec{p} - \vec{A})^2 + A_0} - \frac{\vec{\sigma} \cdot (\nabla \times \vec{A})}{2 \sqrt{M_i^2 + (\vec{p} - \vec{A})^2}}. \]  \hfill (16)
B. Equations of motion for partons and the extended AMPT model

\[ \dot{r} = \mathbf{\nabla}_p H, \]
\[ \dot{p} = -\mathbf{\nabla} H, \]
\[ \dot{\sigma} = -i[\bar{\sigma}, H]. \]

\[ \frac{d r_k}{d t} = \frac{p_{k}^*}{E_i} + \frac{1}{2} \frac{p_{k}^*}{E_i^*} [\bar{\sigma} \cdot (\nabla \times \bar{A})], \]
\[ \frac{d p_{k}^*}{d t} = -M_i \frac{\partial M_i}{\partial r_k} + \frac{p_{j}^*}{E_i^*} \frac{\partial A_j}{\partial r_k} - \frac{\partial A_0}{\partial r_k} - \frac{\partial A_k}{\partial t} \]
\[ - \dot{r}_j \frac{\partial A_k}{\partial r_j} - \frac{1}{2} [\bar{\sigma} \cdot (\nabla \times \bar{A})] \frac{M_i}{E_i^*} \frac{\partial M_i}{\partial r_k} \]
\[ + \frac{1}{2} [\bar{\sigma} \cdot (\nabla \times \bar{A})] \frac{p_{j}^*}{E_i^*} \frac{\partial A_j}{\partial r_k}, \]
\[ + \frac{\bar{\sigma}}{2E_i^*} \cdot (\nabla \times \frac{\partial A}{\partial r_k}) \]
\[ \frac{d \sigma}{d t} = \frac{\sigma \times (\nabla \times \bar{A})}{E_i^*}, \]

Detailed EOMs