Workshop on AMPT for relativistic heavy-ion collisions
Sichuan University, Jul. 24-27, 2017

An extended AMPT model with mean-field potentials

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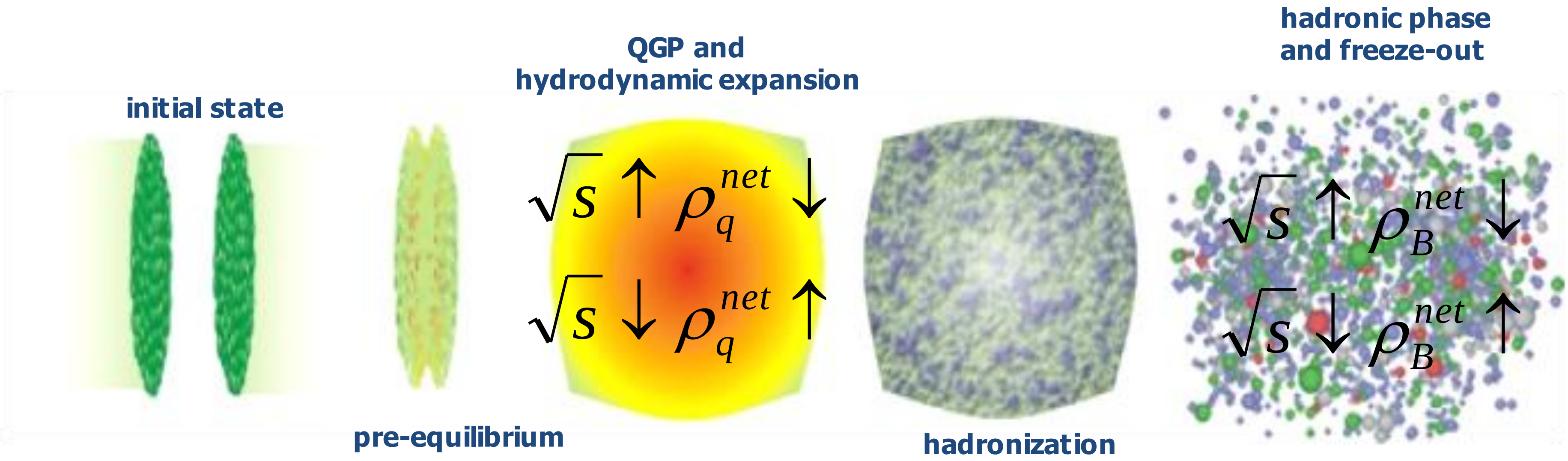
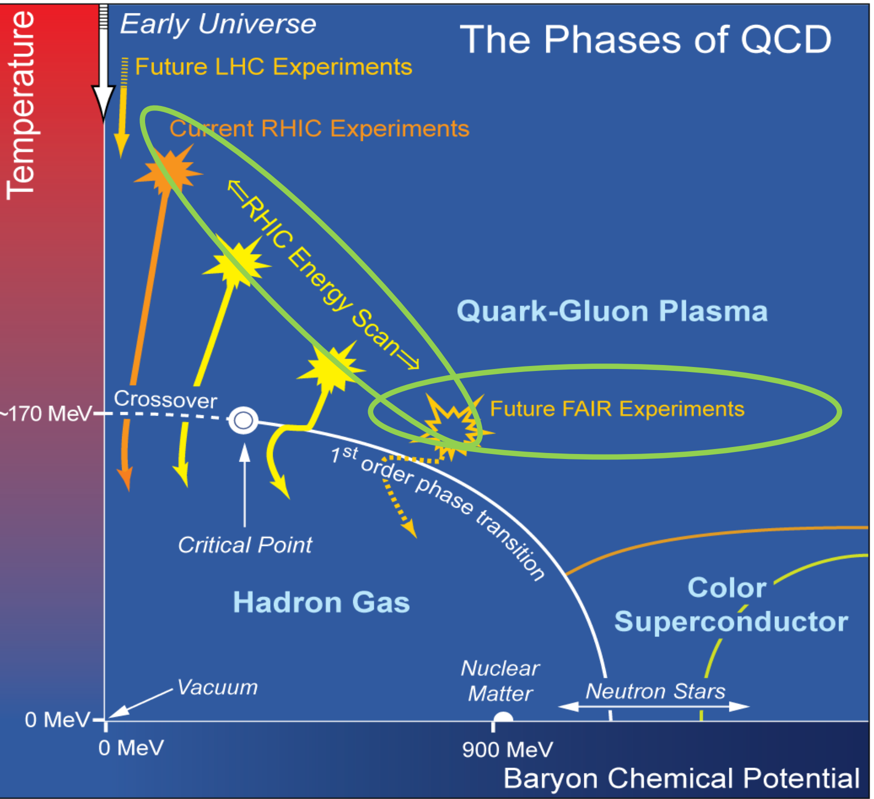
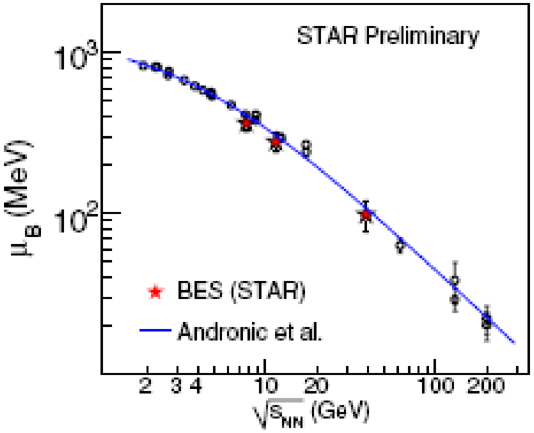
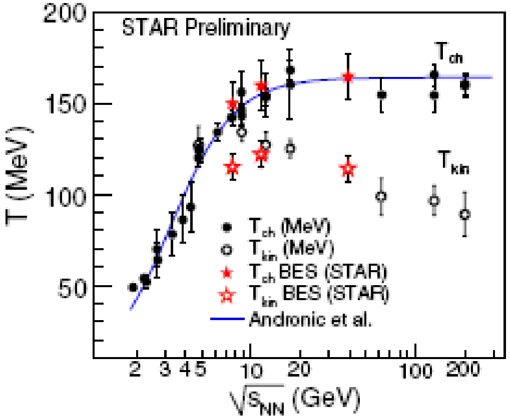
Chun-Jian Zhang (SINAP), He Liu (SINAP), Zhang-Zhu Han (SINAP), Chong-Qiang Guo (SINAP)









Search for signals of critical point at finite μ_B!

RHIC-BES:

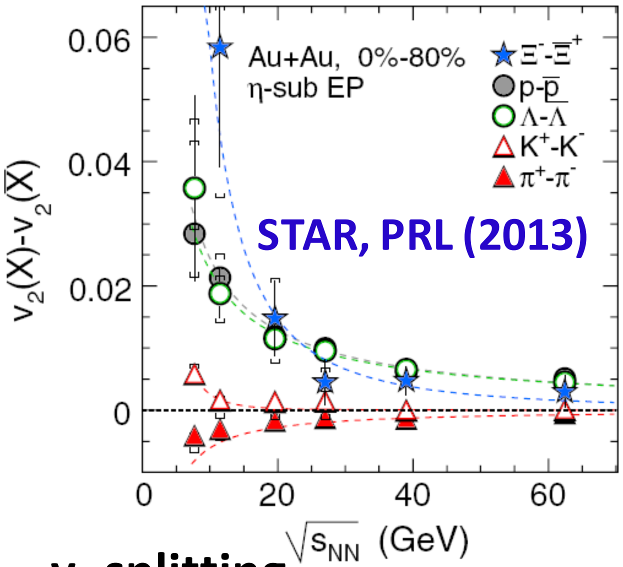
$\sqrt{s} \sim 7.7\text{-}39$ GeV

FAIR-CBM:

$\sqrt{s}$ <12 GeV

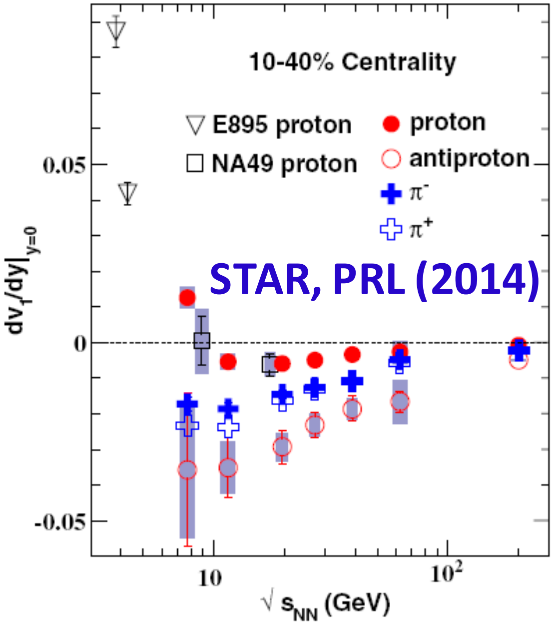
Highlights in RHIC-BES I

v_2 splitting



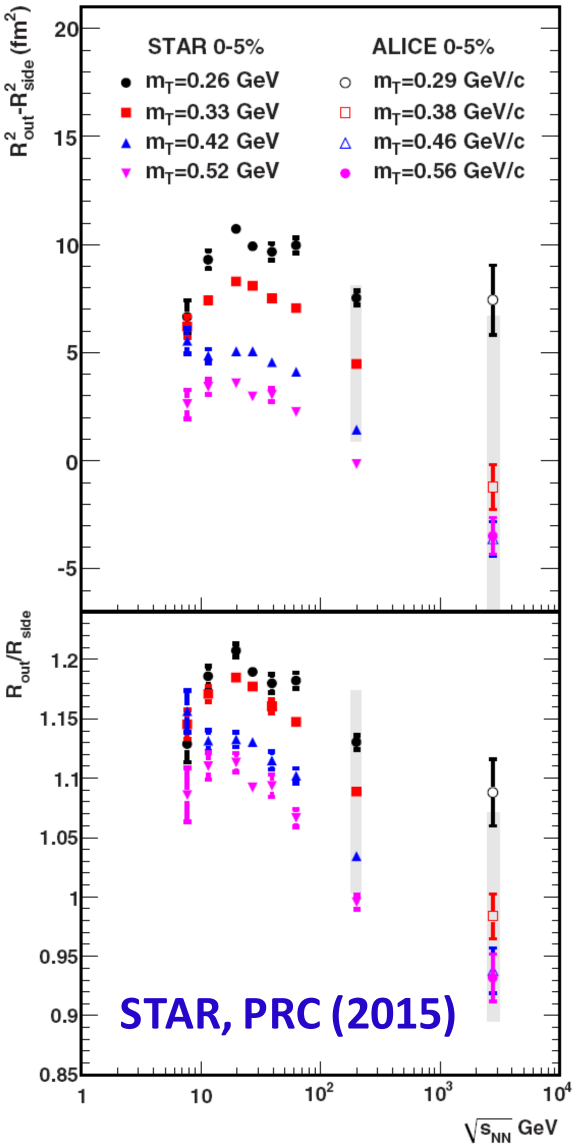
STAR, PRL (2013)

v_1 splitting



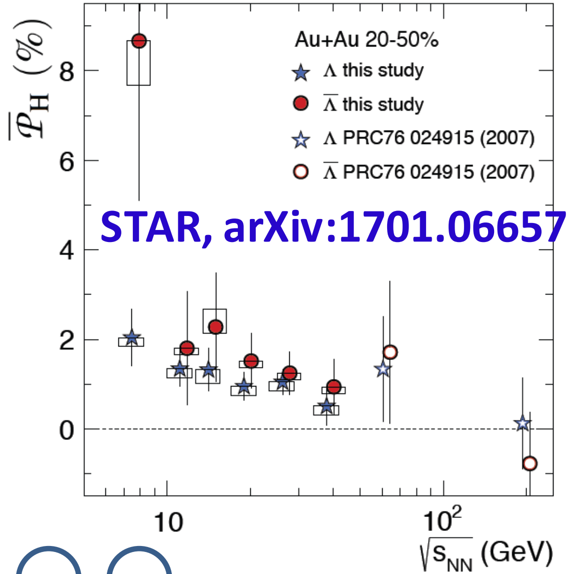
STAR, PRL (2014)

HBT correlation

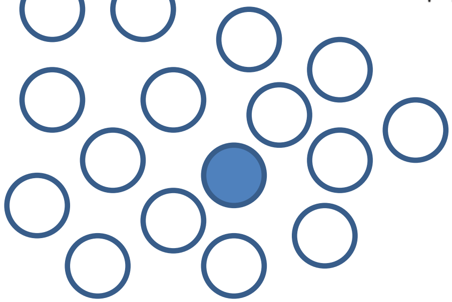


STAR, PRC (2015)

Baryon-Antibaryon polarization



STAR, arXiv:1701.06657



$$U_i \sim \sum_{i\neq j} V_{ij} + \ldots$$

Effects of mean-field potentials on these observables?

Transport model simulations of intermediate-energy heavy-ion collisions

Heavy-ion experiments

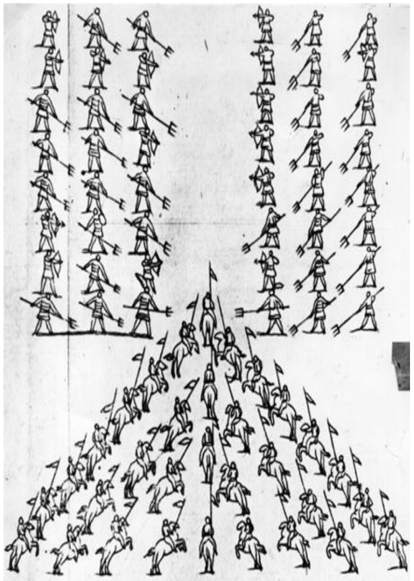
Transport simulations

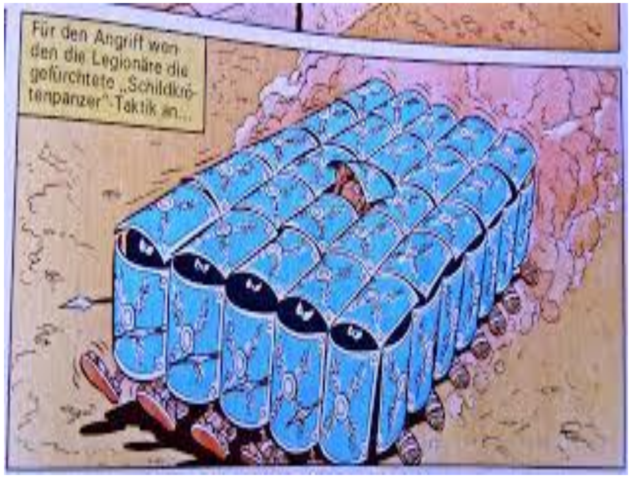
Mean-field potential

Nuclear EOS, E_{sym}

Main approaches:

Boltzman transport and Quantum Molecular Dynamics





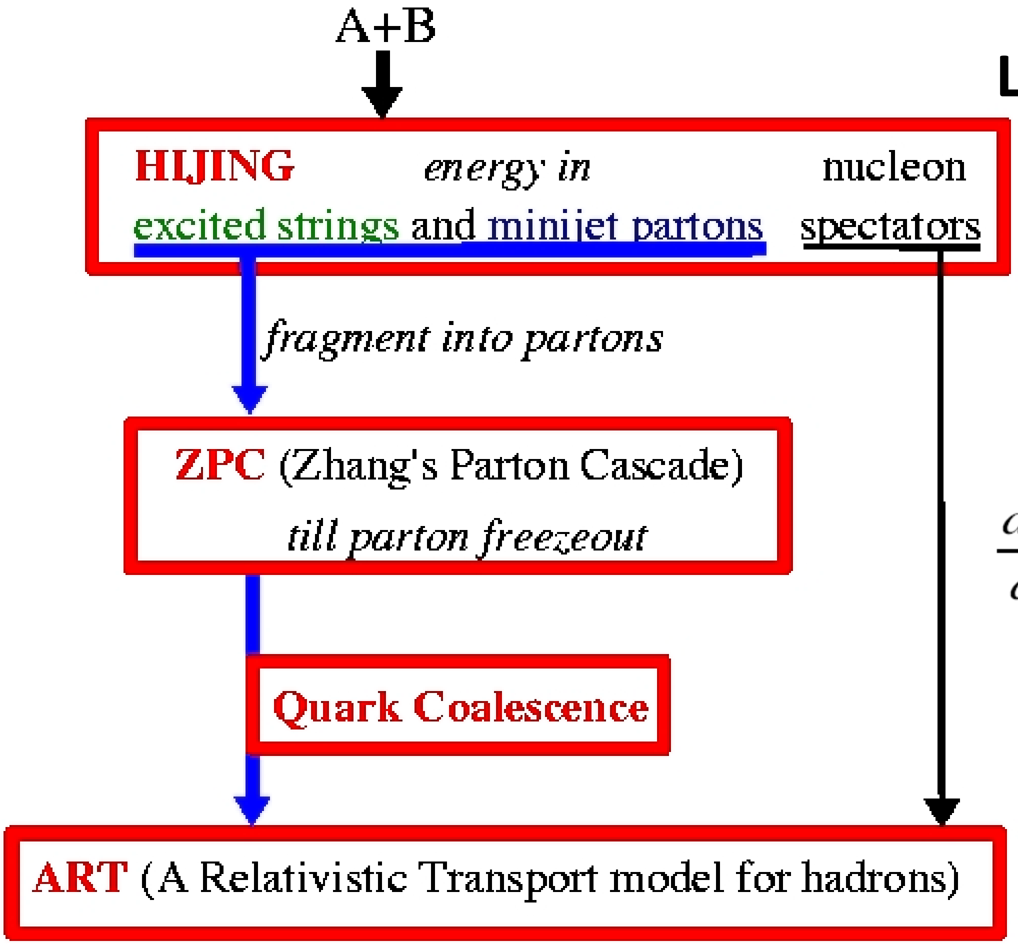
Initialization

Mean-field potential

NN scatterings

A multiphase transport (AMPT) model with string melting

Structure of AMPT model with string melting



A+B

HIJING *energy in* nucleon

excited strings and minijet partons spectators

fragment into partons

ZPC (Zhang's Parton Cascade)

till parton freezeout

Quark Coalescence

ART (A Relativistic Transport model for hadrons)

Lund string fragmentation function

$$f(z) \approx z^{-1}(1-z)^{a}\exp\left[-\frac{b(m^2+p_t^2)}{z}\right]$$

z : light-cone momentum fraction

Parton scattering cross section

$$\frac{d\sigma}{dt} \approx \frac{9\pi\alpha^2}{2s^2}\left(1+\frac{\mu^2}{s}\right)\left(\frac{1}{t-\mu^2}\right)^2, \quad \sigma \approx \frac{9\pi\alpha^2}{2\mu^2}$$

α: strong coupling constant

μ: screening mass

a, b: particle multiplicity

α, μ: partonic interaction

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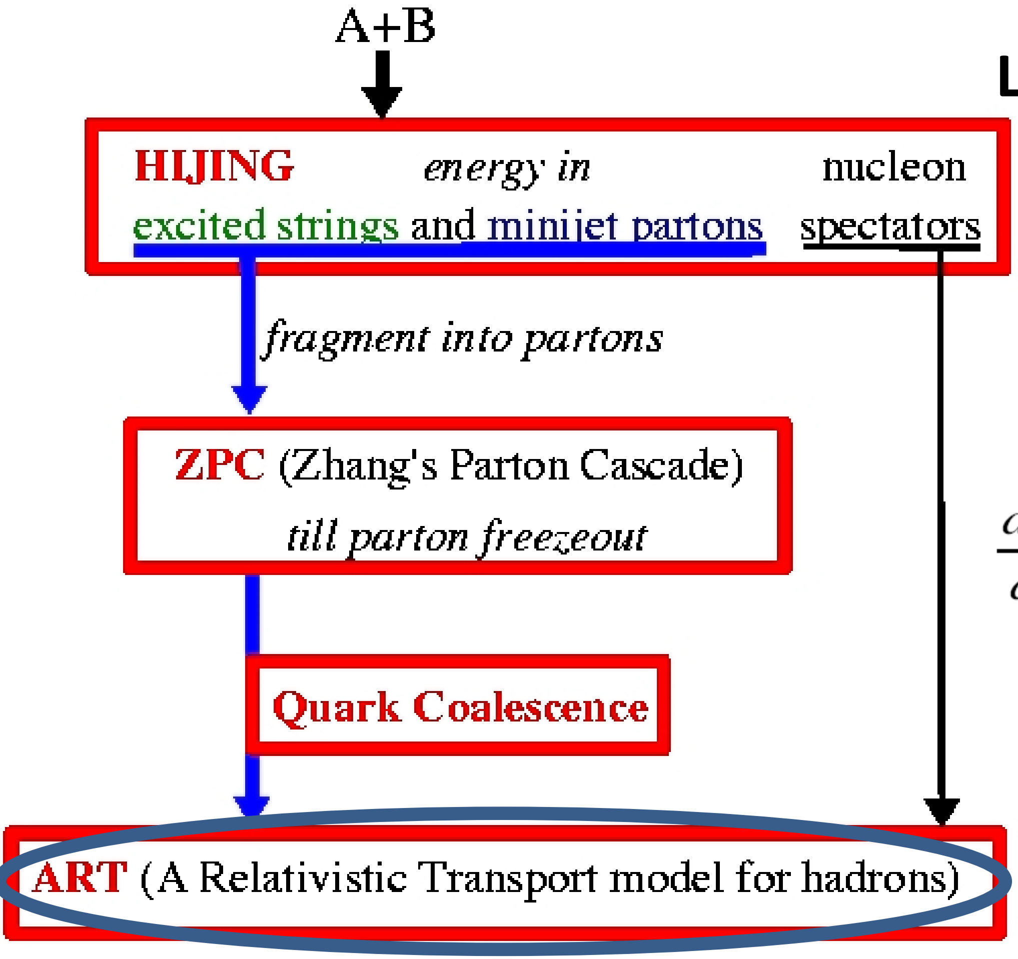
Structure of AMPT model with string melting



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Turn on hadronic mean-field potentials

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α: strong coupling constant

μ: screening mass

a, b: particle multiplicity

α, μ: partonic interaction

hadronic potentials for particles and antiparticles

Nucleon and antinucleon potential

$$\mathcal{L} = \overline{\psi}[i\gamma_\mu\partial^\mu - m - g_\sigma\sigma - g_\omega\gamma_\mu\omega^\mu]\psi + \frac{1}{2}(\partial^\mu\sigma)^2 - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}b\sigma^3 - \frac{1}{4}c\sigma^4 - \frac{1}{4}(\partial_\mu\omega^\nu - \partial_\nu\omega^\mu)^2 + \frac{1}{2}m_\omega^2\omega^{\mu 2},$$

G.Q. Li, C.M. Ko, X.S. Fang, and Y.M. Zheng, PRC (1994)

$$\Sigma_s = g_\sigma\langle\sigma\rangle, \qquad \Sigma_{v\mu} = g_\omega\langle\omega_\mu\rangle$$

$$U_{N,\bar{N}} = \Sigma_s(\rho_B, \rho_{\bar{B}}) \pm \Sigma_v^0(\rho_B, \rho_{\bar{B}})$$

$$U_{\Lambda,\overline{\Lambda}} \sim \frac{2}{3}U_{N,\overline{N}}, U_{\Xi,\overline{\Xi}} \sim \frac{1}{3}U_{N,\overline{N}}$$

Vector potential changes sign for antiparticles!

(e$^+$e$^-$ exchange γ)

Kaon and antikaon potential

$$\omega_{K,\bar{K}} = \sqrt{m_K^2 + p^2 - a_K\rho_s + (b_K\rho_B^{\rm net})^2} \pm b_K\rho_B^{\rm net}$$

$$U_{K(\overline{K})} = \omega_{K(\overline{K})} - \omega_0 \qquad \omega_0 = \sqrt{m_K^2 + p^2}$$

G.Q. Li, C.H. Lee, and G.E. Brown, PRL (1997); NPA (1997)

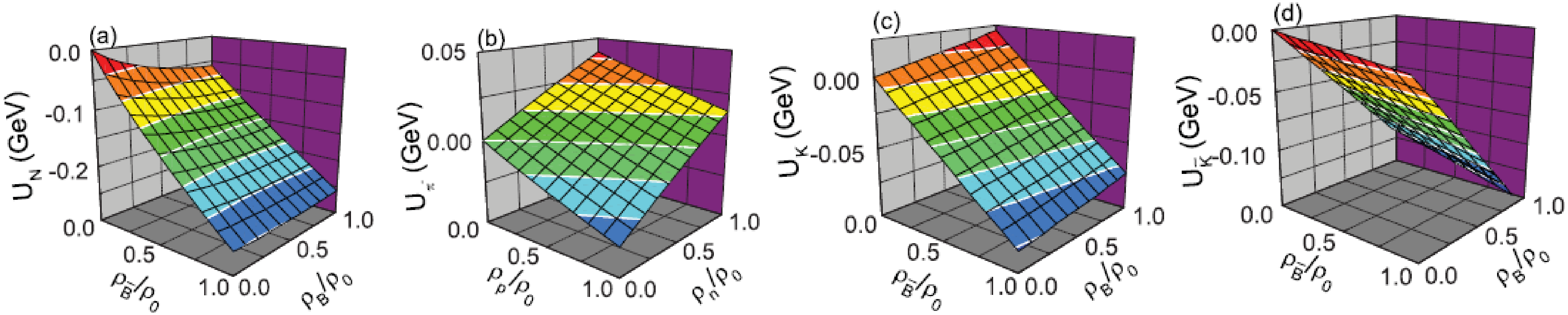
Pion s-wave potential

$$\begin{aligned}
\Pi_s^-(\rho_p, \rho_n) &= \rho_n[T_{\pi N}^- - T_{\pi N}^+] - \rho_p[T_{\pi N}^- + T_{\pi N}^+] \\
&\quad + \Pi_{\rm rel}^-(\rho_p, \rho_n) + \Pi_{\rm cor}^-(\rho_p, \rho_n) \\
\Pi_s^+(\rho_p, \rho_n) &= \Pi_s^-(\rho_n, \rho_p) \\
\Pi_s^0(\rho_p, \rho_n) &= -(\rho_p + \rho_n)T_{\pi N}^+ + \Pi_{\rm cor}^0(\rho_p, \rho_n).
\end{aligned}$$

$$U_{\pi^{\pm 0}} = \Pi_s^{\pm 0}/(2m_\pi)$$

N. Kaiser and W. Weise, PLB (2001)

hadronic potentials for particles and antiparticles



In baryon-rich and neutron-rich matter:

Introduced with test-particle method

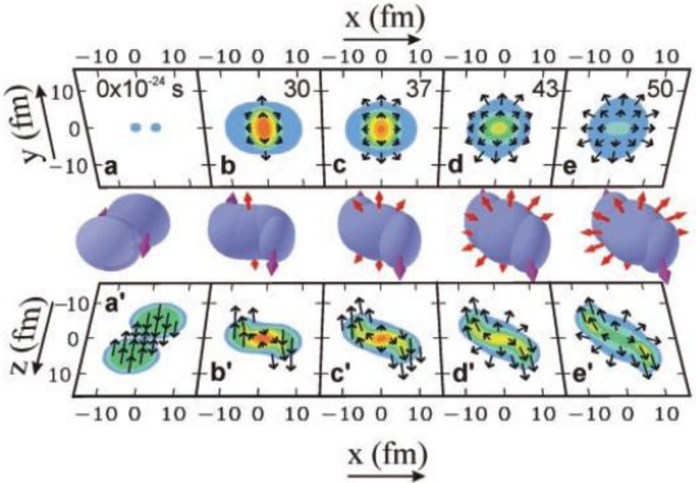
- **Baryon potential: weakly attractive**
- **Antibaryon potential: deeply attractive**
- **K^+ potential: weakly repulsive**
- **K^- potential: deeply attractive**

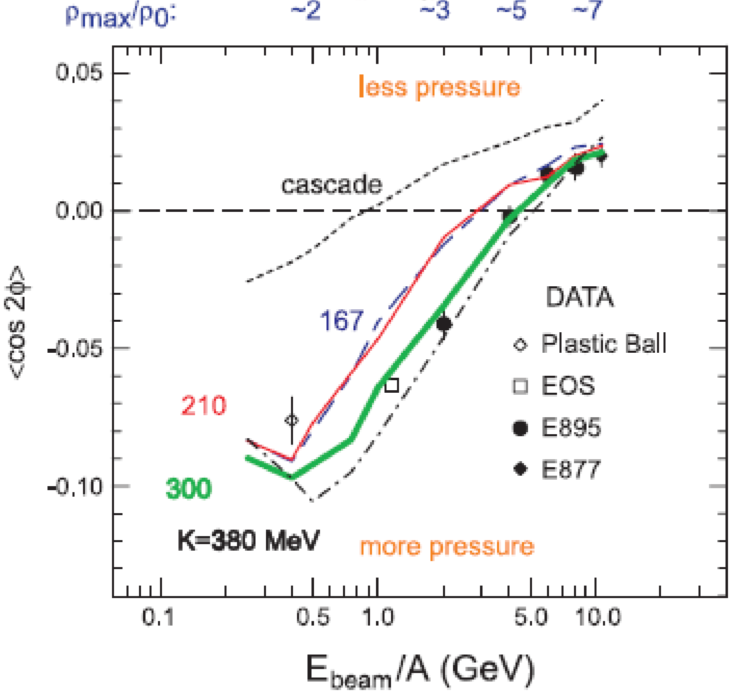
Sub threshold particle production

- **π^+ potential: weakly attractive**
- **π^- potential: weakly repulsive**

Chiral perturbation theory

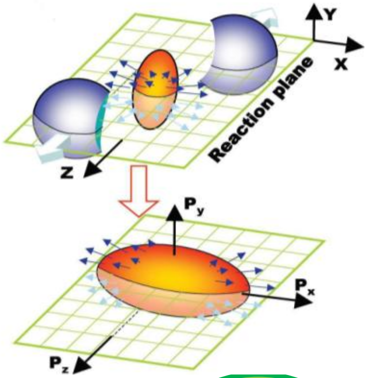
Effects of mean-field potentials on elliptic flow





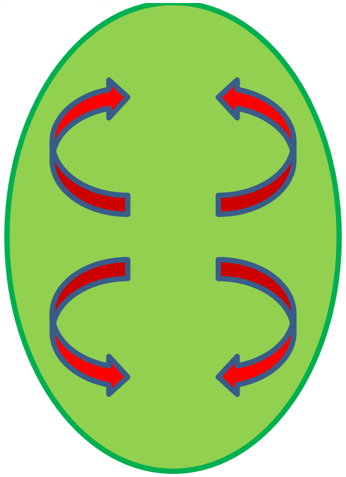
P. Danielewicz, R. Lacey,
and W. G. Lynch, Science (2002).

$$v_2 = \frac{\left\langle p_x^2 - p_y^2 \right\rangle}{\left\langle p_x^2 + p_y^2 \right\rangle}$$



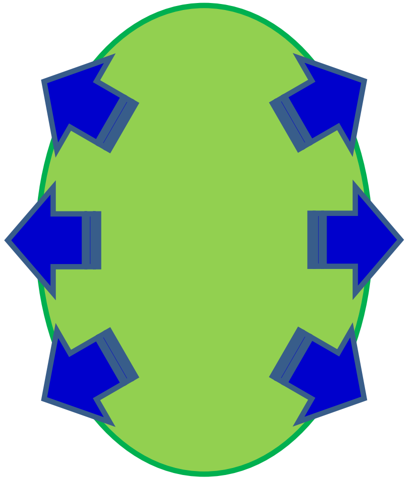
Particles with attractive potentials are more likely to be trapped in the system

v_2 decrease

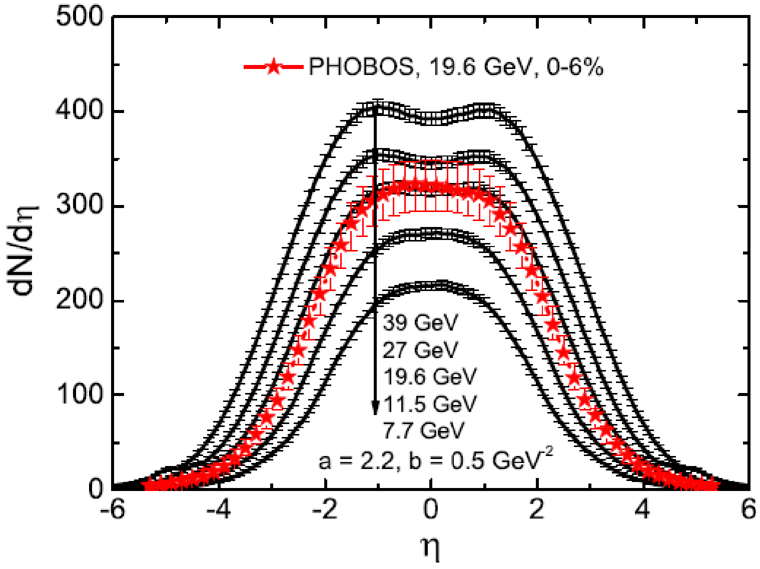


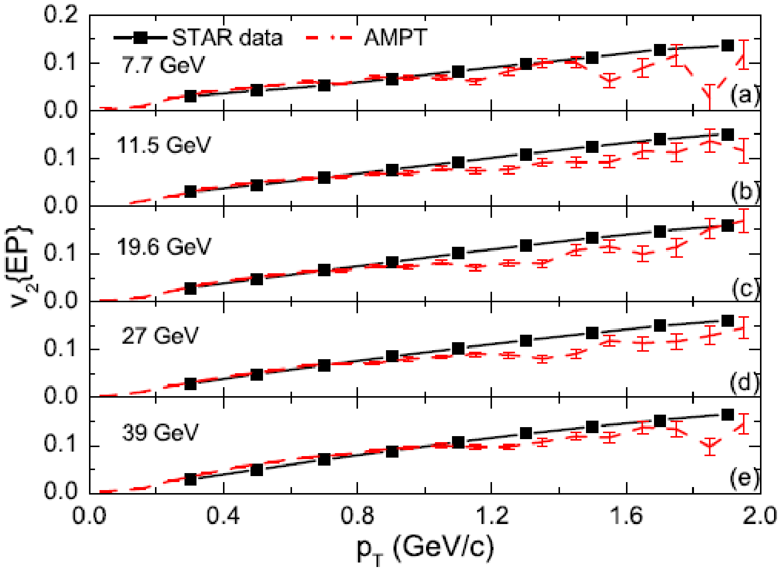
Particles with repulsive potentials are more likely to leave the system

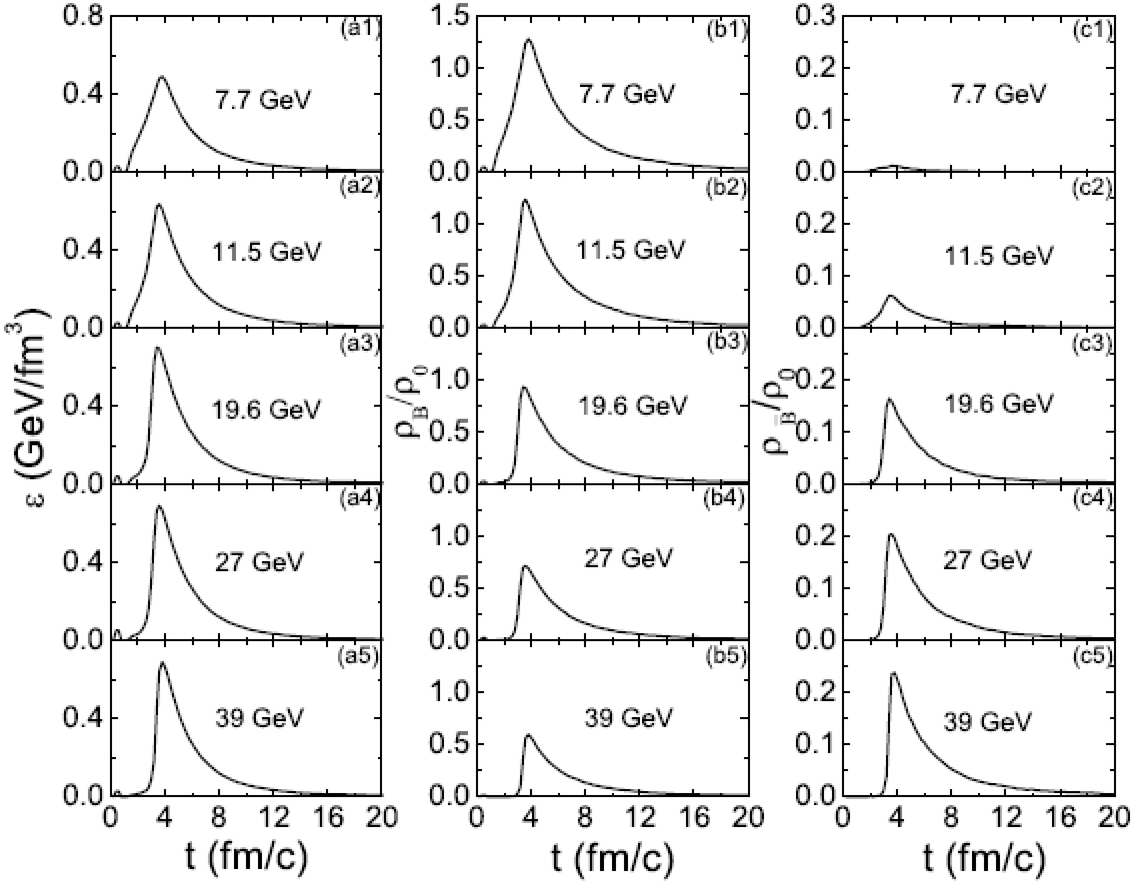
v_2 increase



Fit AMPT parameters at RHIC-BES energies



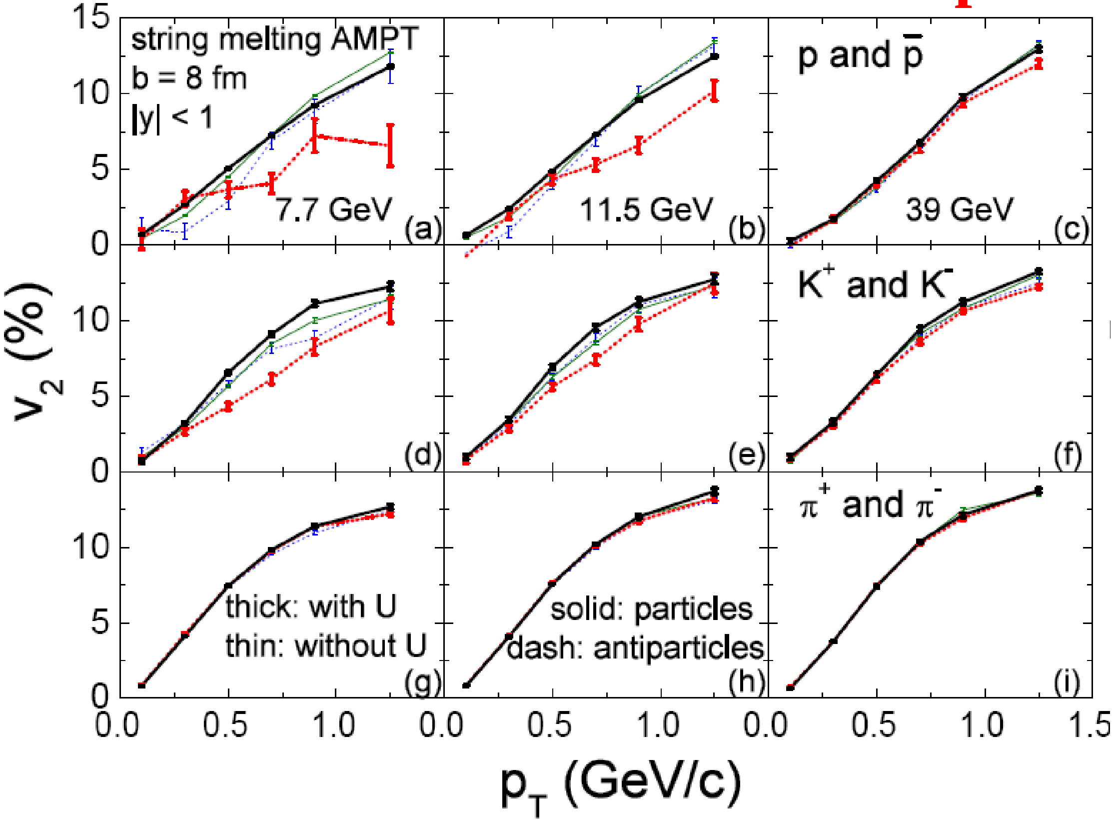


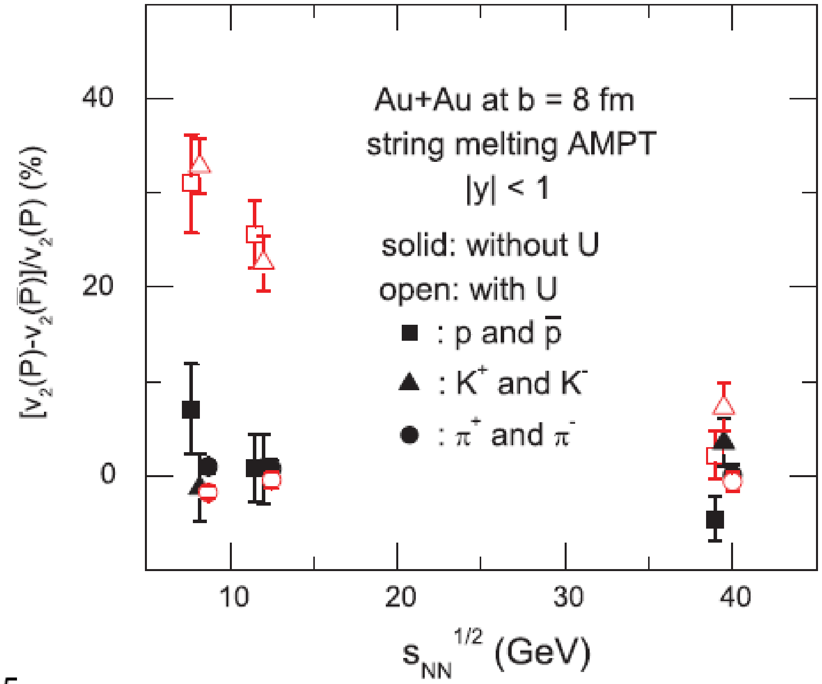


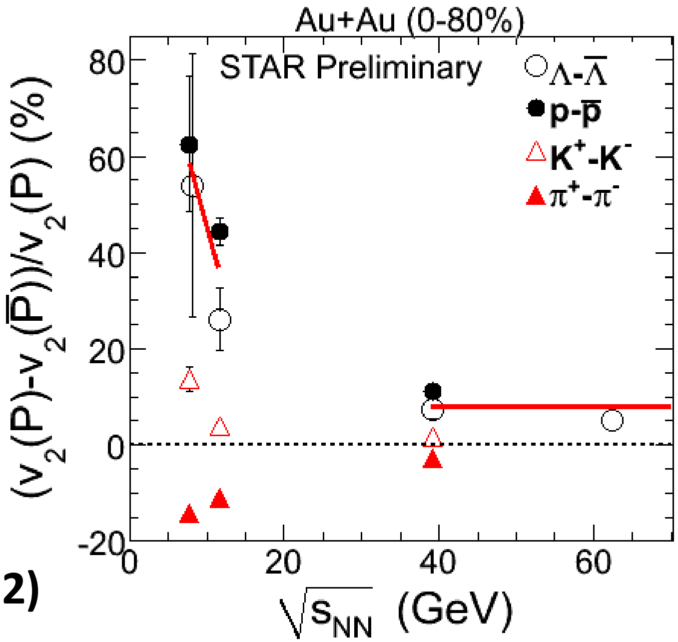
Lund string fragmentation parameters,
Parton scattering cross section, parton life time

Pseudorapidity distribution, elliptic flow,
energy density at chemical freeze-out

Effects of hadronic mean-field potentials on elliptic flow splitting







Qualitatively consistent

proton and antiproton: underestimate

K^+ and K^-: overestimate

π^+ and π^-: underestimate

JX, L. W. Chen, C. M. Ko, and Z. W. Lin, PRC 85, 041901(R) (2012)

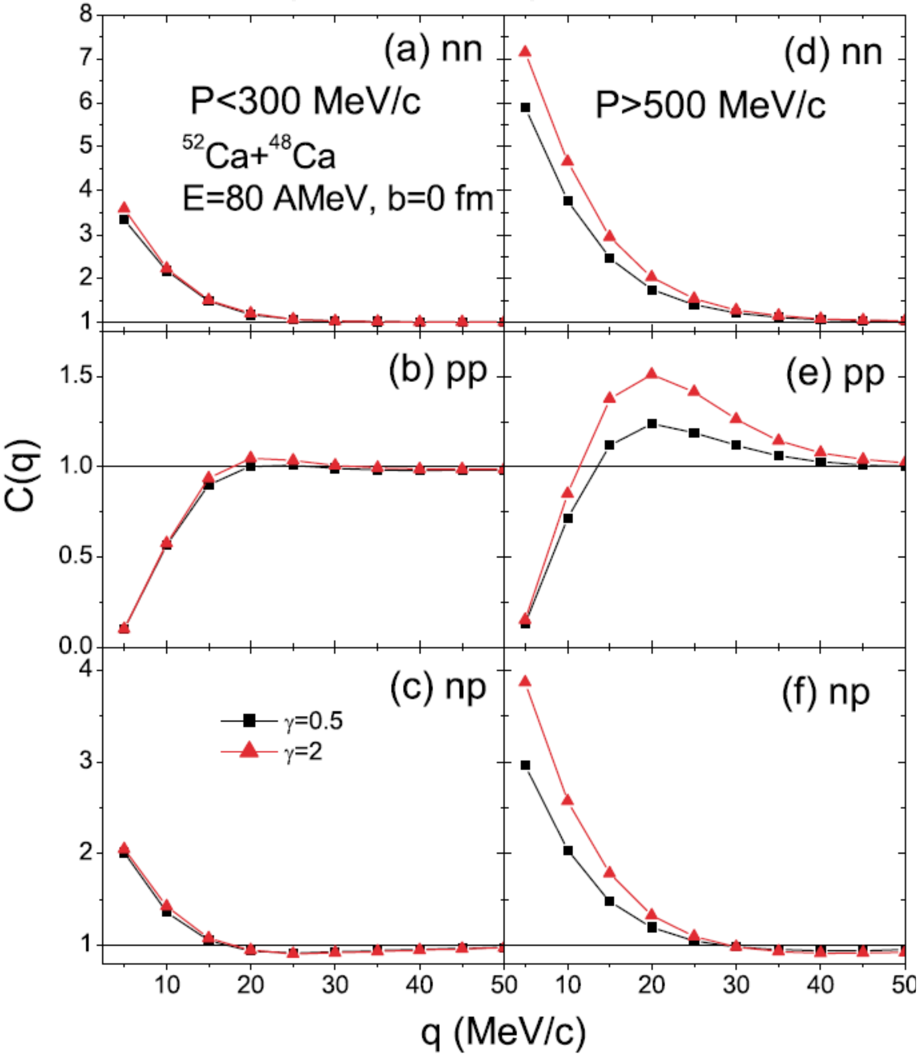
Effects of mean-field potentials on HBT correlation

a probe of neutron-proton U difference based on IBUU

$$E_{\text{sym}}(\rho) = E_{\text{sym}}(\rho_0) \cdot u^{\gamma}$$

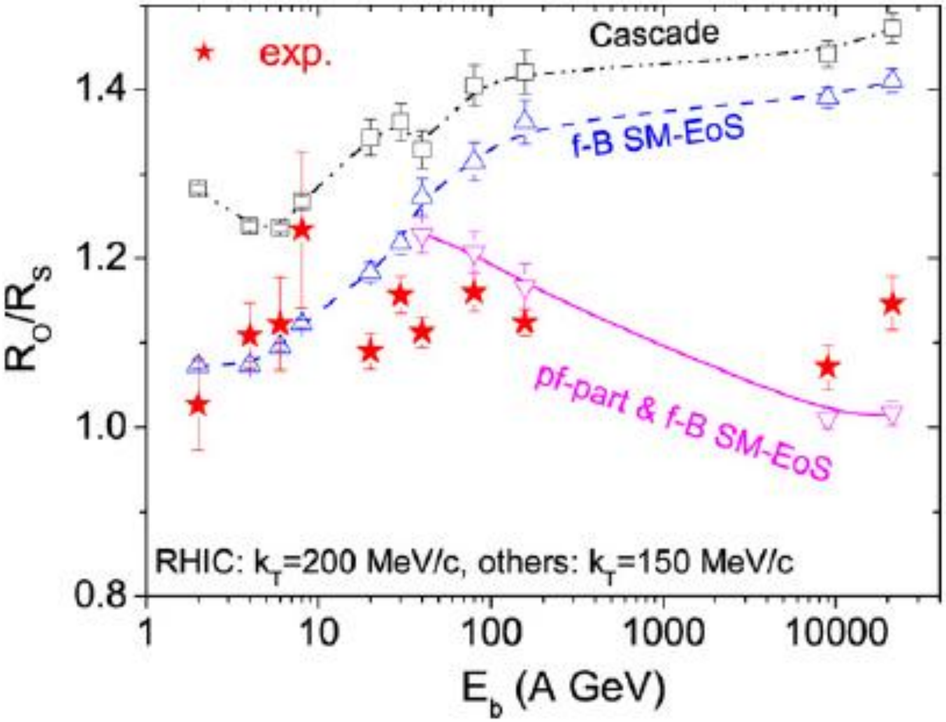
$$C(\mathbf{k}^*) = \frac{\int S(\mathbf{r}^*, \mathbf{k}^*) \left| \Psi^{S(+)}_{-\mathbf{k}^*}(\mathbf{r}^*) \right|^2 d^4\mathbf{r}^*}{\int S(\mathbf{r}^*, \mathbf{k}^*) d^4\mathbf{r}^*}$$

$$C(\vec{q}) = (1-\lambda) + \lambda K_{\text{coul}}(q_{\text{inv}}) \times \left(1 + e^{-q_o^2 R_o^2 - q_s^2 R_s^2 - q_l^2 R_l^2 - 2 q_o q_s R_{os}^2 - 2 q_o q_l R_{ol}^2}\right)$$



L.W. Chen, V. Greco, C.M. Ko, and B.A. Li, PRL (2003)

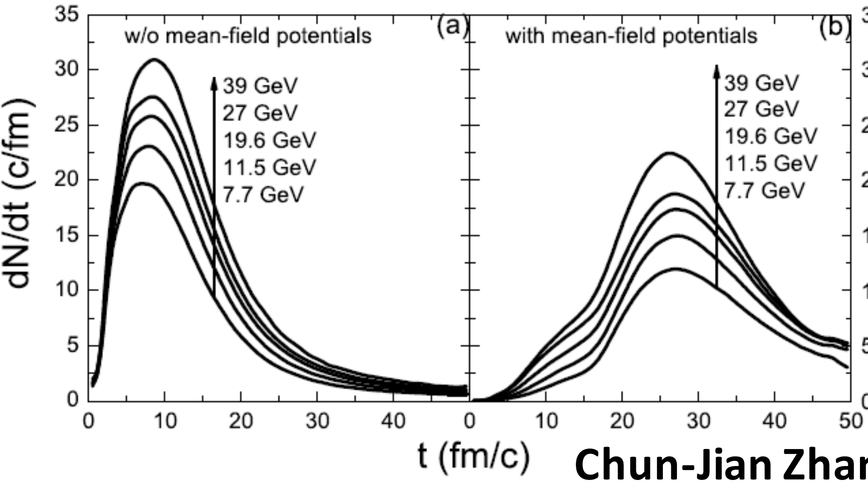
affect the HBT radii based on UrQMD



Q.F. Li, M. Bleicher, and H. Stocker, PLB (2008)

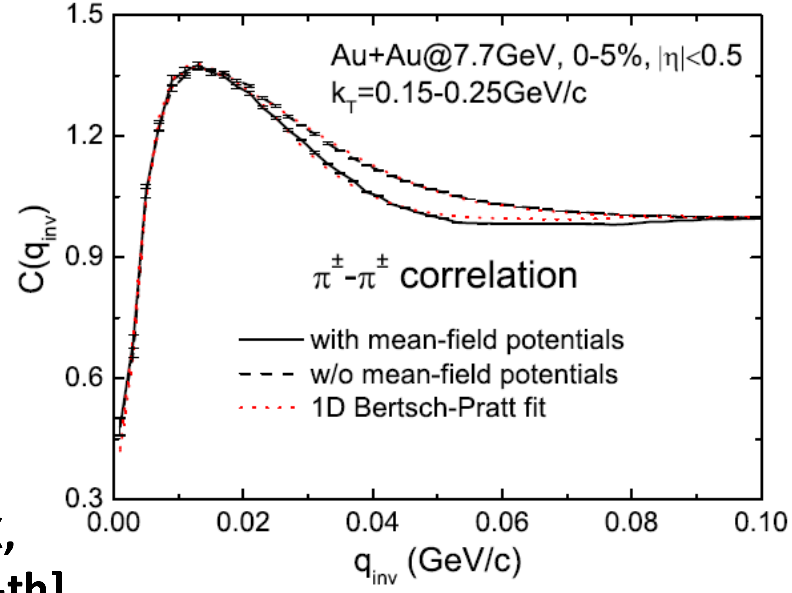
Effects of hadronic mean-field potentials on HBT correlation

later emission and broader emission time distribution

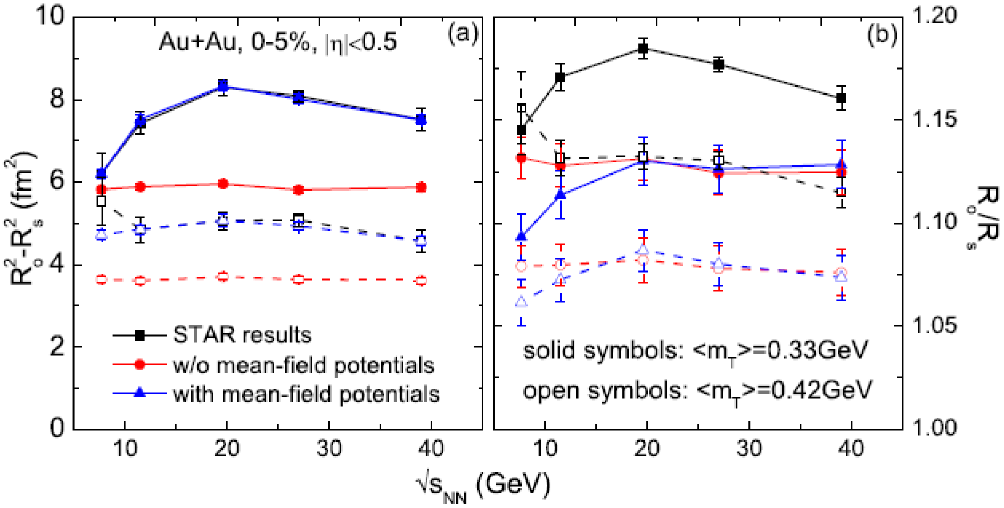


Chun-Jian Zhang and JX, arXiv:1707.07272 [nucl-th]

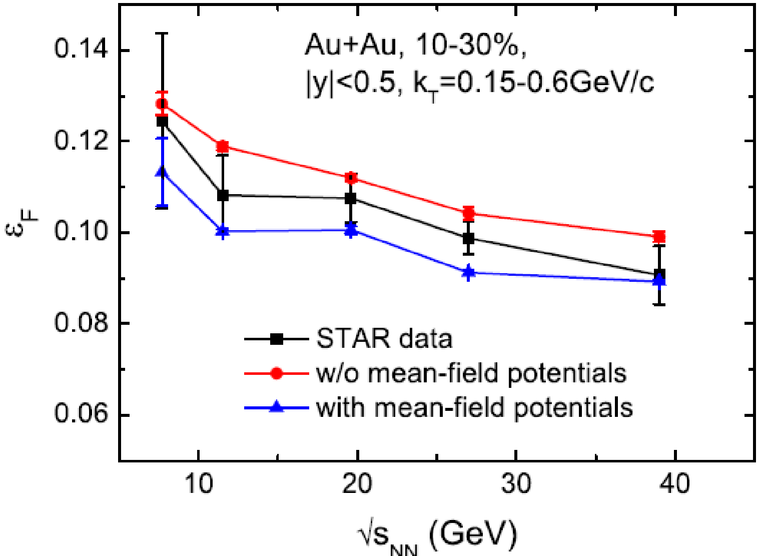
larger HBT radius



Affect R_{out} and R_{side}

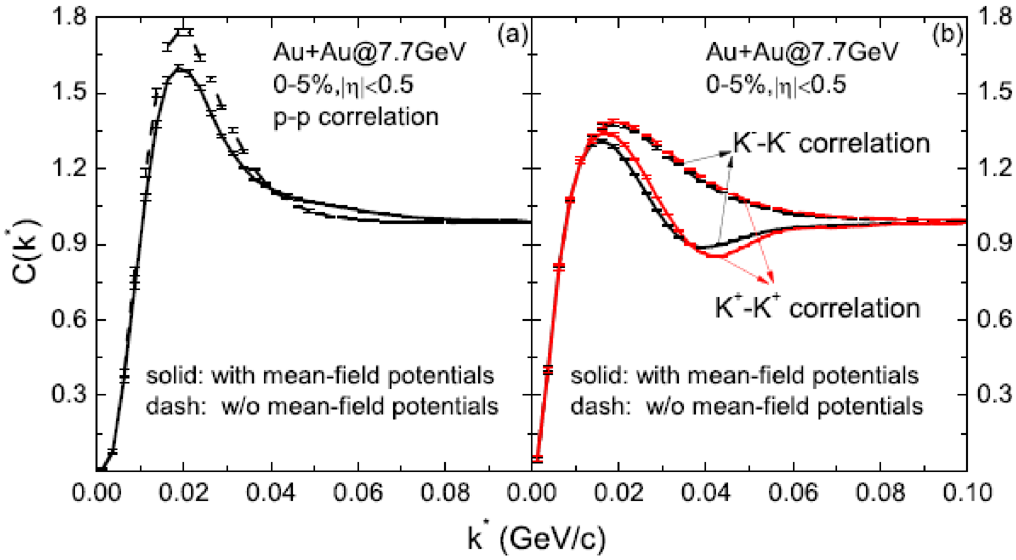


Lead to a smaller freeze-out eccentricity



Effects of hadronic mean-field potentials on HBT correlation

Affect correlation for indentified particles

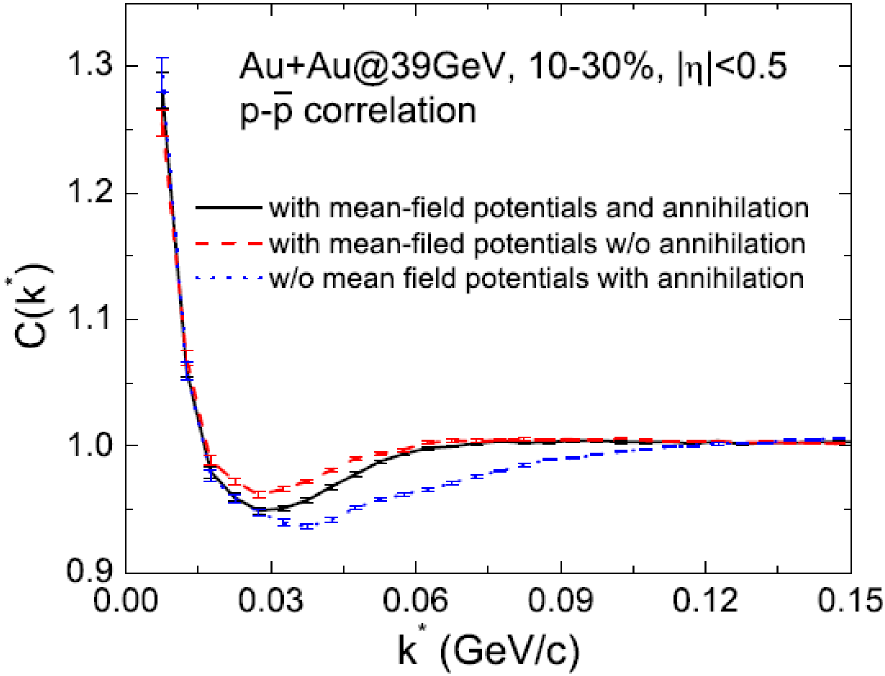


Affect global system evolution

+

Affect emission of individual particles

Interplay with p-pbar annihilation



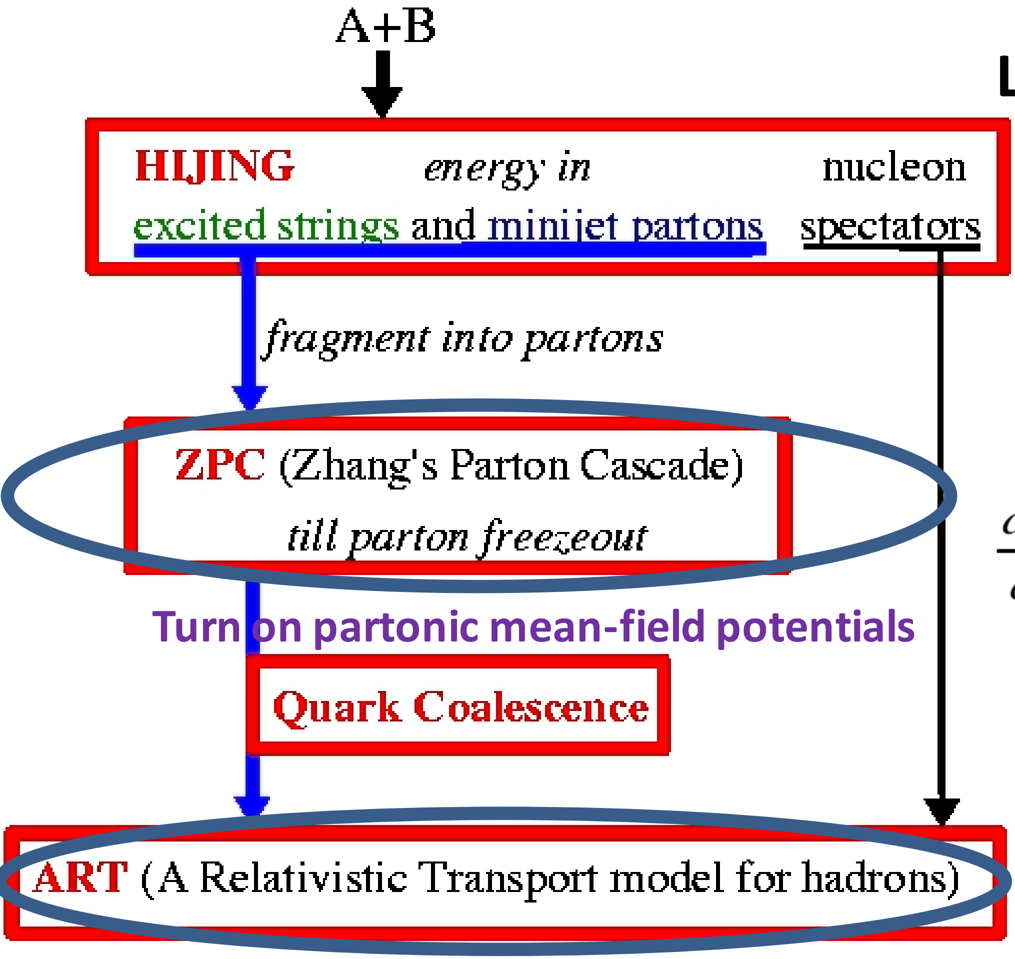
Attractive potential => correlation ↑

Annihilation => correlation ↓

Chun-Jian Zhang and JX,
arXiv:1707.07272 [nucl-th]

A multiphase transport (AMPT) model with string melting

Structure of AMPT model with string melting



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z : light-cone momentum fraction

Parton scattering cross section

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α: strong coupling constant

μ: screening mass

a, b: particle multiplicity

α, μ: partonic interaction

3-flavor Nambu-Jona-Lasinio transport model

Lagrangian:

scalar pseudoscalar vector pseudovector

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - M)\psi + \frac{G}{2}\sum_{a=0}^{8}[(\bar{\psi}\lambda^a\psi)^2 + (\bar{\psi}i\gamma_5\lambda^a\psi)^2] - \sum_{a=0}^{8}\left[\frac{G_V}{2}(\bar{\psi}\gamma_\mu\lambda^a\psi)^2 + \frac{G_A}{2}(\bar{\psi}\gamma_\mu\gamma_5\lambda^a\psi)^2\right]$$

Kobayashi-Maskawa-t'Hooft interaction

$$- K[\det_f(\bar{\psi}(1+\gamma_5)\psi) + \det_f(\bar{\psi}(1-\gamma_5)\psi)]$$

Yoichiro *Nambu*

2008年
Nobel Prize
For Physics

Parameters taken from
M. Lutz, S. Klimt, and W. Weise, NPA (1992)
to reproduce meson properties

Boltzmann equation:

$$\frac{\partial}{\partial t}f + \vec{v}\cdot\nabla_x f - \nabla_x H \cdot \nabla_p f = \mathcal{C}$$

$$\langle\bar{q}_i q_i\rangle = -2M_i N_c \int \frac{d^3p}{(2\pi)^3 E_i}(1 - f_i - \bar{f}_i) \;\; (i = u, d, s)$$

$$\rho^\mu = \langle\bar{\psi}\gamma^\mu\psi\rangle = 2N_c \sum_{i=u,d,s}\int \frac{d^3p}{(2\pi)^3 E_i} p^\mu (f_i - \bar{f}_i)$$

Single-quark Hamiltonian:

$$H = \sqrt{M^{*2} + p^{*2}} \boxed{\pm} g_V \rho^0$$

$$M_u = m_u - 2G\langle\bar{u}u\rangle + 2K\langle\bar{d}d\rangle\langle\bar{s}s\rangle$$
$$M_d = m_d - 2G\langle\bar{d}d\rangle + 2K\langle\bar{s}s\rangle\langle\bar{u}u\rangle$$
$$M_s = m_s - 2G\langle\bar{s}s\rangle + 2K\langle\bar{u}u\rangle\langle\bar{d}d\rangle$$

$$\mathbf{p}^* = \mathbf{p} \mp g_V \boldsymbol{\rho} \qquad g_V \equiv (2/3)G_V$$

Equations of motion:

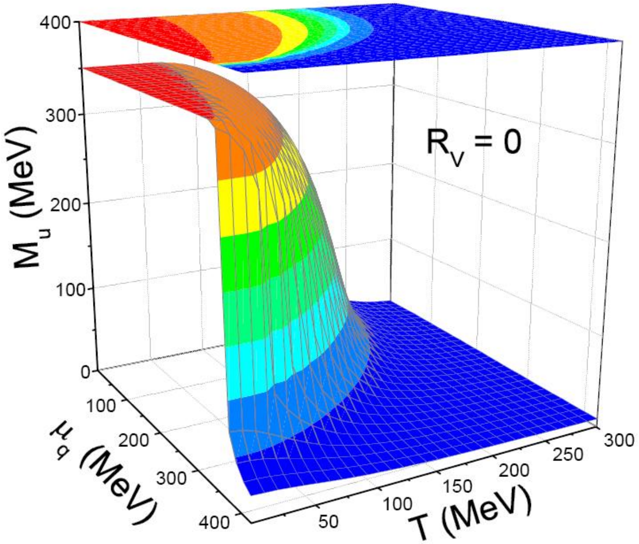
$$\frac{dx_i}{dt} = \frac{\partial H}{\partial p_i} = \frac{p_i^*}{E^*}$$

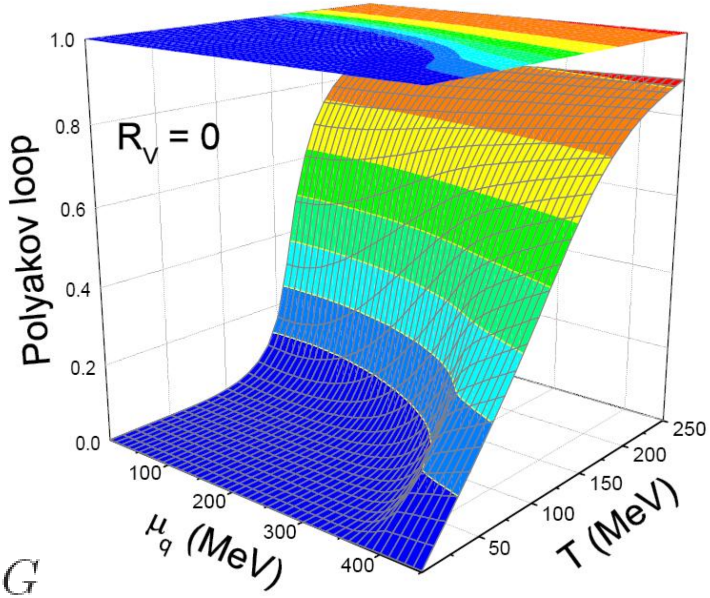
$$\frac{dp_i}{dt} = -\frac{\partial H}{\partial x_i}$$

$$= -\frac{M^*}{E^*}\frac{\partial M^*}{\partial x_i} \pm g_V\left(v_j\frac{\partial \rho_j}{\partial x_i} - \frac{\partial \rho_0}{\partial x_i}\right)$$

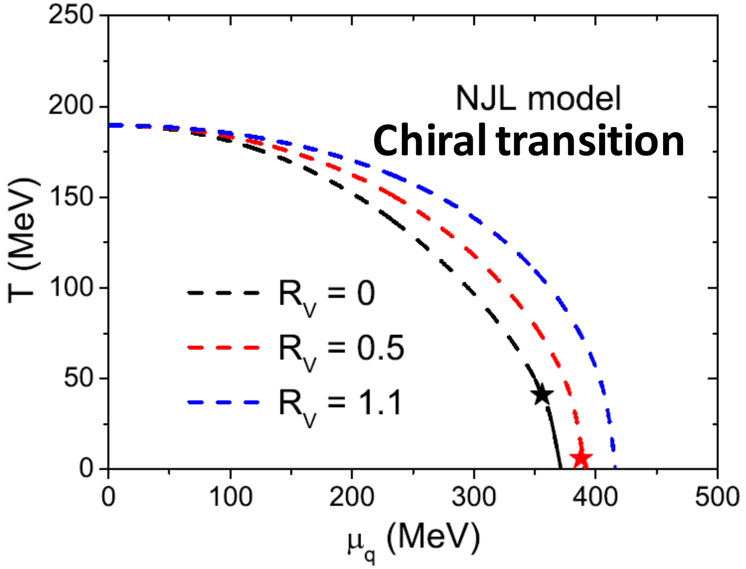
Solve with
test particle method

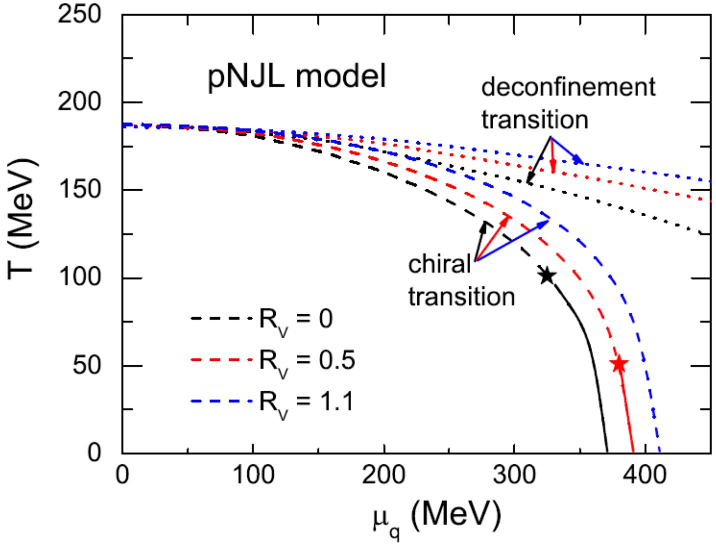
Phase diagram from NJL model





$$R_V = G_V/G$$





Fierz transformation: $R_V = 0.5$

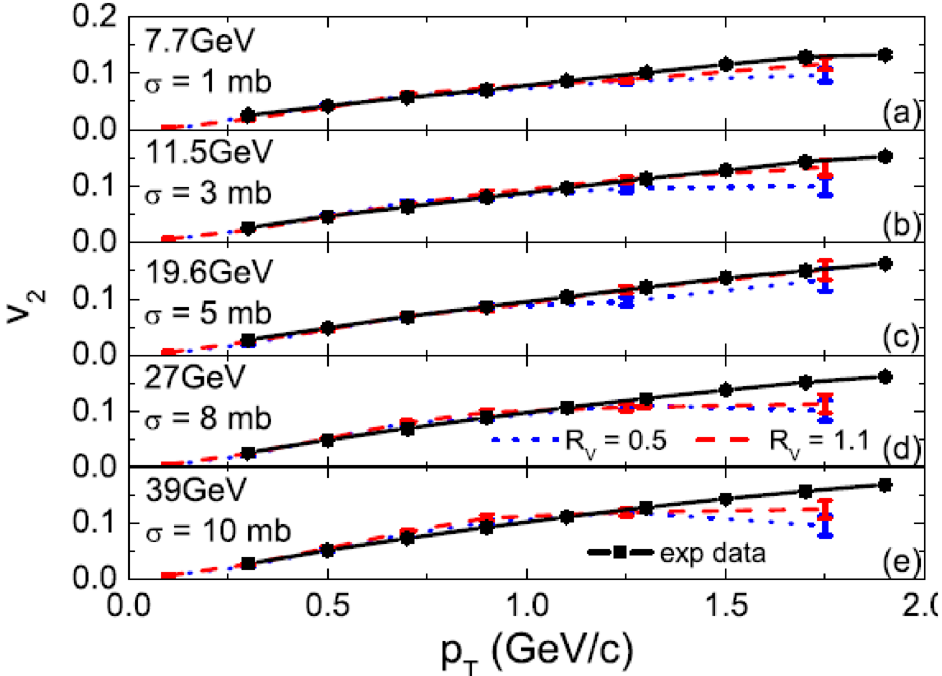
Vector meson-mass spectrum: $R_V = 1.1$

Hadronization happens when chiral symmetry is broken, i.e., $M^* > M_{vac}/2$

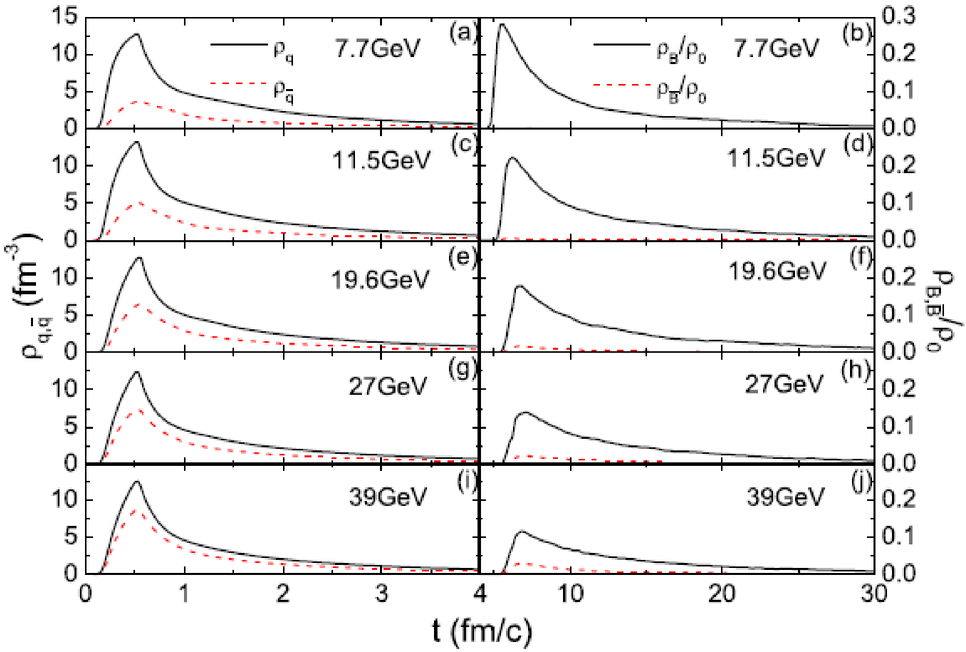
Fit the parton scattering cross section with charged-particle v_2

global hadronization

Total v_2 is less sensitive to R_V

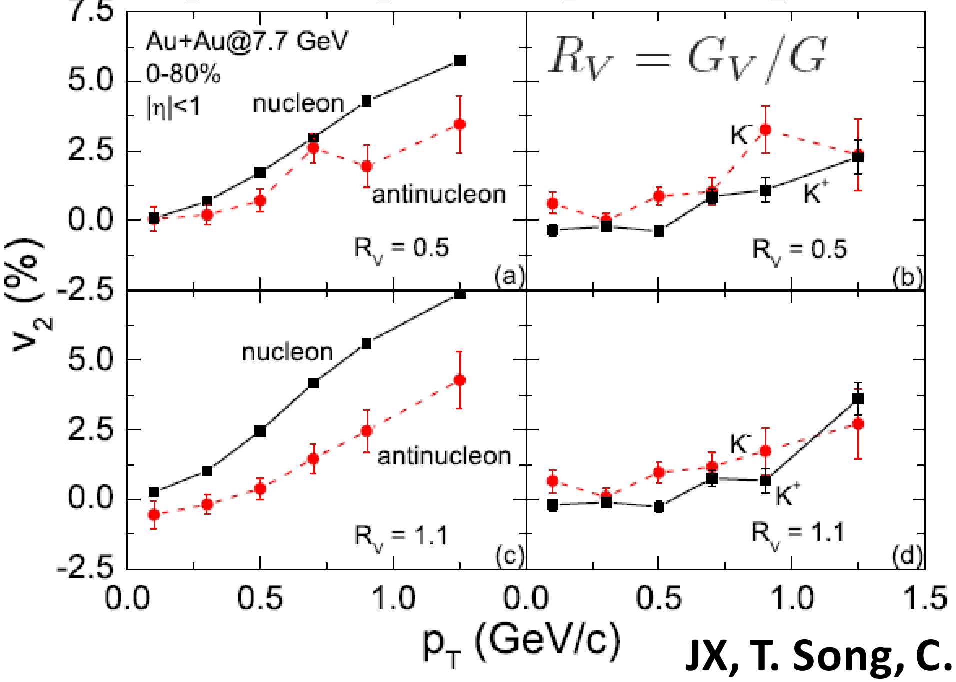


Density evolution in the partonic and hadronic phase



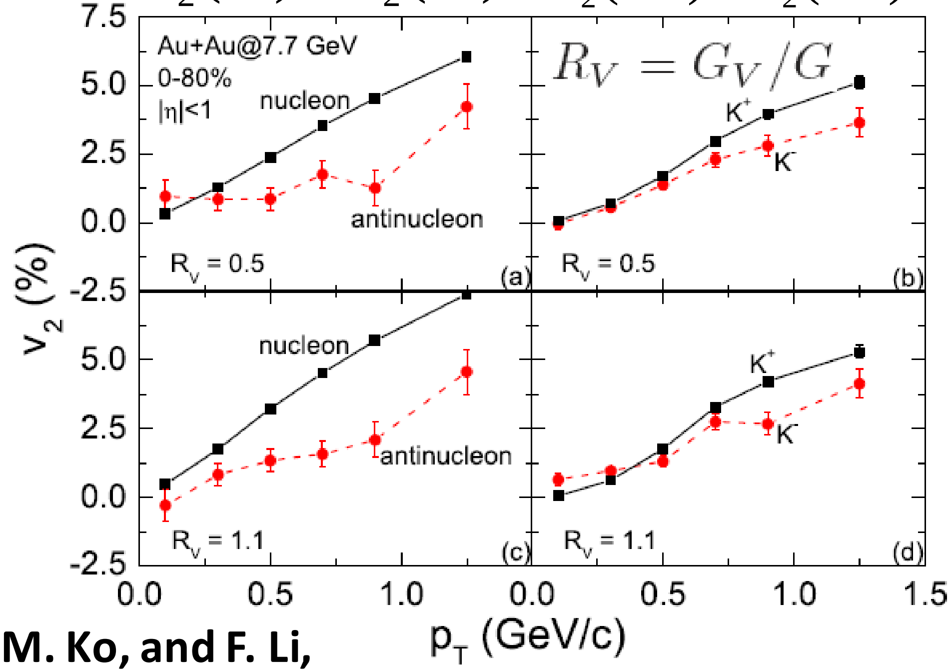
v_2 right after hadronization

$v_2(N) > v_2(\bar{N})$ $v_2(K^+) < v_2(K^-)$

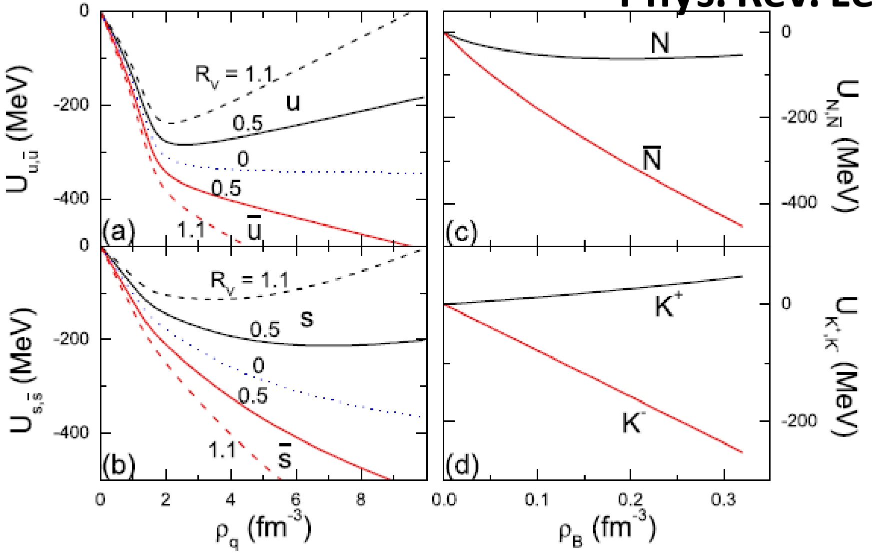


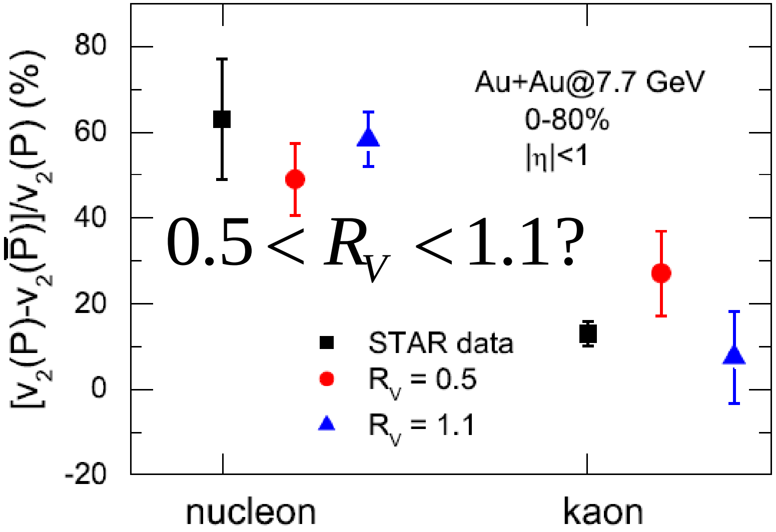
Final v_2

$v_2(N) > v_2(\bar{N})$ $v_2(K^+) > v_2(K^-)$



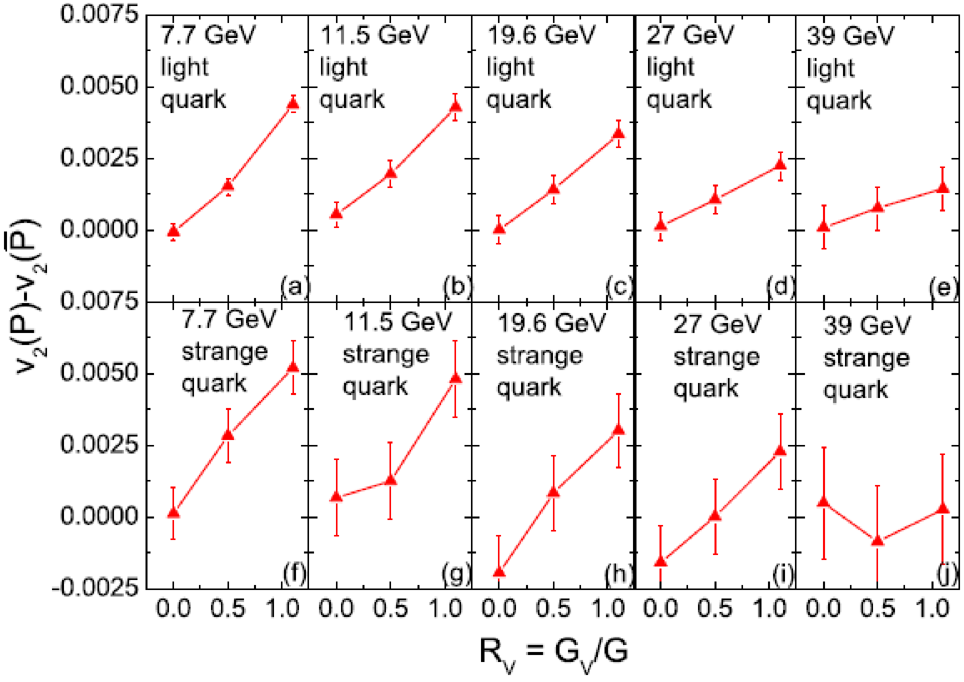
JX, T. Song, C. M. Ko, and F. Li,
Phys. Rev. Lett. 110, 012301 (2014)





$0.5 < R_V < 1.1?$

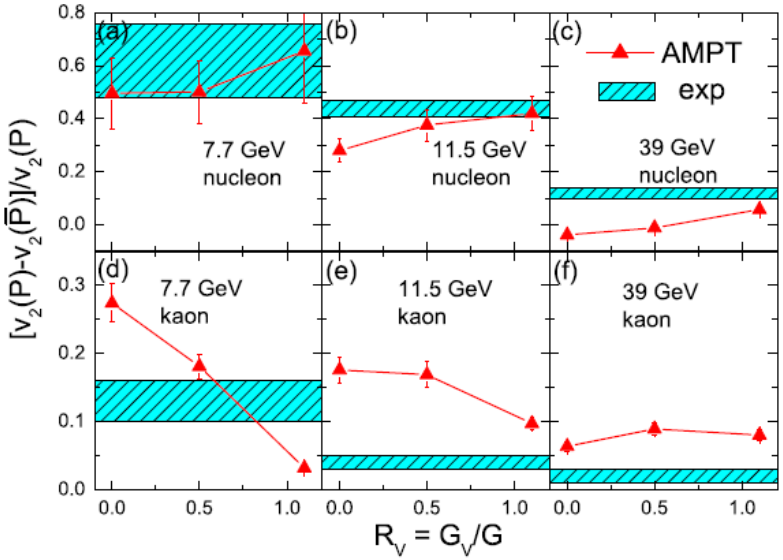
Collision energy dependence of v_2 splitting



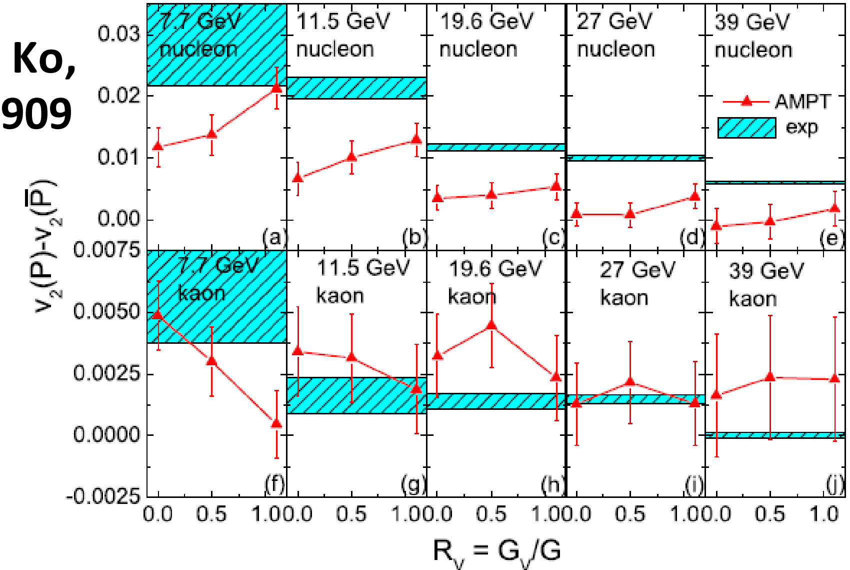
Difficult to reproduce quantitatively v_2 splitting at all collision energies at RHIC-BES

Effect of vector potential still holds

Energy dependence of v_2 splitting is qualitatively consistent with experimental observation

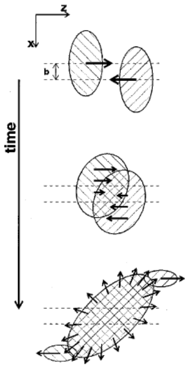


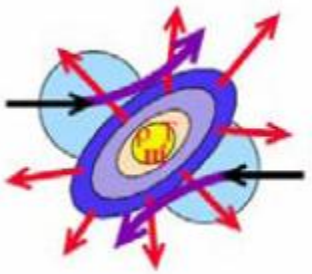
JX and C.M. Ko, PRC 94, 054909 (2016)

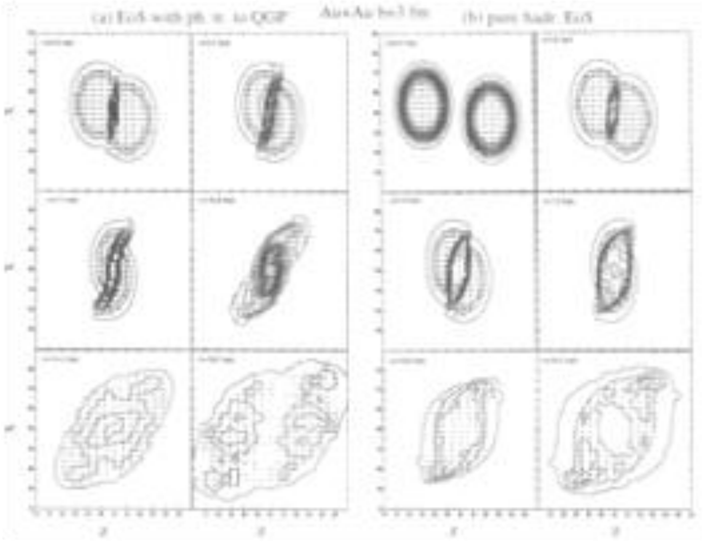


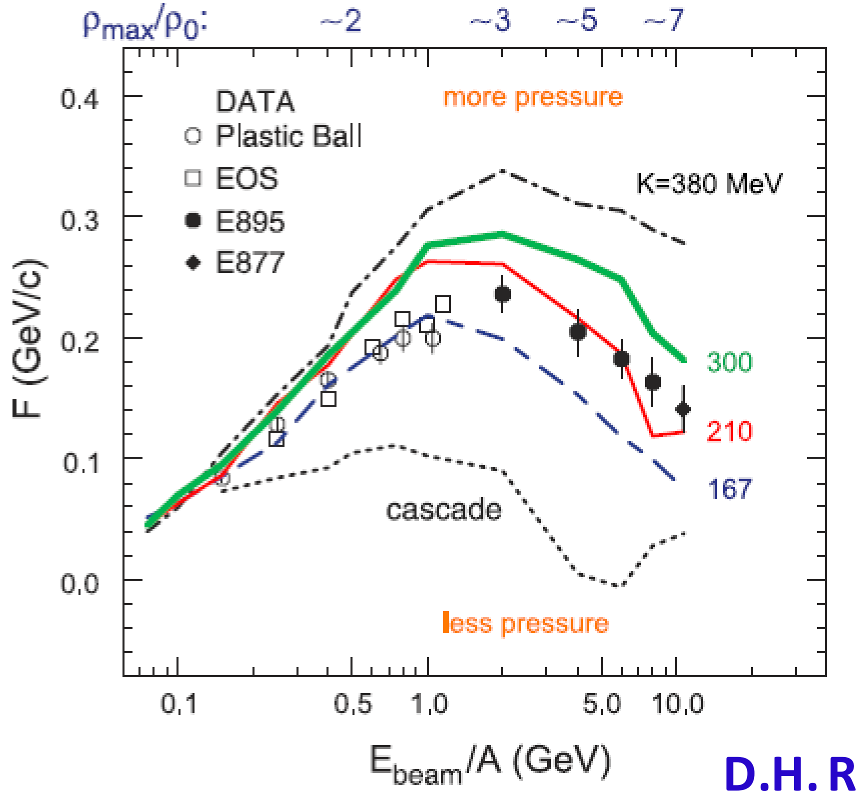
EOS effects on the directed flow

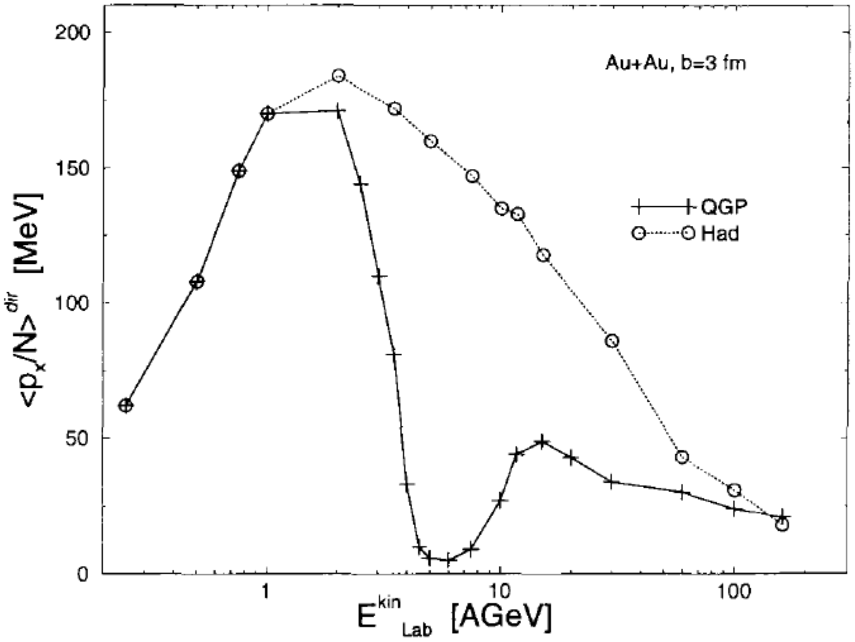
$$v_1 = \left\langle \frac{p_x}{p_T} \right\rangle$$











D.H. Rischke et al., APH N.S. Heavy Ion Physics (1995)

P. Danielewicz, R. Lacey, and W.G. Lynch, Science (2002)

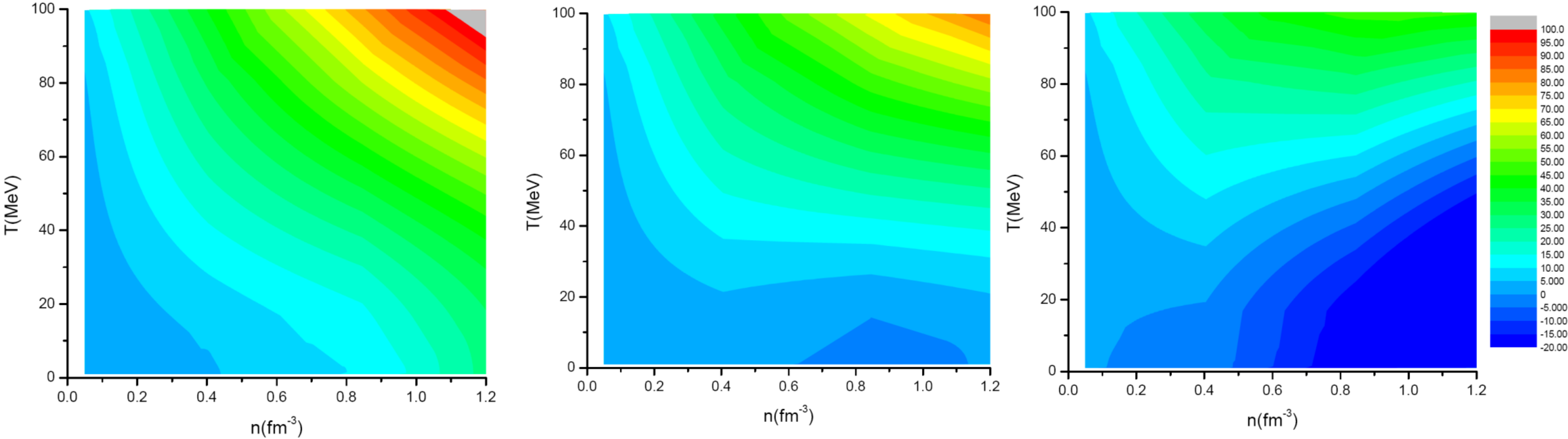
EOS of quark phase from NJL

$$\Omega_{\mathrm{NJL}} = -2N_c \sum_{i=u,d,s} \int_0^{\Lambda} \frac{d^3p}{(2\pi)^3} [E_i + T\ln(1+e^{-\beta(E_i-\tilde{\mu}_i)}) + T\ln(1+e^{-\beta(E_i+\tilde{\mu}_i)})] + G_S(\sigma_u^2+\sigma_d^2+\sigma_s^2) - 4K\sigma_u\sigma_d\sigma_s - \frac{1}{3}G_V(\rho_u+\rho_d+\rho_s)^2$$

$$P = -\Omega_{NJL}$$

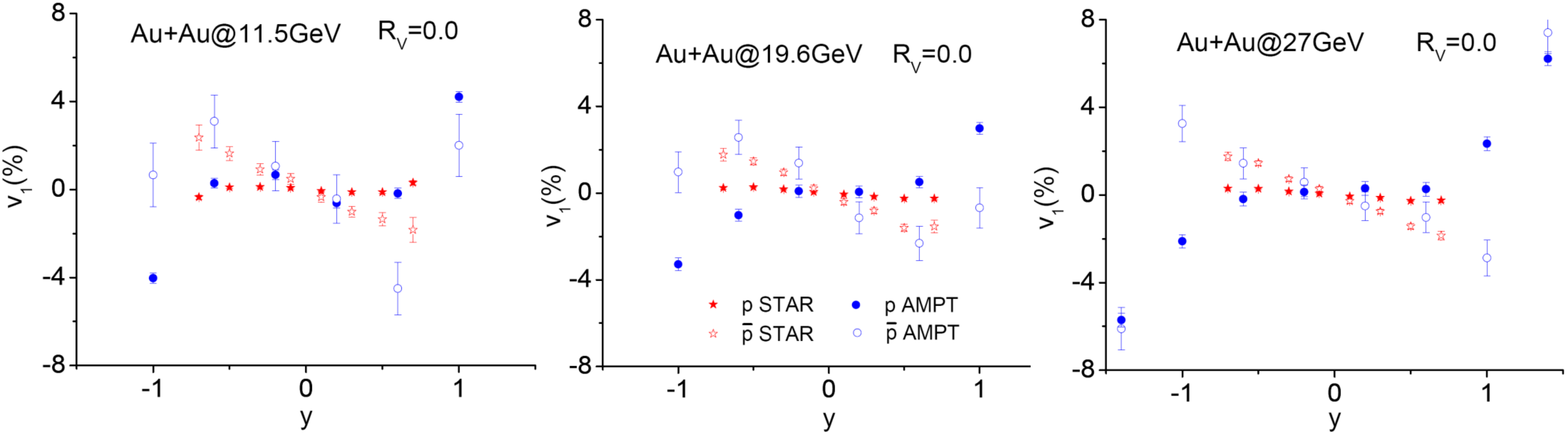
Pressure in the temperature-density (T-n) plane

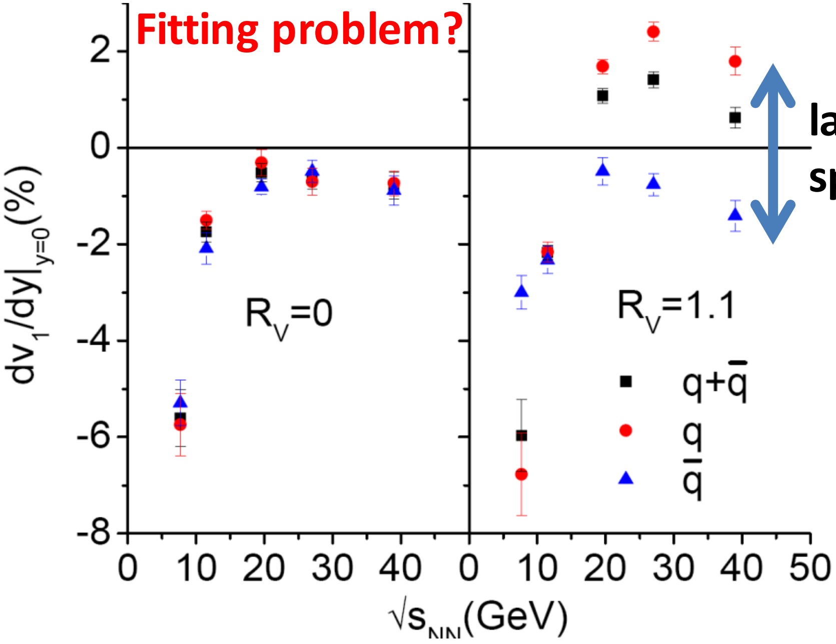
G_V = 1.1G **G_V = 0** **G_V = -1.1G?**



Directed flow from AMPT+NJL

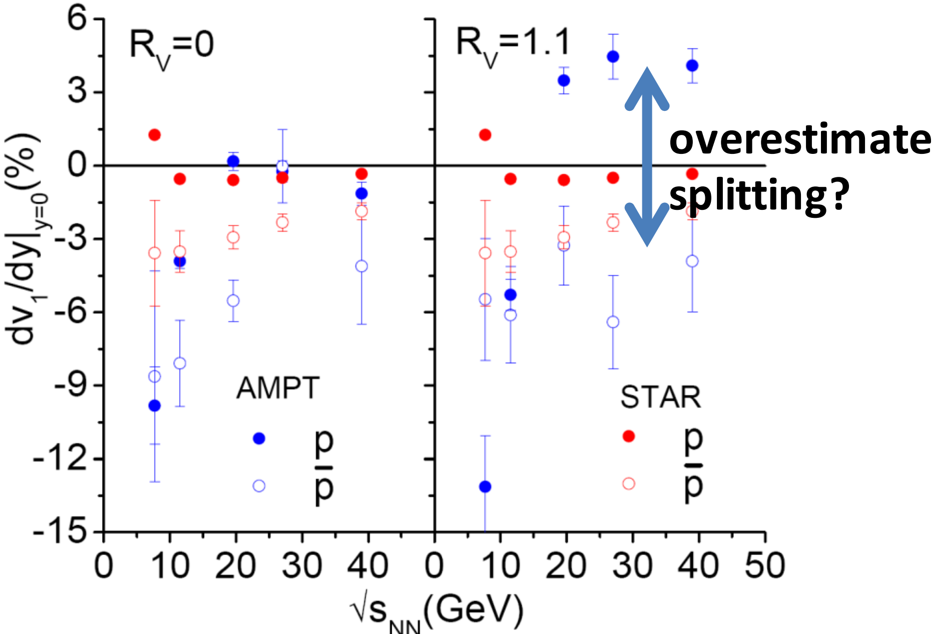
preliminary





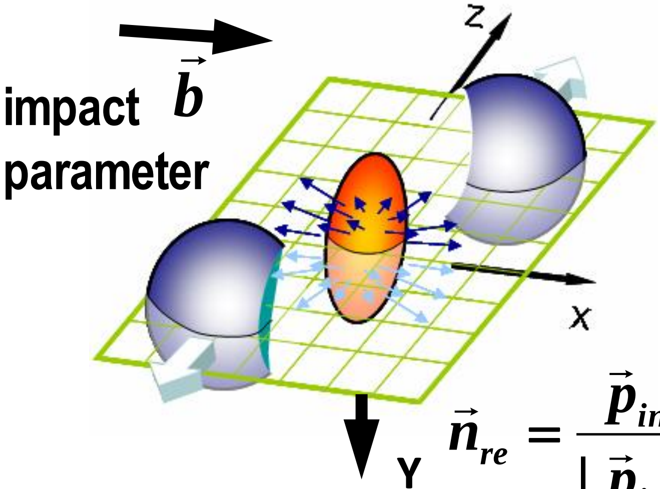
Fitting problem?

large splitting



overestimate splitting?

Spin effects on high-energy nuclear reactions - polarization



$$\vec{n}_{re} = \frac{\vec{p}_{in} \times \vec{b}}{|\vec{p}_{in} \times \vec{b}|}$$

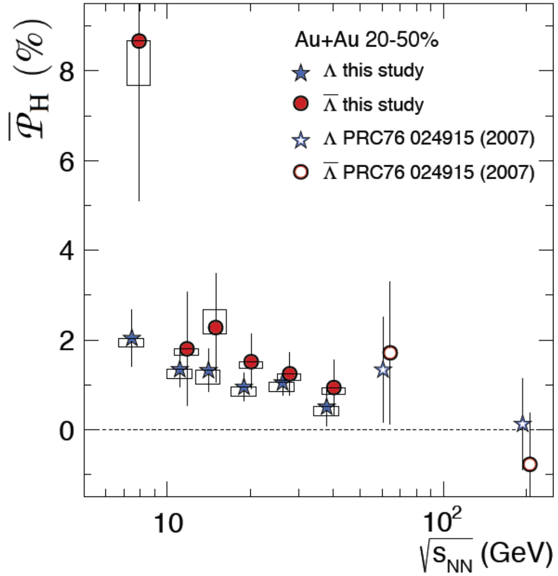
perpendicular to the reaction plane

Z. T. Liang and X. N. Wang, Phys. Rev. Lett., 2005
Phys. Lett. B, 2005

Λ polarization

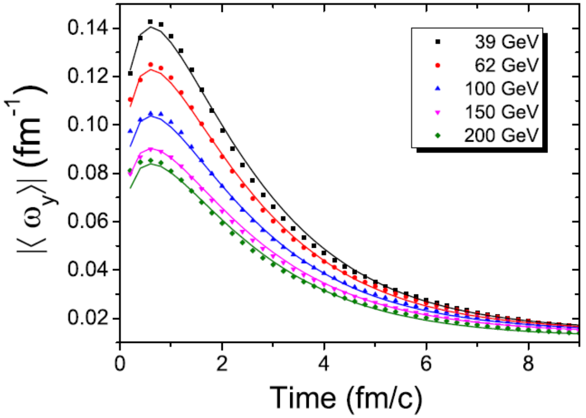
$$\overline{P}_{\rm H} = \frac{8}{\pi\alpha}\frac{\langle \sin(\phi_{\rm p}^{*} - \Psi_{\rm EP}^{(1)})\rangle}{R_{\rm EP}^{(1)}}$$

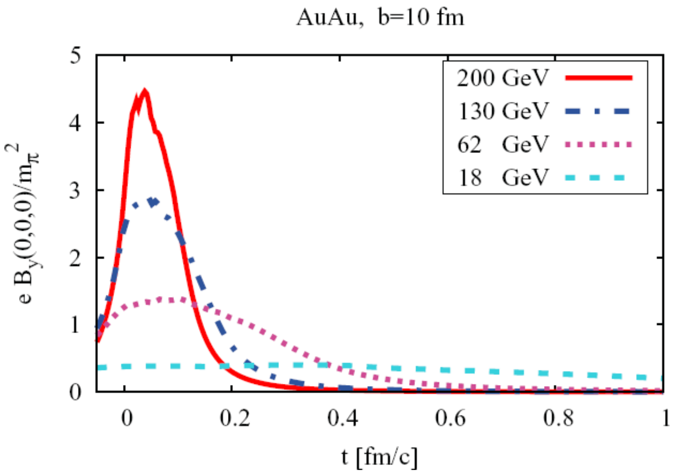
$$\frac{d\sigma}{d\Omega} \sim 1+\alpha_H P_H \cos\theta \qquad P_H = \frac{\Lambda^{\uparrow} - \Lambda^{\downarrow}}{\Lambda^{\uparrow} + \Lambda^{\downarrow}}$$



STAR, arXiv:1701.06657

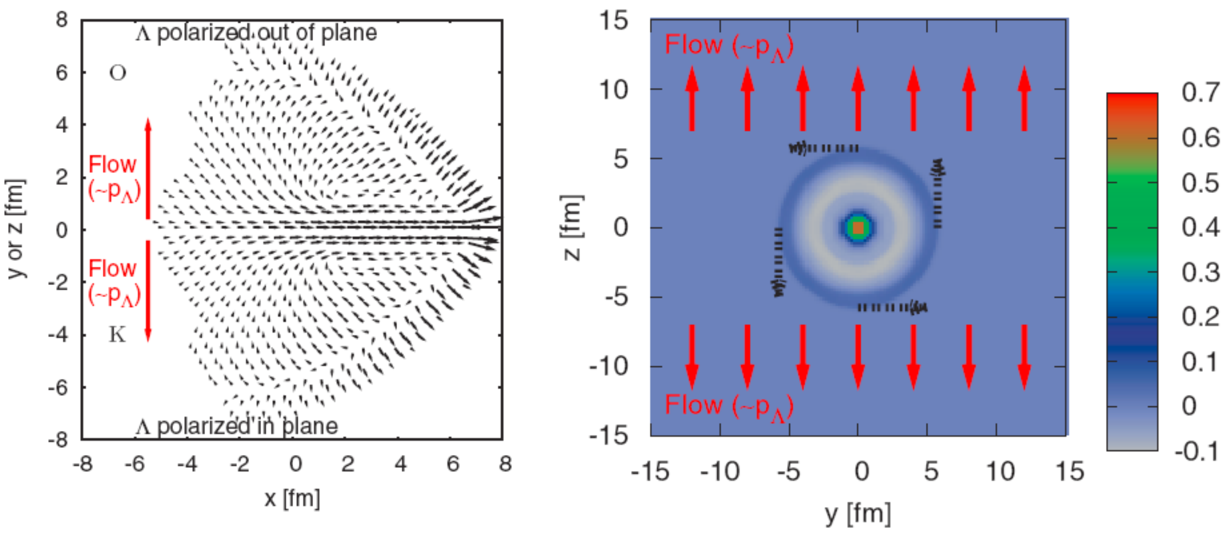
Larger $\overline{\Lambda}$ spin polarization than Λ ?



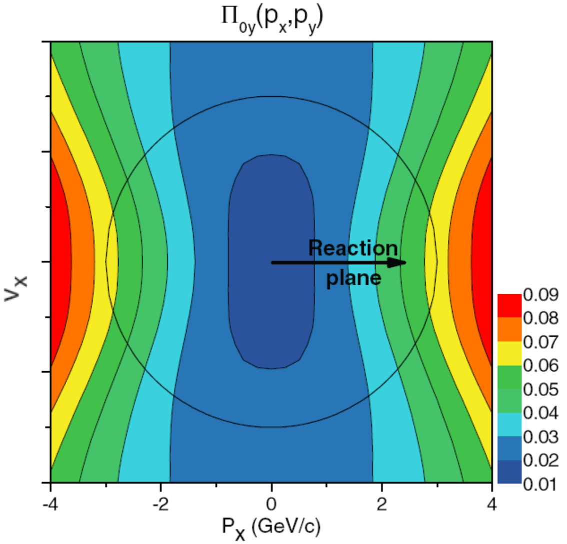


Y. Jiang et al., Phys. Rev. C, 2016 **V. Voronyuk et al., Phys. Rev. C, 2011**

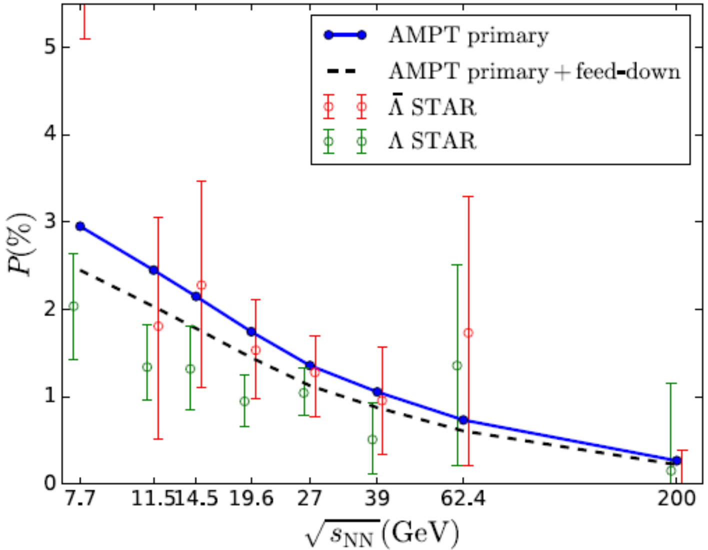
Vorticity leads to same $\Lambda(\bar{\Lambda})$ polarization



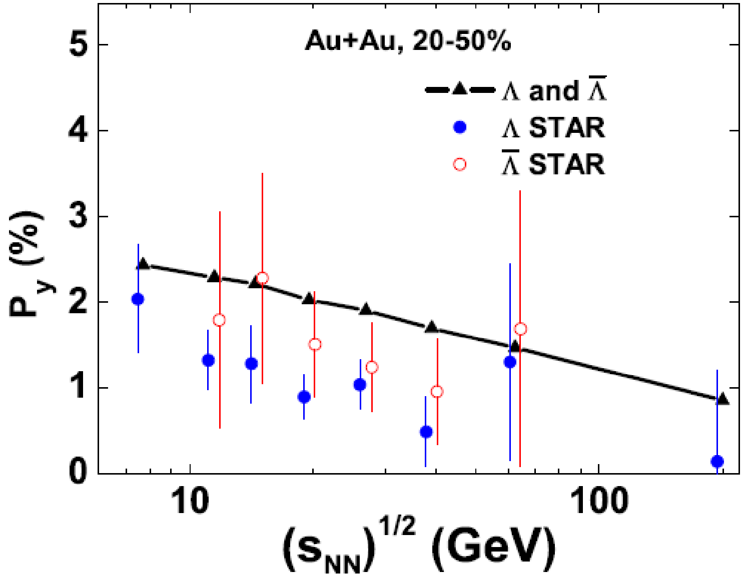
B. Betz et al., Phys. Rev. C, 2007



F. Becattini et al., Phys. Rev. C, 2013



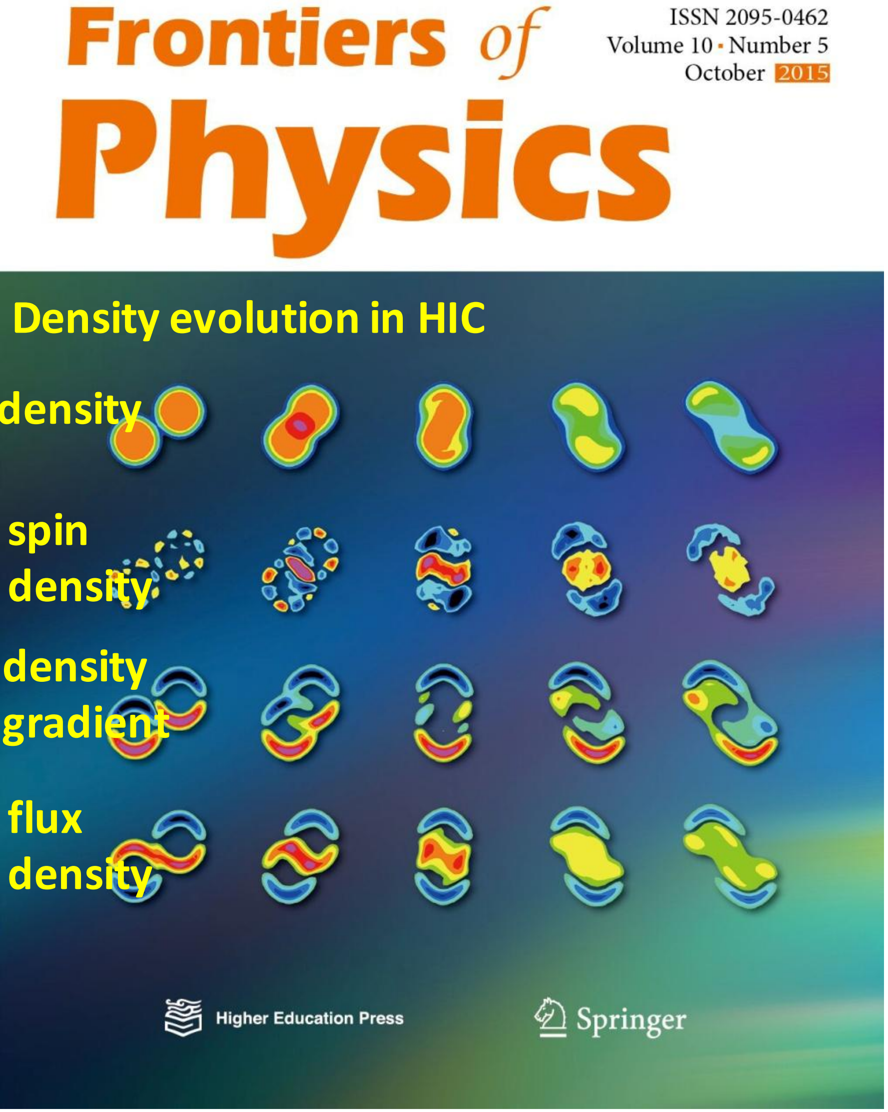
H. Li, L.G. Pang, Q. Wang, and X.L. Xia, arXiv: 1704.01507



Y.F. Sun and C.M. Ko, arXiv: 1706.09467

Spin dynamics in intermediate- and low-energy heavy-ion collisions

SIBUU12



Frontiers of Physics

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Density evolution in HIC

invited review, selected as cover story

JX, B.A. Li, W.Q. Shen, and Y. Xia, Front. Phys. (2015)

Boltzmann-Uehling-Uhlenbeck equation

$$\frac{\partial f}{\partial t}+v\cdot\nabla_r f-\nabla U\cdot\nabla_p f=-\int\frac{d^3p_2d^3p_1'd^3p_2'}{(2\pi)^9}\sigma v_{12}[ff_2(1-f_{1'})(1-f_{2'})$$
$$-f_{1'}f_{2'}(1-f)(1-f_2)](2\pi)^3\delta^3(p+p_2-p_1'-p_2')$$

test-particle method

C.Y. Wong, PRC 25, 1460 (1982)

equation of motion
$$\frac{d\vec{r}}{dt}=\frac{\vec{p}}{m} \qquad \frac{d\vec{p}}{dt}=-\nabla U$$

Spin-dependent Boltzmann-Uehling-Uhlenbeck eq

$$\hat{\varepsilon}(\vec{r},\vec{p}) = \varepsilon(\vec{r},\vec{p})\hat{I}+\vec{h}(\vec{r},\vec{p})\cdot\vec{\sigma},$$
$$\hat{f}(\vec{r},\vec{p}) = f_0(\vec{r},\vec{p})\hat{I}+\vec{g}(\vec{r},\vec{p})\cdot\vec{\sigma}.$$
$$\frac{\partial\hat{f}}{\partial t}+\frac{i}{\hbar}\left[\hat{\varepsilon},\hat{f}\right]+\frac{1}{2}\left(\frac{\partial\hat{\varepsilon}}{\partial\vec{p}}\cdot\frac{\partial\hat{f}}{\partial\vec{r}}+\frac{\partial\hat{f}}{\partial\vec{r}}\cdot\frac{\partial\hat{\varepsilon}}{\partial\vec{p}}\right)-\frac{1}{2}\left(\frac{\partial\hat{\varepsilon}}{\partial\vec{r}}\cdot\frac{\partial\hat{f}}{\partial\vec{p}}+\frac{\partial\hat{f}}{\partial\vec{p}}\cdot\frac{\partial\hat{\varepsilon}}{\partial\vec{r}}\right)=I_c$$

test-particle method

Y. Xia, JX, B.A. Li, and W.Q. Shen, Phys. Lett. B (2016)

spin-dependent equation of motion

$$\frac{d\vec{r}}{dt}=\frac{\vec{p}}{m}+\nabla_p(\varepsilon+\vec{h}\cdot\vec{n}) \qquad \frac{d\vec{p}}{dt}=-\nabla(\varepsilon+\vec{h}\cdot\vec{n})$$

$$\frac{d\vec{n}}{dt}=2\vec{h}\times\vec{n}$$

$\vec{n}$ spin expectation direction

Spin polarization from vector interactions

Consider quark spin in NJL Hamiltonian

$$H \approx \sqrt{M_i^2 + (\vec{p} - \vec{A})^2} + A_0 \boxed{- \frac{\vec{\sigma}\cdot(\nabla\times\vec{A})}{2\sqrt{M_i^2 + (\vec{p} - \vec{A})^2}}}$$

spin-orbit coupling

$$\begin{aligned} A_0 &= B_i g_V \rho_0 + Q_i e\varphi, \\ \vec{A} &= B_i g_V \vec{\rho} + Q_i e \vec{A}_m, \end{aligned}$$

effective magnetic field

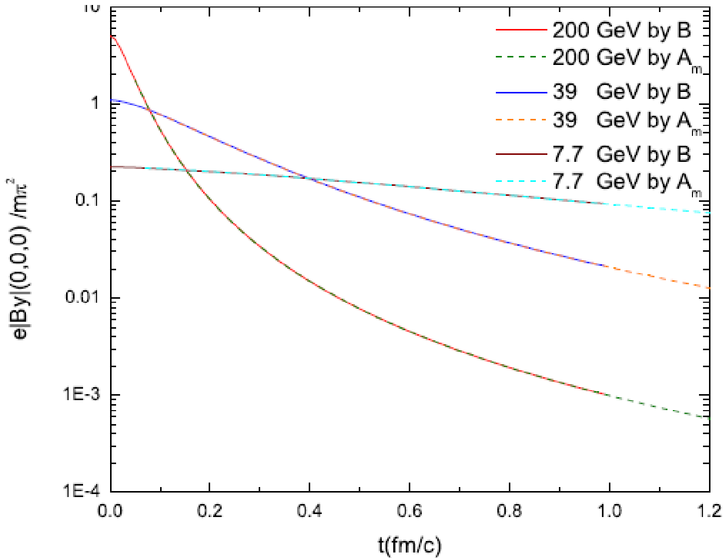
real magnetic field

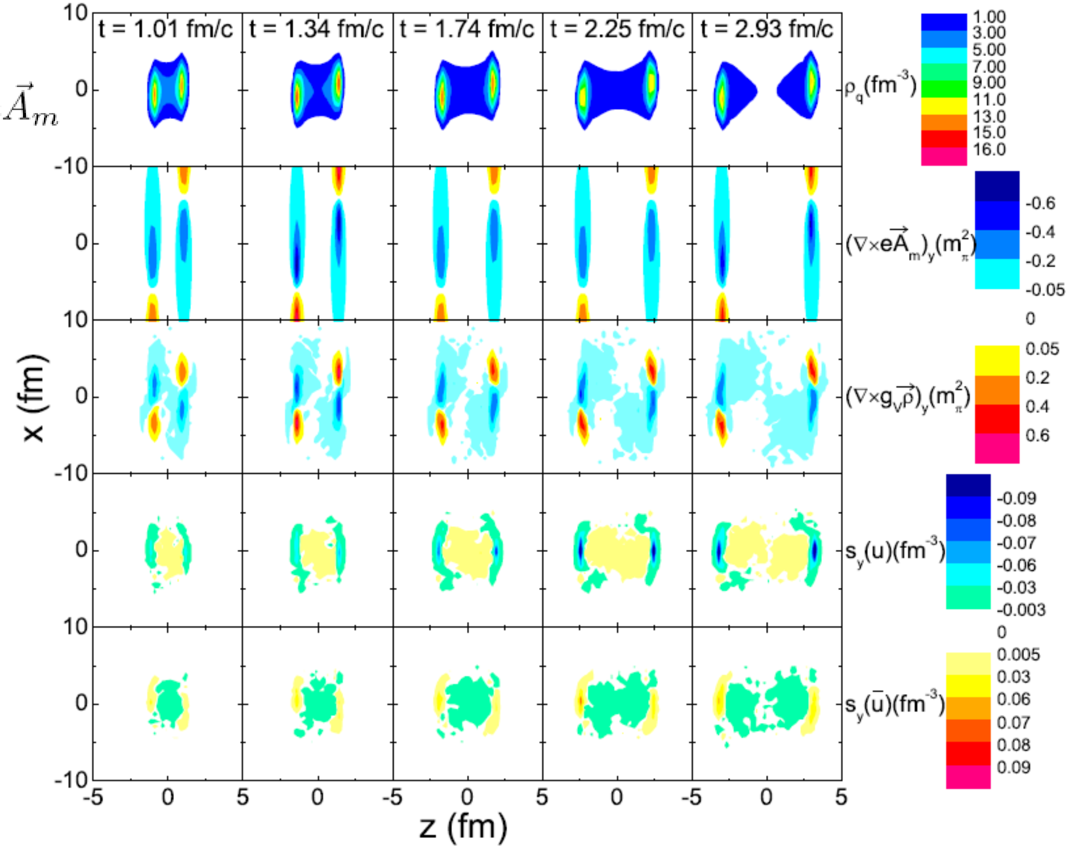
B_i: baryon number; Q_i: charge number

Comparing the real magnetic field from two calculation methods

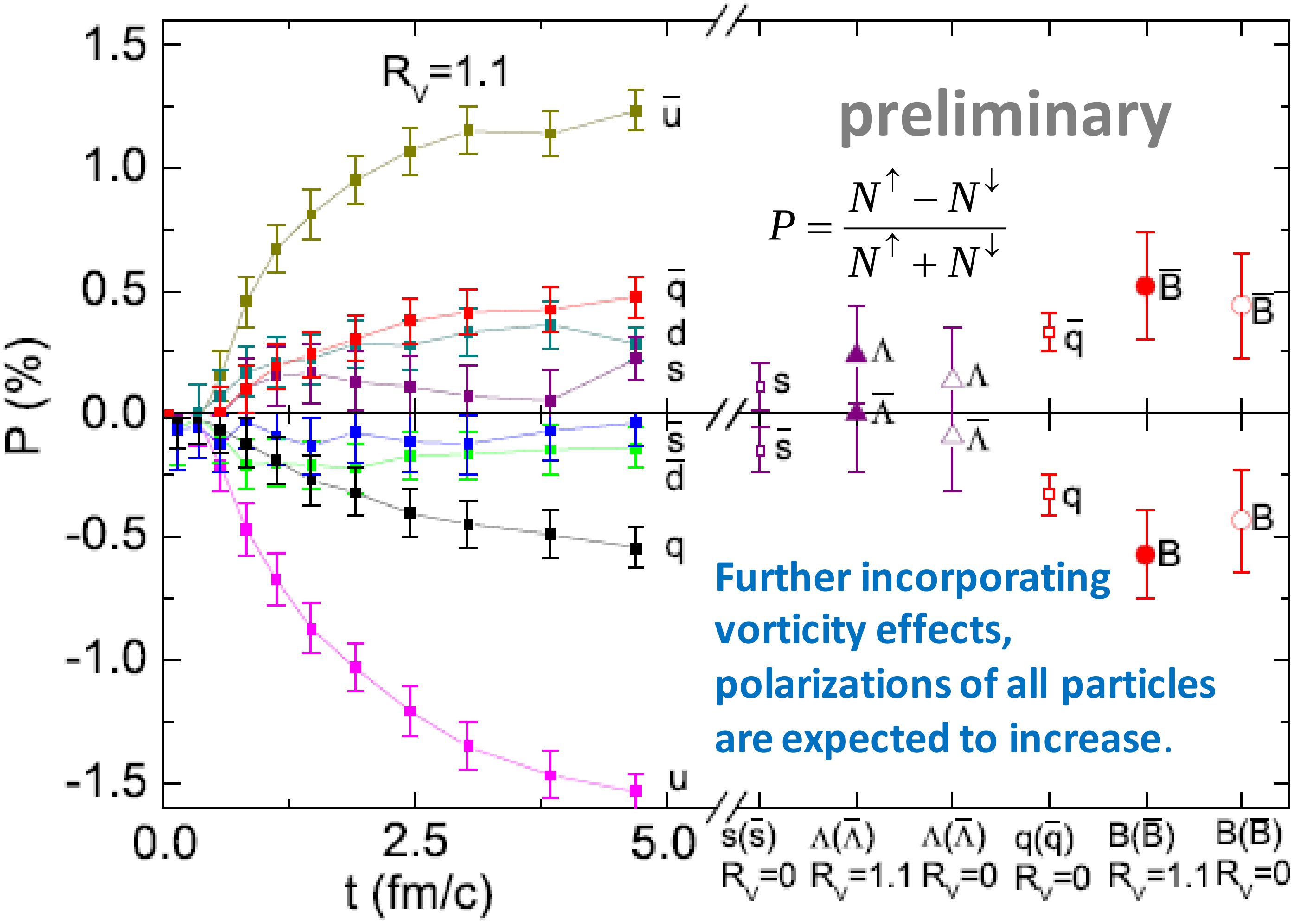
$$e\vec{A}_m(t,\vec{r}) = \frac{e^2}{4\pi}\sum_n Z_n \frac{\vec{v}_n}{R_n - \vec{v}_n\cdot\vec{R}_n} \qquad e\vec{B} = \nabla\times e\vec{A}_m$$

$$\vec{B}(t,\vec{r}) = \frac{e}{4\pi}\sum_n Z_n \frac{\vec{v}_n\times\vec{R}_n}{(R_n - \vec{v}_n\cdot\vec{R}_n)^3}(1-v_n^2)$$





Effects on the different baryon and antibaryon polarizations



$$P = \frac{N^{\uparrow} - N^{\downarrow}}{N^{\uparrow} + N^{\downarrow}}$$

Further incorporating vorticity effects, polarizations of all particles are expected to increase.

$$\Lambda^{\uparrow(\downarrow)} \sim uds^{\uparrow(\downarrow)}$$

$$\overline{\Lambda}^{\uparrow(\downarrow)} \sim \overline{u}\overline{d}\overline{s}^{\uparrow(\downarrow)}$$

$$P_{\Lambda} > P_{\overline{\Lambda}}$$

$$B^{\uparrow(\downarrow)} \sim qqq^{\uparrow(\downarrow)}$$

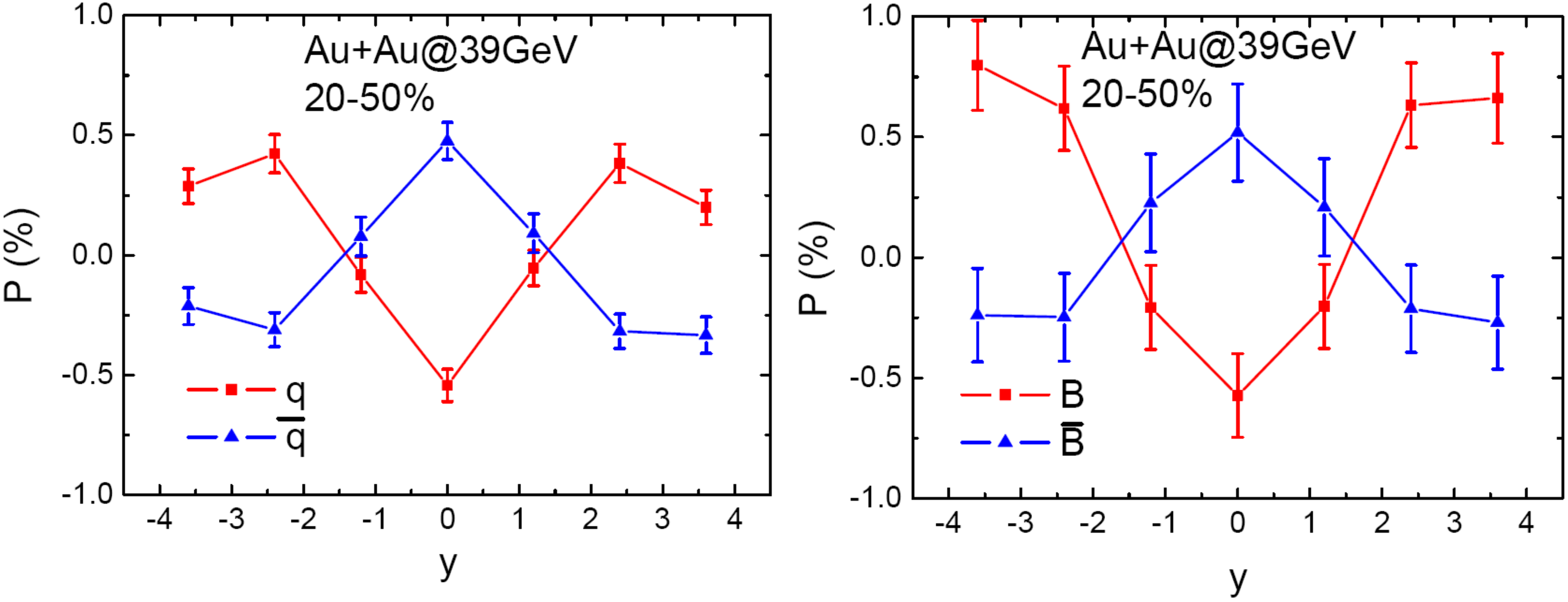
$$\overline{B}^{\uparrow(\downarrow)} \sim \overline{q}\overline{q}\overline{q}^{\uparrow(\downarrow)}$$

$$P_{B} < P_{\overline{B}}$$

Baryon spin: quark spin, gluon spin, quark angular momentum, …

Zhang-Zhu Han and JX, arXiv:1707.07262 [nucl-th]

Rapidity dependence of polarization



It is of great interest to confirm experimentally whether baryons have a stronger polarization than antibaryons at large rapidities.

Zhang-Zhu Han and JX, arXiv:1707.07262 [nucl-th]

Summary

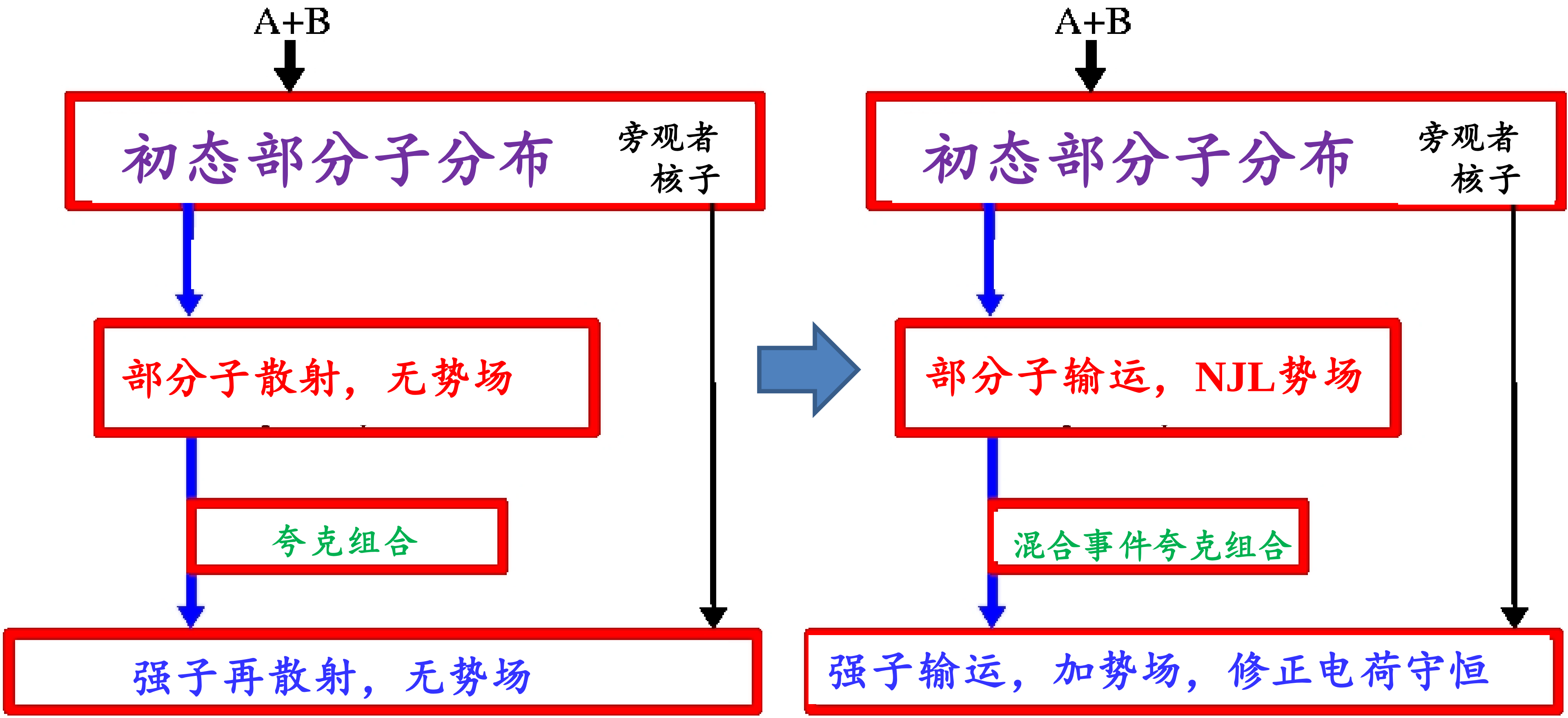
Effects of mean-field potentials on

- Elliptic flow splitting between particles and antiparticles
- Directed flow splitting between particles and antiparticles
- HBT correlation, radii, and chaoticity parameter
- Different polarizations of quarks (baryons) and antiquarks (antibaryons)

对多相输运模型(AMPT)的改进

原始AMPT模型的结构

改进后AMPT模型的结构



A+B

初态部分子分布

旁观者核子

部分子散射，无势场

夸克组合

强子再散射，无势场

A+B

初态部分子分布

旁观者核子

部分子输运，NJL势场

混合事件夸克组合

强子输运，加势场，修正电荷守恒

适用于LHC能区、RHIC能区

适用于RHIC束流能量扫描能区、FAIR能区

Needs further improvements on

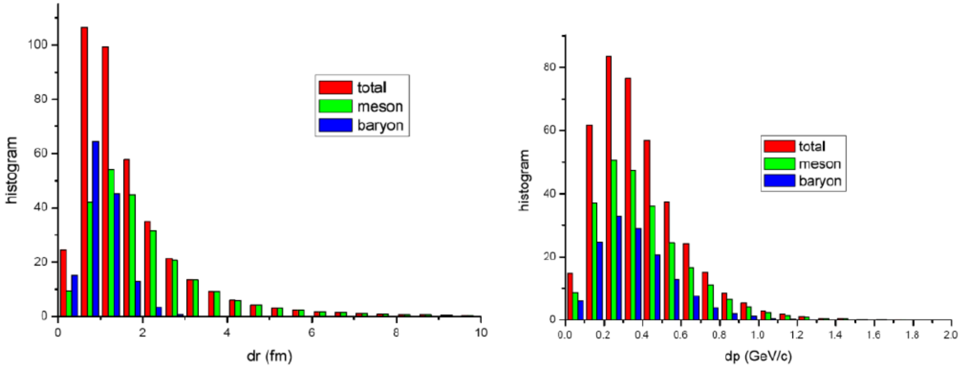
- Coalescence to reproduce better baryon/antibaryon ratio
- Finite thickness for initial partons and spectator nucleons
- Initial energy conservation

and others …

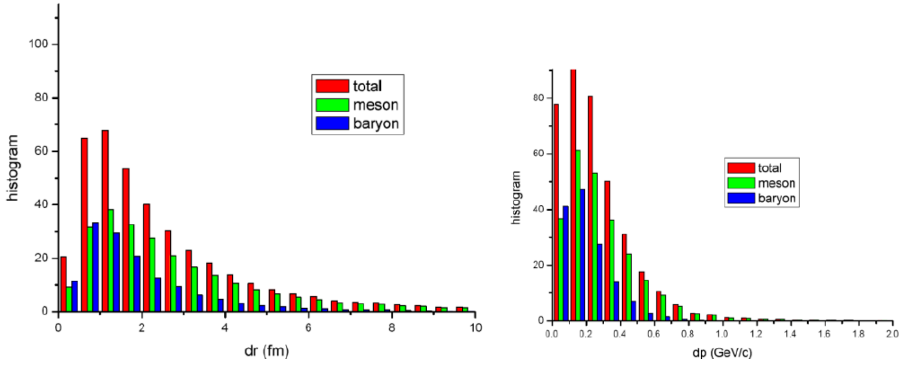
AMPT coalescence

Histogram of coalescence distance in coordinate space (dr) and in momentum space (dp)

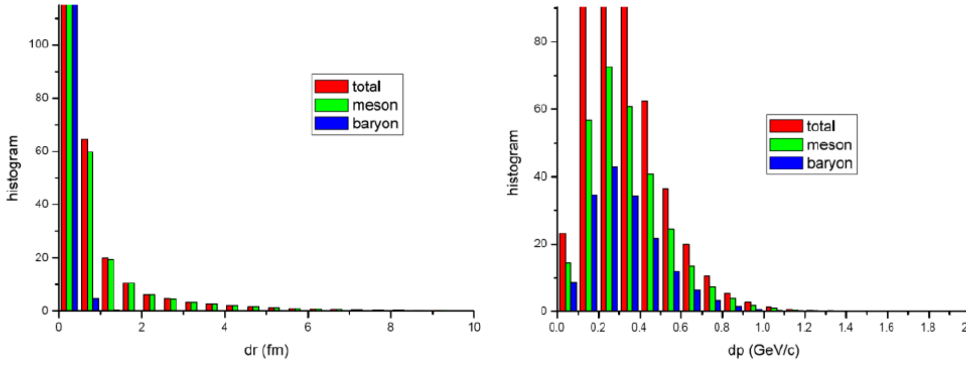
Single event coalescence, dr criterion



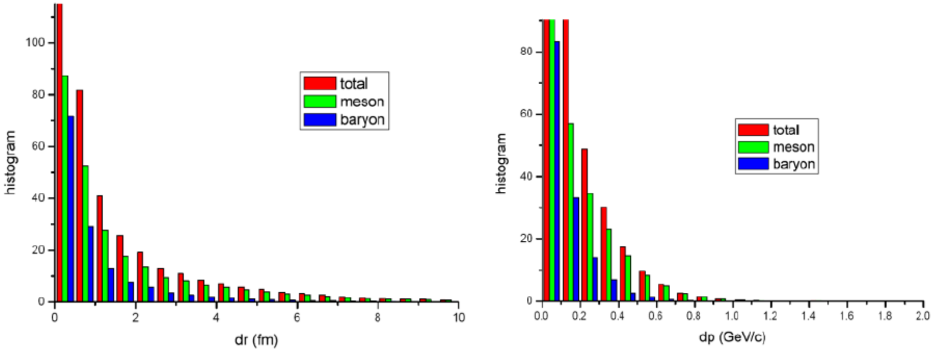
Single event coalescence, dr*dp criterion



Mix event coalescence, dr criterion

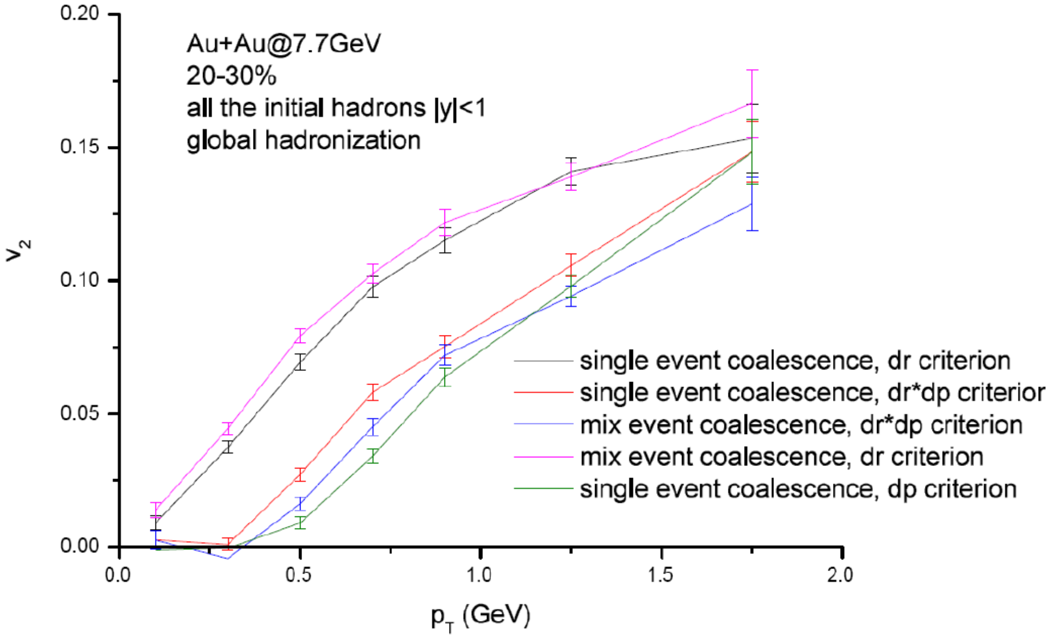


Mix event coalescence, dr*dp criterion

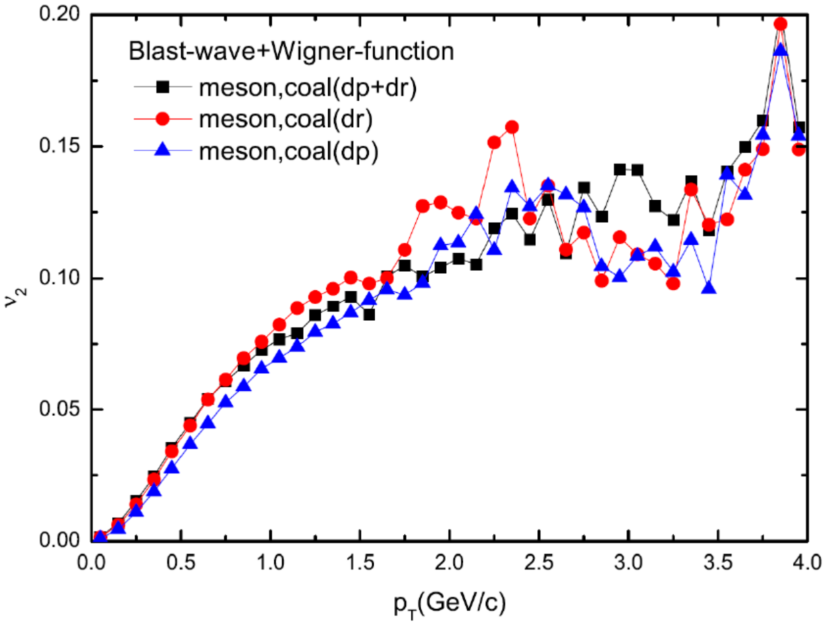


v_2 from different coalescence treatments

Elliptic flow of initial hadrons from different coalescence treatments:

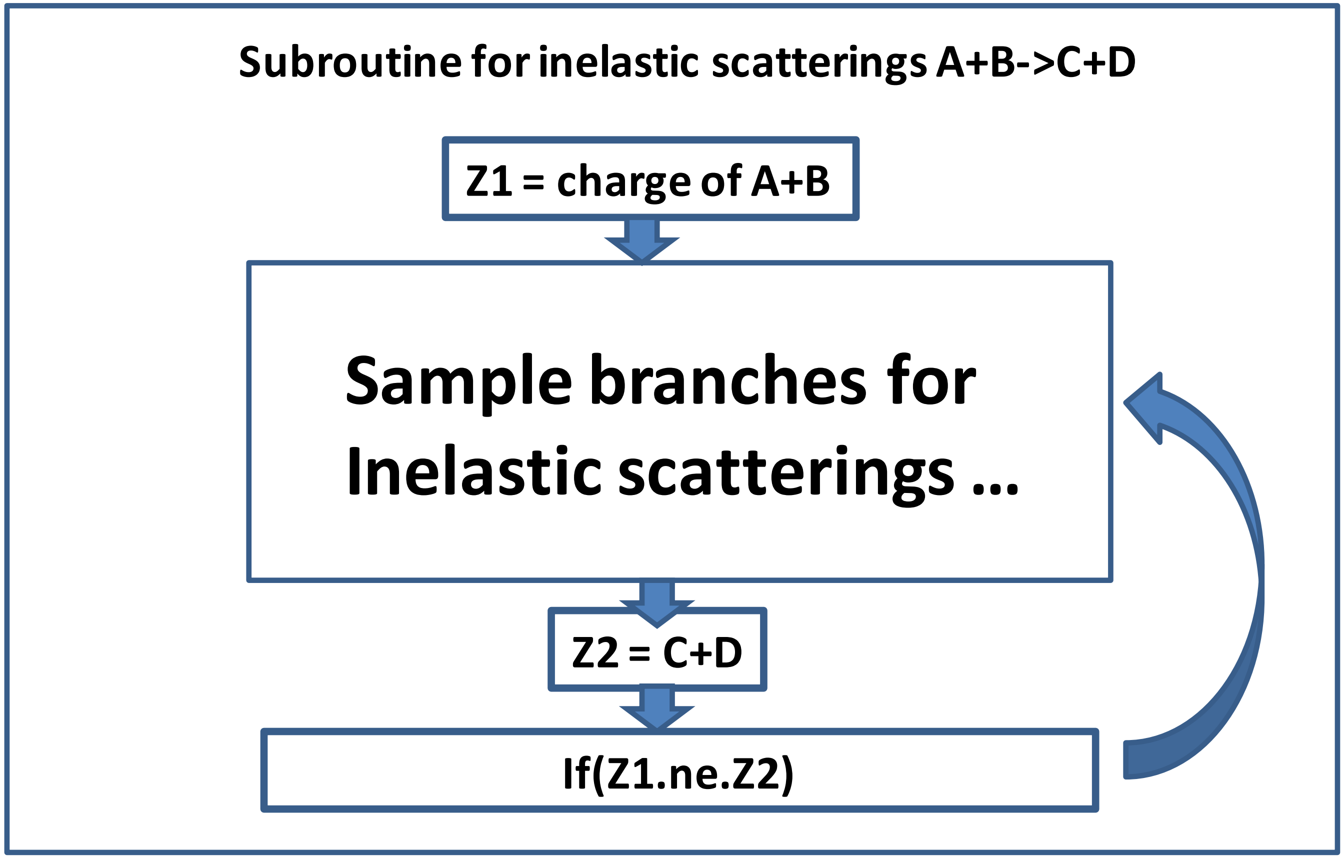


V2 from blast wave + Wigner function coalescence with dr+dp (Gaussian in r and p space), dr (Gaussian only in r space), dp (Gaussian only in p space)



Correct ART/AMPT charge violation

1) Correct inelastic channels where charges are not conserved



Subroutine for inelastic scatterings A+B->C+D

Z1 = charge of A+B

Sample branches for Inelastic scatterings …

Z2 = C+D

If(Z1.ne.Z2)

2) Correct charge violation by turning off K^0 and K^0-bar

$\sqrt{s_{NN}}$ (GeV)		λ	R_o (fm)	R_s (fm)	R_l (fm)
7.7	cascade	0.673 ± 0.007	5.58 ± 0.03	4.75 ± 0.03	4.39 ± 0.03
	mean-field	0.719 ± 0.007	6.16 ± 0.04	5.53 ± 0.04	5.51 ± 0.04
	expt.	0.532 ± 0.007	5.57 ± 0.13	4.93 ± 0.10	5.01 ± 0.11
11.5	cascade	0.663 ± 0.007	5.59 ± 0.03	4.72 ± 0.03	4.39 ± 0.03
	mean-field	0.704 ± 0.007	6.33 ± 0.04	5.54 ± 0.04	5.67 ± 0.04
	expt.	0.508 ± 0.004	5.68 ± 0.07	4.79 ± 0.05	5.43 ± 0.07
19.6	cascade	0.656 ± 0.007	5.60 ± 0.03	4.78 ± 0.03	4.43 ± 0.03
	mean-field	0.698 ± 0.008	6.39 ± 0.04	5.48 ± 0.04	5.76 ± 0.04
	expt.	0.498 ± 0.002	5.84 ± 0.05	4.84 ± 0.03	5.80 ± 0.05
27	cascade	0.651 ± 0.007	5.60 ± 0.03	4.78 ± 0.03	4.49 ± 0.03
	mean-field	0.689 ± 0.008	6.41 ± 0.04	5.51 ± 0.04	5.88 ± 0.04
	expt.	0.492 ± 0.002	5.82 ± 0.03	4.89 ± 0.02	5.99 ± 0.04
39	cascade	0.655 ± 0.007	5.63 ± 0.03	4.79 ± 0.03	4.55 ± 0.03
	mean-field	0.678 ± 0.008	6.42 ± 0.04	5.53 ± 0.04	5.95 ± 0.04
	expt.	0.491 ± 0.004	5.86 ± 0.07	4.97 ± 0.05	6.18 ± 0.08

Equations of motion for solving Boltzmann equation

Substitute

$$f(rp,t)=\int\frac{\mathrm{d}r_0\,\mathrm{d}p_0\,\mathrm{d}s}{(2\pi\hbar)^3}\exp\{\mathrm{i}s\cdot[p-P(r_0p_0s,t)]/\hbar\}\,\delta[r-R(r_0p_0s,t)]\,f(r_0p_0,t_0)$$

into the Boltzmann-Vlasov equation

$$\frac{\partial f(rp,t)}{\mathrm{d}t}+\frac{p}{m}\cdot\nabla_r f(rp,t)-\frac{2}{\hbar}\sin\left\{\frac{\hbar}{2}\nabla_r^V\cdot\nabla_p^f\right\}V(r,t)\,f(rp,t)=0$$

First term:

$$\begin{aligned}\frac{\partial f(rp,t)}{\partial t}=\int\frac{\mathrm{d}r_0\,\mathrm{d}p_0\,\mathrm{d}s}{(2\pi\hbar)^3}f(r_0p_0,t_0)\Bigg[&\frac{(-\mathrm{i}s)}{\hbar}\cdot\frac{\partial P}{\partial t}\\&\times\exp\{\mathrm{i}s\cdot[p-P(r_0p_0s,t)]/\hbar\}\delta[r-R(r_0p_0s,t)]\\&+\exp\{\mathrm{i}s\cdot[p-P(r_0p_0s,t)]/\hbar\}\\&\times(\nabla_R\delta[r-R(r_0p_0s,t)])\cdot\partial R(r_0p_0s,t)/\partial t\Bigg].\end{aligned}$$

Noting that

$$\nabla_R \delta[r - R(r_0 p_0 s, t)] = -\nabla_r \delta[r - R(r_0 p_0 s, t)]$$

So

$$\begin{aligned}\int \frac{\mathrm{d}r_0\,\mathrm{d}p_0\,\mathrm{d}s}{(2\pi\hbar)^3} & f(r_0 p_0, t_0)\exp\{\mathrm{i}s\cdot[p - P(r_0 p_0 s, t)]/\hbar\} \\ & \times (-\nabla_r \delta[r - R(r_0 p_0 s, t)]) \cdot \partial R(r_0 p_0 s, t)/\partial t \\ = & -\frac{\partial R(r_0 p_0 s, t)}{\partial t} \cdot \nabla_r f(rp, t),\end{aligned}$$

The potential term:

$$\begin{aligned}\frac{2}{\hbar}\sin\left\{\frac{\hbar}{2}\nabla_r^V \cdot \nabla_p^f\right\} & V(r, t) f(rp, t) \\ = \frac{1}{\hbar}\int & \frac{\mathrm{d}r_0\,\mathrm{d}p_0\,\mathrm{d}s}{(2\pi\hbar)^3} f(r_0 p_0, t_0) \\ & \times \exp\{\mathrm{i}s\cdot[p - P(r_0 p_0 s, t)]/\hbar\}\delta[r - R(r_0 p_0 s, t)] \\ & \times \frac{[V(r - \frac{s}{2}, t) - V(r + \frac{s}{2}, t)]}{\mathrm{i}}.\end{aligned}$$

Put everything together:

$$
\begin{aligned}
&\left[-\frac{\partial R(r_0p_0s,t)}{\partial t}+\frac{p}{m}\right]\cdot\nabla_r f(rp,t)\\
&\quad+\int\frac{\mathrm{d}r_0\,\mathrm{d}p_0\,\mathrm{d}s}{(2\pi\hbar)^3}f(r_0p_0,t_0)\left[\frac{(-\mathrm{i}s)}{\hbar}\cdot\frac{\partial P}{\partial t}\right.\\
&\quad\left.-\frac{[V(r-\frac{s}{2},t)-V(r+\frac{s}{2},t)]}{\mathrm{i}\hbar}\right]\\
&\quad\times\exp\{\mathrm{i}s\cdot[p-P(r_0p_0s,t)]/\hbar\}\\
&\quad\times\delta[r-R(r_0p_0s,t)]=0.
\end{aligned}
$$

➡ **Equations of motion:**

$$\frac{\partial R}{\partial t}=\frac{p}{m},$$

$$s\cdot\frac{\partial P}{\partial t}=V\left(R-\frac{s}{2},t\right)-V\left(R+\frac{s}{2},t\right).$$

Momentum-dependent potential:
One more term in BV equation

$$+\frac{2}{\hbar}\sin\left(\frac{\hbar}{2}\nabla_p^V\cdot\nabla_r^f\right)V(R,P,t)\,f(R,P,t)$$

$$\frac{\partial R}{\partial t}=\frac{p}{m}+\nabla_pV$$

$$\Longrightarrow\quad\frac{\partial P}{\partial t}\approx-\nabla_RV(R,t)$$

Calculate phase-space distribution function $f(\vec{r},\vec{p};t)=\frac{1}{N_{\mathrm{TP}}}\sum_{i=1}^{N_{\mathrm{TP}}A}g[\vec{r}-\vec{r}_i(t)]\tilde{g}[\vec{p}-\vec{p}_i(t)]$

Euler-Lagrange equation,

$$[\gamma^{\mu}(i\partial_{\mu} - A_{\mu}) - M_i]\psi_i = 0. \qquad (8)$$

Space and time components of the vector potential,

$$\begin{aligned} A_0 &= B_i g_V \rho_0 + Q_i e\varphi, && (9)\\ \vec{A} &= B_i g_V \vec{\rho} + Q_i e\vec{A}_m, && (10) \end{aligned}$$

with $g_V = \frac{2}{3}G_V$, $\rho_0 = \langle\bar{\psi}\gamma^0\psi\rangle$ and

$\vec{\rho} \equiv \langle\bar{\psi}\vec{\gamma}\psi\rangle$

The scalar and vector potential of the real electromagnetic field.

$$\begin{aligned} e\varphi(t,\vec{r}) &= \frac{e^2}{4\pi}\sum_n Z_n \frac{1}{R_n - \vec{v}_n\cdot\vec{R}_n}, && (11)\\ e\vec{A}_m(t,\vec{r}) &= \frac{e^2}{4\pi}\sum_n Z_n \frac{\vec{v}_n}{R_n - \vec{v}_n\cdot\vec{R}_n}, && (12) \end{aligned}$$

Calculated from Eq. (8),

$$i\partial_t\psi_i = [\gamma^0\gamma^k(-i\partial_k + A_k) + \gamma^0 M_i + A_0]\psi_i. \qquad (13)$$

Hamiltonian operator,

$$\hat{H} = \gamma^0\gamma^k(-\hat{p}_k + A_k) + \gamma^0 M_i + A_0, \qquad (14)$$

Single-particle Hamiltonian,

$$H = \sqrt{(\vec{p} - \vec{A})^2 + M_i^2 - \vec{\sigma}\cdot(\nabla\times\vec{A})} + A_0. \qquad (15)$$

$$\vec{\sigma}\cdot(\nabla\times\vec{A}) \ll (\vec{p}-\vec{A})^2 + M_i^2$$

Single-particle Hamiltonian can be further expressed as,

$$H \approx \sqrt{M_i^2 + (\vec{p} - \vec{A})^2} + A_0 - \frac{\vec{\sigma}\cdot(\nabla\times\vec{A})}{2\sqrt{M_i^2 + (\vec{p} - \vec{A})^2}}. \qquad (16)$$

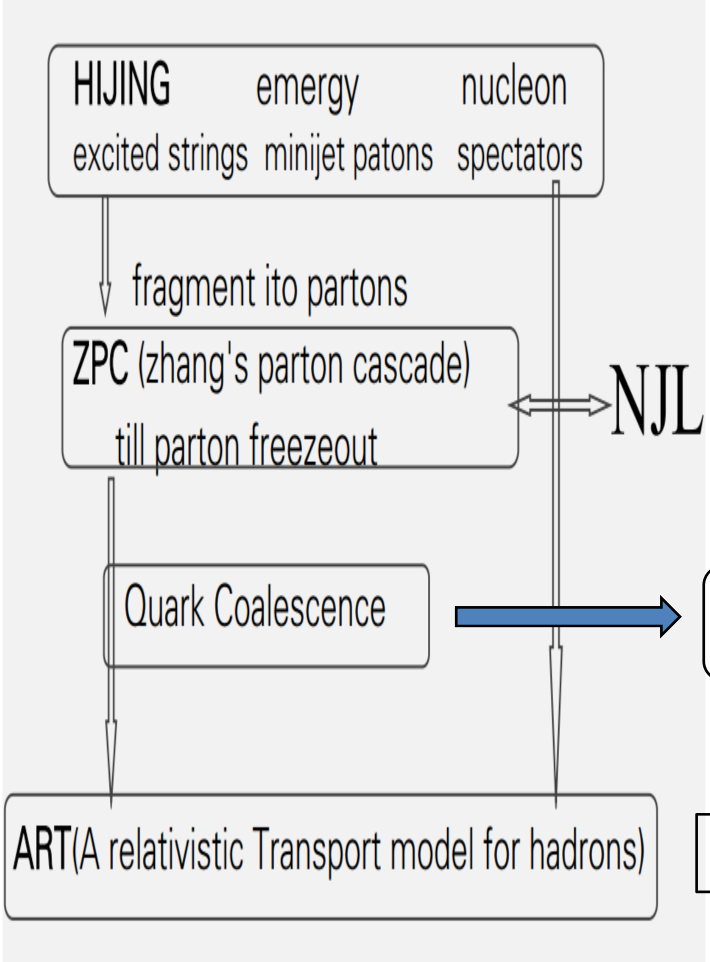
B. Equations of motion for partons and the extended AMPT model

$$
\begin{aligned}
\dot{\vec{r}} &= \vec{\nabla}_{\vec{p}} H, \\
\dot{\vec{p}} &= -\vec{\nabla} H, \\
\dot{\vec{\sigma}} &= -i[\vec{\sigma}, H].
\end{aligned}
$$

$$
\begin{aligned}
\frac{dr_k}{dt} &= \frac{p_k^*}{E_i^*} + \frac{1}{2}\frac{p_k^*}{E_i^{*3}}[\vec{\sigma} \cdot (\nabla \times \vec{A})], \\
\frac{dp_k^*}{dt} &= -\frac{M_i}{E_i^*}\frac{\partial M_i}{\partial r_k} + \frac{p_j^*}{E_i^*}\frac{\partial A_j}{\partial r_k} - \frac{\partial A_0}{\partial r_k} - \frac{\partial A_k}{\partial t} \\
&\quad - \dot{r}_j \frac{\partial A_k}{\partial r_j} - \frac{1}{2}[\vec{\sigma} \cdot (\nabla \times \vec{A})]\frac{M_i}{E_i^{*3}}\frac{\partial M_i}{\partial r_k} \\
&\quad + \frac{1}{2}[\vec{\sigma} \cdot (\nabla \times \vec{A})]\frac{p_j^*}{E_i^{*3}}\frac{\partial A_j}{\partial r_k} \\
&\quad + \frac{\vec{\sigma}}{2E_i^*} \cdot (\nabla \times \frac{\partial \vec{A}}{\partial r_k}) \\
\frac{d\vec{\sigma}}{dt} &= \frac{\vec{\sigma} \times (\nabla \times \vec{A})}{E_i^*},
\end{aligned}
$$

Detailed EOMs

EXTENDED AMPT MODEL



dynamical

neglect