Vorticity in high-energy heavy-ion collisions

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Workshop on AMPT for Relativistic Heavy Ion Collisions
四川大学-成都
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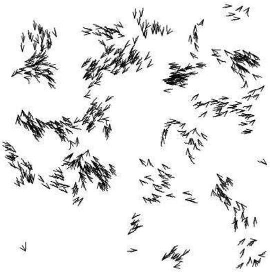
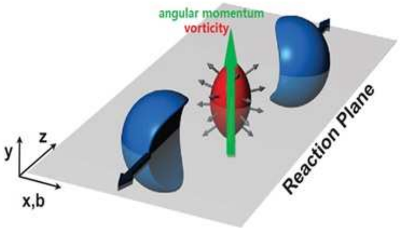
Outline

T. Vicsek et al., PRL 75,1226(1995)





1. **Vicsek flock with external fluid forcing.**
2. **Vorticity induced by angular momentum in HIC.**

PRL **106,** 192301 (2011) PHYSICAL REVIEW LETTERS week ending 13 MAY 2011

200 *A* GeV Au + Au Collisions Serve a Nearly Perfect Quark-Gluon Liquid

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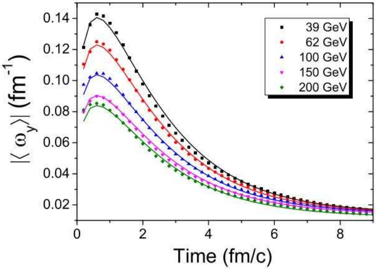
(Received 11 November 2010; published 9 May 2011)

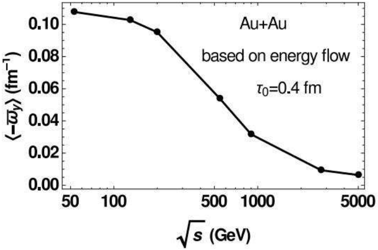
A new robust method to extract the specific shear viscosity $(\eta/s)_{QGP}$ of a quark-gluon plasma (QGP) at temperatures $T_c < T \lesssim 2T_c$ from the centrality dependence of the eccentricity-scaled elliptic flow v_2/ε measured in ultrarelativistic heavy-ion collisions is presented. Coupling viscous fluid dynamics for the QGP with a microscopic transport model for hadronic freeze-out we find for 200 *A* GeV Au + Au collisions that v_2/ε is a universal function of multiplicity density $(1/S)(dN_{ch}/dy)$ that depends only on the viscosity but not on the model used for computing the initial fireball eccentricity ε. Comparing with measurements we find $1 < 4\pi(\eta/s)_{QGP} < 2.5$ where the uncertainty range is dominated by model uncertainties for the values of ε used to normalize the measured v_2.

Introduction of vorticity in HIC

Y. Jiang, Z. W. Lin, J. Liao,
PRC 94,044910(2016)

X. G. Huang and W. T. Deng,
PRC 93,064907(2016)

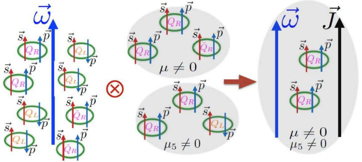




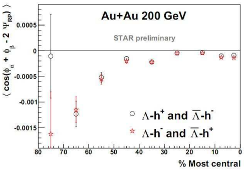
1. **Vorticity as a function of time at various impact parameter.**
2. **The double-averaged vorticity is dependent on collision energy.**

Introduction of vorticity in HIC

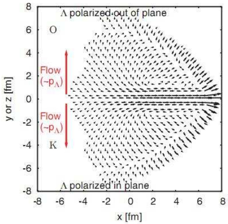
D.E. Kharzeev et al., PPNP 88,1(2016)



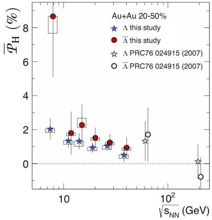
G. Wang, L. Wen, 1609.05506



B. Betz et al., PRC 76,044901(2007)



STAR, 1701.06657(2017)



1. **What are the observations of vorticity fields? May be the charged particles separation, Lambda polarization, or others.**

Method: smearing function in AMPT

It is begin the particle distribution function $f(r, p)$ in AMPT

$$f(r,p) = \frac{1}{N}\sum_i (2\pi)^3 \delta^3[p - p_i(t)]\Phi(r, r_i) \quad (1)$$

where $N = \int dr\Phi(r, r_i)$, and $\Phi(r, r_i)$ is a Gaussian function,

$$\Phi(r, r_i) = \frac{K}{\tau_0\sqrt{2\pi\sigma_\eta^2 2\pi\sigma_r^2}} exp[-\frac{(x-x_i)^2 + (y-y_i)^2}{2\sigma_r^2} - \frac{(\eta-\eta_i)^2}{2\sigma_\eta^2}] \quad (2)$$

Then the energy-momentum tensor and particle number current are given by:

$$T^{\mu\nu} = \int \frac{d^3p}{(2\pi)^3}\frac{p^\mu p^\nu}{p^0} f(r,p) = \frac{1}{N}\sum_i \frac{p_i^\mu p_i^\nu}{p_i^0}\Phi(r, r_i), \quad (3)$$

$$J^\mu = \int \frac{d^3p}{(2\pi)^3}\frac{p^\mu}{p^0} f(r,p) = \frac{1}{N}\sum_i \frac{p_i^\mu}{p_i^0}\Phi(r, r_i). \quad (4)$$

W. T. Deng and X. G. Huang, PRC 93,064907(2016)

Method: smearing function in AMPT

These give

$$T^{0a} = \frac{1}{N}\sum_i p_i^a \Phi(r, r_i), \tag{5}$$

$$T^{\mu\nu} = \frac{1}{N}\sum_i p_i^0 \Phi(r, r_i), \tag{6}$$

$$T^{ab} = \frac{1}{N}\sum_i \frac{p_i^a p_i^b}{p_i^0} \Phi(r, r_i), \tag{7}$$

$$J^0 = \frac{1}{N}\sum_i \Phi(r, r_i), \tag{8}$$

$$J^a = \frac{1}{N}\sum_i \frac{p_i^a}{p_i^0} \Phi(r, r_i). \tag{9}$$

W. T. Deng and X. G. Huang, PRC 93,064907(2016)

Method: smearing function in AMPT

Thus we can identify the velocity of the particle flow and the velocity of energy flow

$$\nu_1 = \frac{J^a}{J^0}, \quad v_1^a(r) = \frac{1}{\sum_i \Phi(r, r_i)} \sum_i \frac{p_i^a}{p_i^0} \Phi(r, r_i), \tag{10}$$

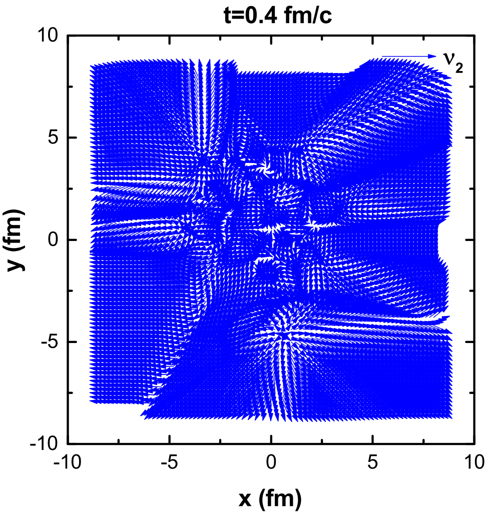
$$\nu_2 = \frac{T^{0a}}{T^{00} + T^{aa}}, \quad v_2^a(r) = \frac{\sum_i p_i^a \Phi(r, r_i)}{\sum_i [p_i^0 + (p_i^a)^2 / p_i^0] \Phi(r, r_i)}. \tag{11}$$

The vorticity field are calculated by:

$$\omega_1 = \nabla \times \nu \tag{12}$$

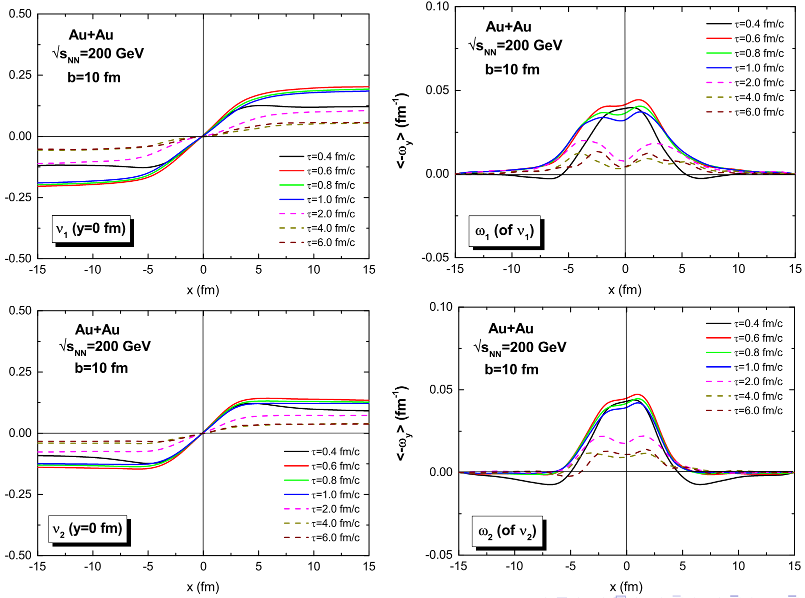
$$\omega_2 = \gamma^2 \nabla \times \nu \tag{13}$$

Calculations: time evolution of vorticity

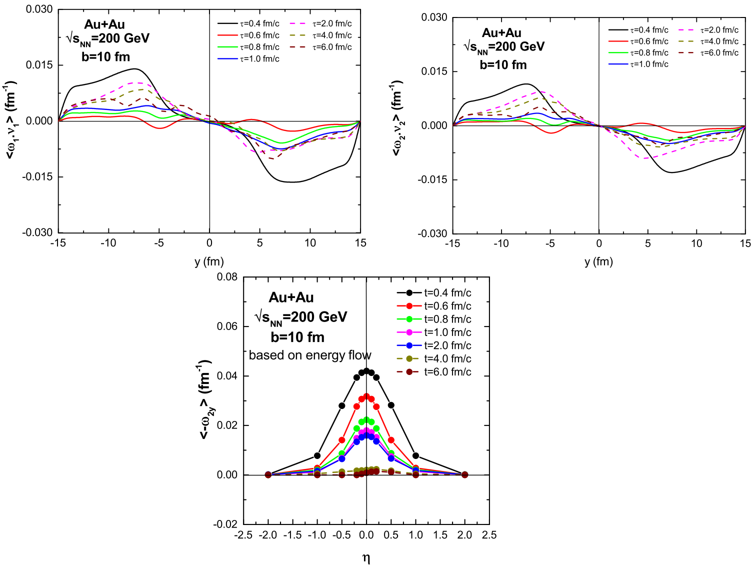


Local vorticity in Au+Au collision at fixed impact parameter (b=10 fm)

Evolution of vorticity



Calculations: time evolution of vorticity



Kinematical vorticity (K-vort) F. Becattini,EPJC 75,406(2015)

$$\omega_{\mu\upsilon} = \partial_\nu u_\mu - \partial_\mu u_\nu \tag{14}$$

Temperature vorticity (T-vort)

$$\Omega_{\mu\upsilon} = \partial_\nu(Tu_\mu) - \partial_\mu(Tu_\nu) \tag{15}$$

Thermal vorticity (th-vort)

$$\widetilde{\omega}_{\mu\upsilon} = \partial_\nu(\beta_\mu) - \partial_\mu(\beta_\nu) \tag{16}$$

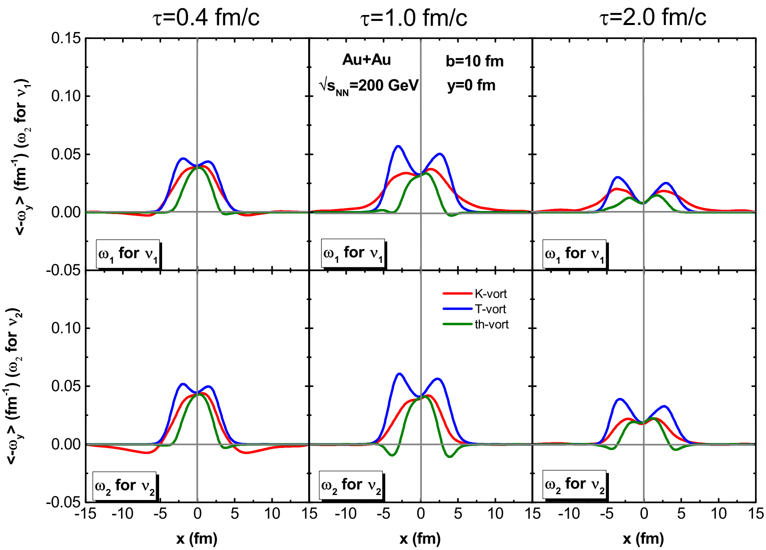
where $\beta = 1/T$, T is the local tempareture.

One can obtain the relations T-vort and K-vort, th-vort and K-vort:

$$\Omega_{\mu\upsilon} = (\partial_\nu T)u_\mu - (\partial_\mu T)u_\nu + T\omega_{\mu\upsilon} \tag{17}$$

$$\widetilde{\omega}_{\mu\upsilon} = -\frac{1}{T^2}\{(\partial_\mu T)u_\nu - (\partial_\nu T)u_\mu\} + \frac{1}{T}\omega_{\mu\upsilon} \tag{18}$$

Calculations: time evolution of vorticity



Polarization of particle at freeze-out time (F. Becattini et al., EPJC 75,406(2015))

$$\Pi^{\mu}(p) = \frac{1}{8m}\frac{\int_{\sigma}\sigma_{\lambda}p^{\lambda}n_F(1-n_F)p_{\sigma}\varepsilon^{\mu\nu\rho\sigma}\partial_{\nu}\beta_{\rho}}{\int_{\sigma}d\sigma_{\lambda}p^{\lambda}n_F} \quad (19)$$

$$= \frac{1}{8m}\frac{\sum_{\sigma}\triangle\sigma_{\lambda}p^{\lambda}n_F(1-n_F)p_{\sigma}\varepsilon^{\mu\nu\rho\sigma}\partial_{\nu}\beta_{\rho}}{\sum_{\sigma}\triangle\sigma_{\lambda}p^{\lambda}n_F} \quad (20)$$

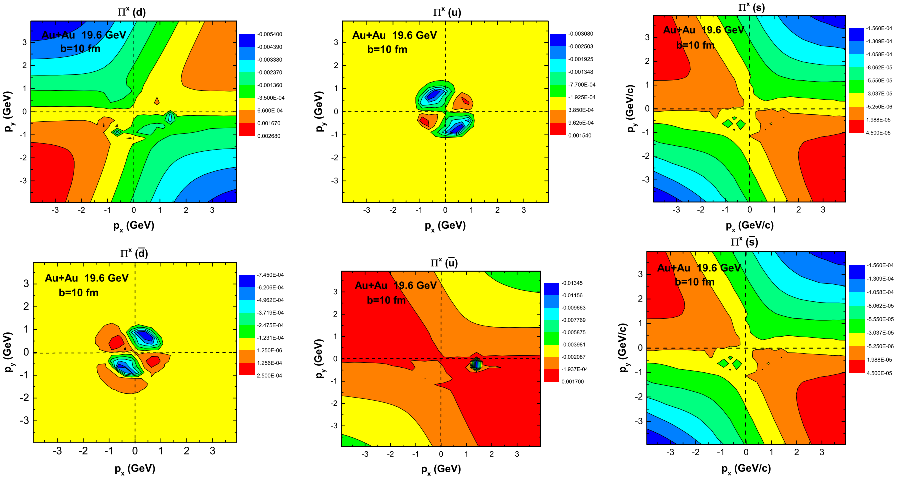
where the Fermi distribution is $n_F = exp[\frac{u^{\mu}p_{\mu}-\mu_i}{T_f}]+1$,
μ_i is the chemical potential (here μ_i=0), and T_f is the freeze-out temperature here we take in 0.137 GeV.
The thermal vorticity is $\Omega_{\mu\upsilon} = \partial_{\nu}\beta_{\mu} - \partial_{\mu}\beta_{\nu}$.
The freeze-out hypersurface is

$$\sigma = [\sigma^0(x,y,\eta_s),\sigma^1(x,y,\eta_s),\sigma^2(x,y,\eta_s),\sigma^3(x,y,\eta_s)] \quad (21)$$

$$= [\tau_f(x,y,\eta_s)cosh\eta_s, x, y, \tau_f(x,y,\eta_s)sinh\eta_s] \quad (22)$$

where τ_f, is the freeze-out time.

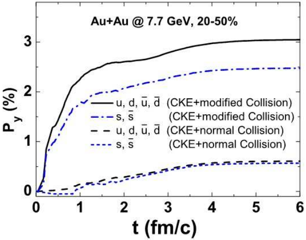
Calculations: partons polarization

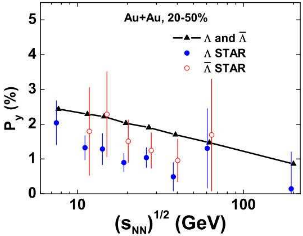


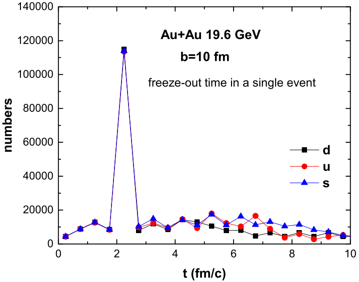
Polarization are difference for various chiral partons

Calculations: partons polarization

Y. Sun and C. M. Ko, 1706.09467





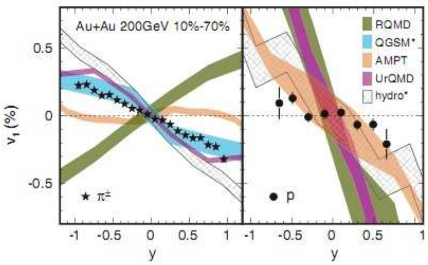


$\Lambda/\bar{\Lambda}$ polarization (S. Shi and H. Li talks)

Calculations: vorticity and directed flow

It is still no good enough for η dependence directed flow on the collision dynamics.

STAR PRL 108,202301(2012)



Calculations: vorticity and directed flow

Gaussian function in $x-\eta$ plane

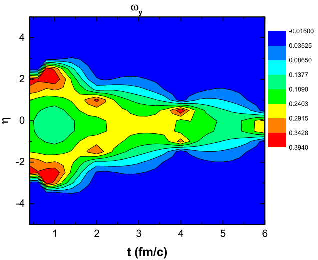
$$\Phi(r,r_i) = \frac{K}{\tau_0\sqrt{2\pi\sigma_\eta^2 2\pi\sigma_r^2}} exp[-\frac{(x-x_i)^2+(0.0-y_i)^2}{2\sigma_r^2} - \frac{(\eta-\eta_i)^2}{2\sigma_\eta^2}] \quad (23)$$

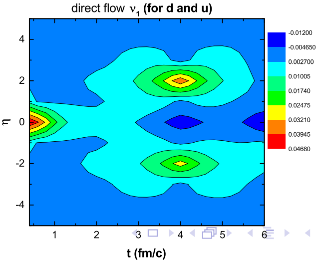
Vorticity and directed flow?

$$\omega_y = \frac{\partial u_x}{\partial \eta} - \frac{\partial u_\eta}{\partial x} \quad (24)$$

$$\nu_1 = \langle cos\phi \rangle = \langle \frac{p_x}{p_T} \rangle \quad (25)$$

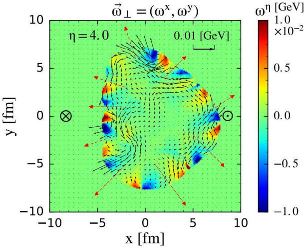
It is still argue that the probe of directed flow is in very earliest stage ($\frac{R}{\gamma} \leq 0.1 fm/c$, NPA 705(PHENIX)) of the collision dynamics.

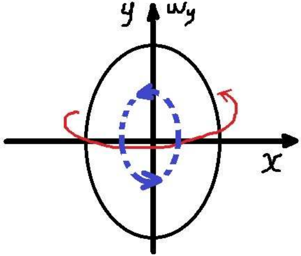


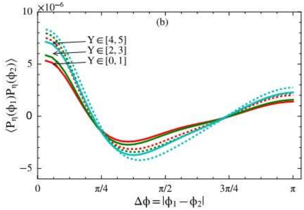


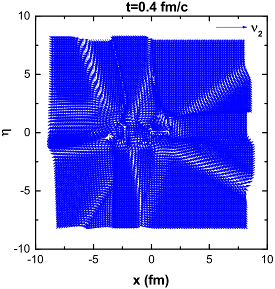
Calculations: longitudinal vorticity

L. G. Pang et al., PRL 117,192301(2016)









Calculations: longitudinal vorticity and charge separation

G. L. Ma, X. G. Huang, PRC 91,054901(2015)

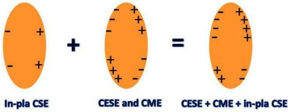
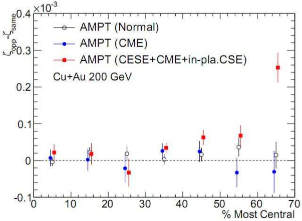
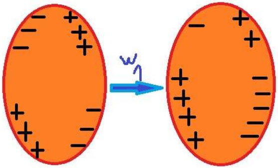


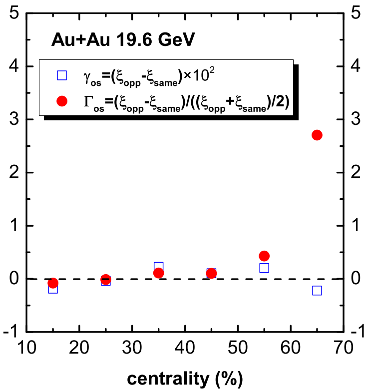
FIG. 3: (Color online) Illustration of the charge configuration of chiral magnetic effect (CME) plus chiral electric separation effect (CESE) plus in-plane charge separation effect (in-plane CSE) in Cu + Au collisions.





What is the observation of longitudinal vorticity field?

Calculations: longitudinal vorticity and charge separation



$$\xi = \langle cos[2(\phi_\alpha + \phi_\beta - 2\Psi_{RP})]\rangle, \quad (26)$$

$$\Gamma_{os} = \frac{\xi_{opp} - \xi_{same}}{(\xi_{opp} + \xi_{same})/2} \quad (27)$$

The relative charge particle separate can be induced by the longitudinal vorticity field.

Summary

1. **There are no only the transverse but the longitudinal vorticity in the HIC, the transverse vorticity fields are mainly stay in the mid-rapidity range.**

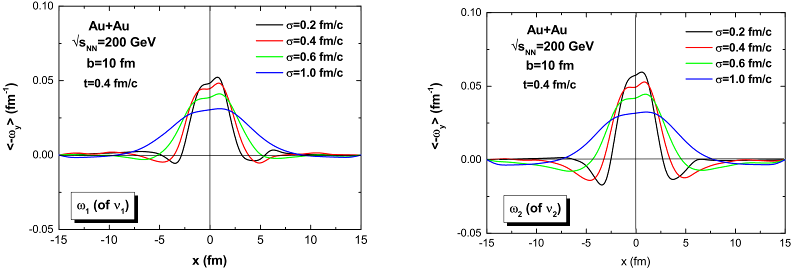
2. **The polarization of partons are chiral dependence.**

3. **Directed flow may be contribute from the vorticity.**

4. **The relative charge particle separate can be induced by the longitudinal vorticity field.**

Thanks for your attention!

backup



Vorticity are sensitive to the Gaussian wave packet width.

the normal vector on the freeze-out hypersurface is

$$d^3\sigma_\alpha = (1, -\frac{\partial\tau_f}{\partial x}, -\frac{\partial\tau_f}{\partial y}, -\frac{\partial\tau_f}{\partial\eta_s})\sqrt{-det\ g}\,dxdyd\eta_s \quad (28)$$

$$= (1, -\frac{\partial\tau_f}{\partial x}, -\frac{\partial\tau_f}{\partial y}, -\frac{\partial\tau_f}{\partial\eta_s})\tau_f dxdyd\eta_s \quad (29)$$

in the Bjorken ($\sigma = (\tau_f, x, y, \eta)$, $p^\mu = (p^\tau, p_T cos\phi, p_T sin\phi, p^{\eta_s})$, where $p^\tau = m_T cosh(y-\eta_s), p^{\eta_s} = m_T sinh(y-\eta_s)$)result , the volume element of hypersurface is given by

$$\triangle\sigma^\tau = s^\tau\tau\triangle x\triangle y\triangle\eta, \quad (30)$$

$$\triangle\sigma^x = s^x\tau\triangle\tau\triangle y\triangle\eta, \quad (31)$$

$$\triangle\sigma^y = s^x\tau\triangle\tau\triangle x\triangle\eta, \quad (32)$$

$$\triangle\sigma^\eta = s^\eta\frac{1}{\tau}\triangle\tau\triangle x\triangle y. \quad (33)$$

where $s^\mu = -sign(\frac{\partial\tau}{\partial x^\mu})$.

in the classical space-time ($\sigma = (\tau_f, x, y, z)$, $p^\mu = (E, p_x, p_y, p_z)$), we can give the volume element of hypersurface as

$$\triangle\sigma^t = s^t t_f \triangle x \triangle y \triangle z, \quad (34)$$

$$\triangle\sigma^x = s^x t_f \triangle t \triangle y \triangle z, \quad (35)$$

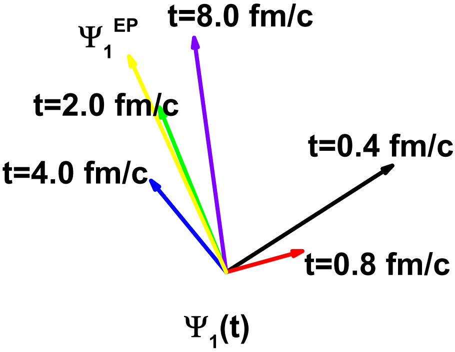
$$\triangle\sigma^y = s^x t_f \triangle t \triangle x \triangle z, \quad (36)$$

$$\triangle\sigma^z = s^z t_f \triangle t \triangle x \triangle y. \quad (37)$$

Then we can consider the volume element of hypersurface is chosen if

$$(f_{i+1} - f^*)(f^* - f_i) \geq 0 \quad (38)$$

where f_{i+1} and f_i respectively temperature field in the adjacent cells, f^* is the freeze-out temperature we take in 0.137 GeV.



Fluctuating of Ψ_i in difference time step.

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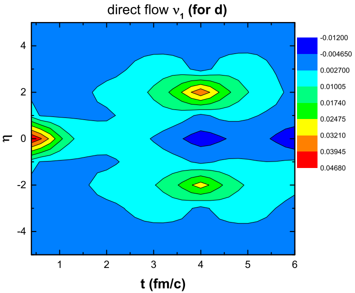
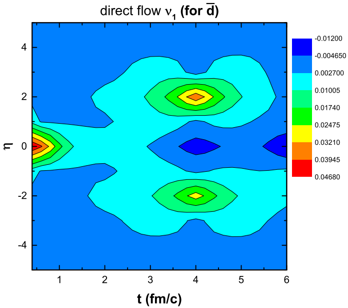

direct flow v_1 (for d)
-0.01200
-0.004650
0.002700
0.01005
0.01740
0.02475
0.03210
0.03945
0.04680
η
4
2
0
-2
-4
1
2
3
4
5
6
t (fm/c)


direct flow v_1 (for $\bar{d}$)
-0.01200
-0.004650
0.002700
0.01005
0.01740
0.02475
0.03210
0.03945
0.04680
η
4
2
0
-2
-4
1
2
3
4
5
6
t (fm/c)