



A Cold Atom Experiment

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Outline

Hydro flow or escape?

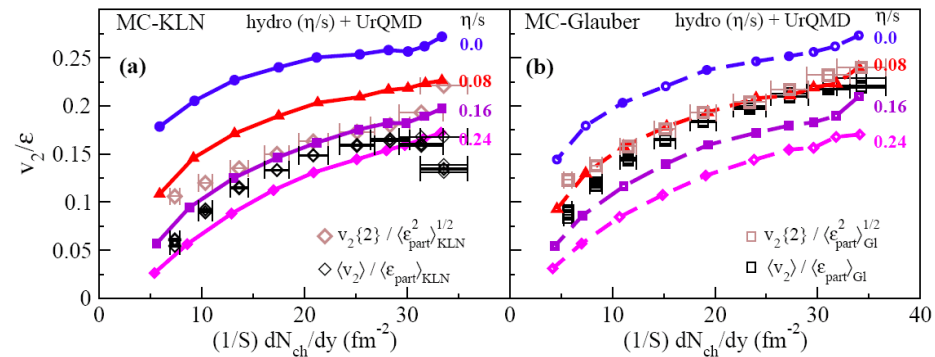
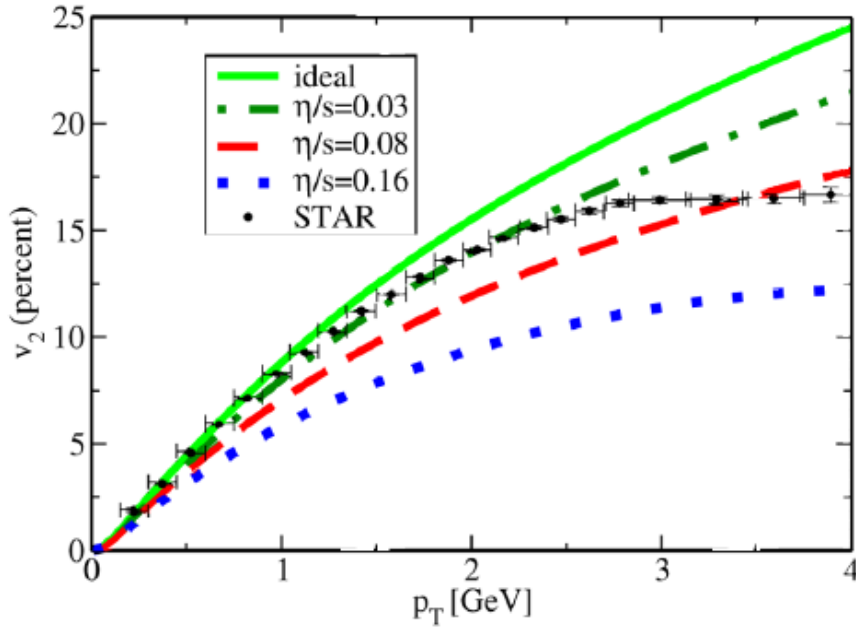
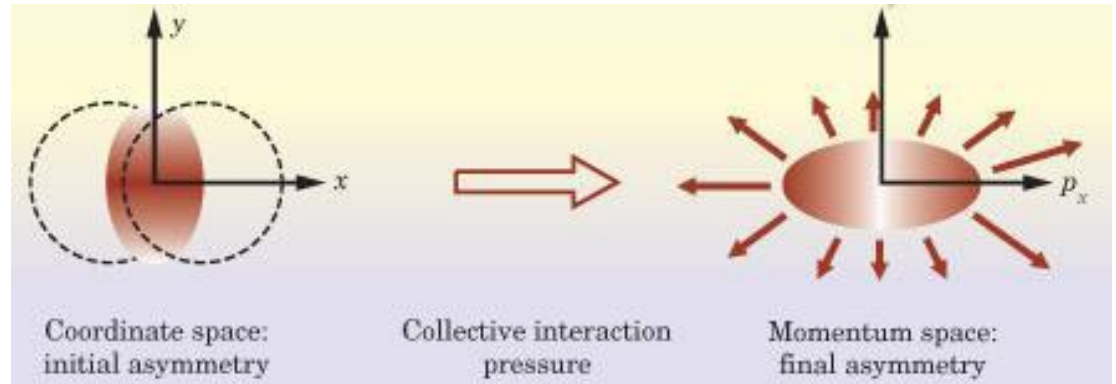
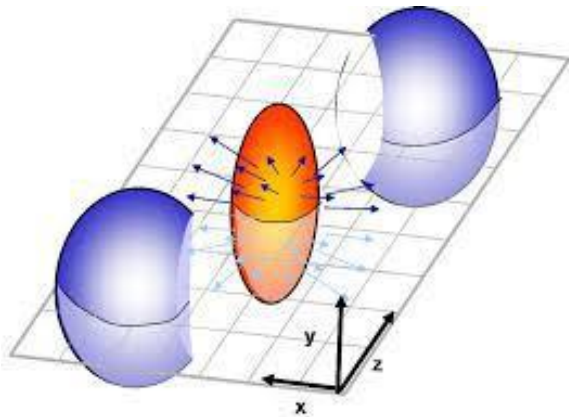
Motivation for a cold atom experiment

Cold atom laboratory

To verify the uncertainty principle*

Summary

Heavy ion experiment



Model: Song *et al.* [arXiv:1011.2783](https://arxiv.org/abs/1011.2783)

The Hydrodynamics Paradigm



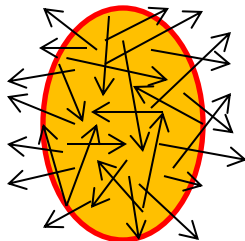
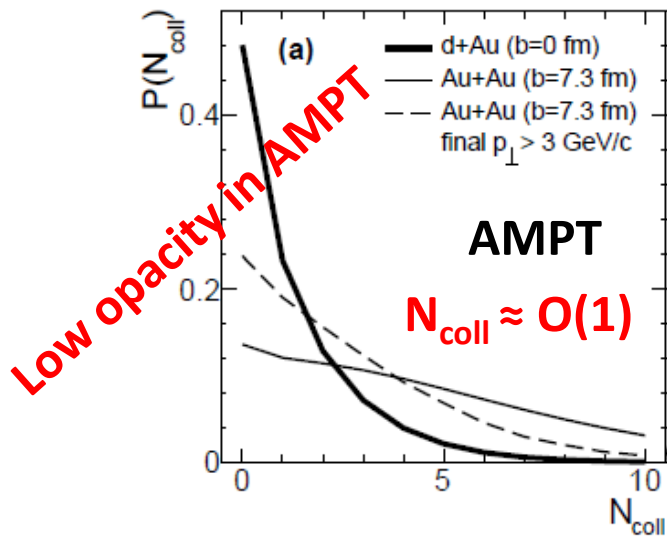
Is it really hydro?

Mean free path: $L_{\text{mfp}} = 1/\rho\sigma$
 Ncoll = Opacity = $L/L_{\text{mfp}} = \rho\sigma L$

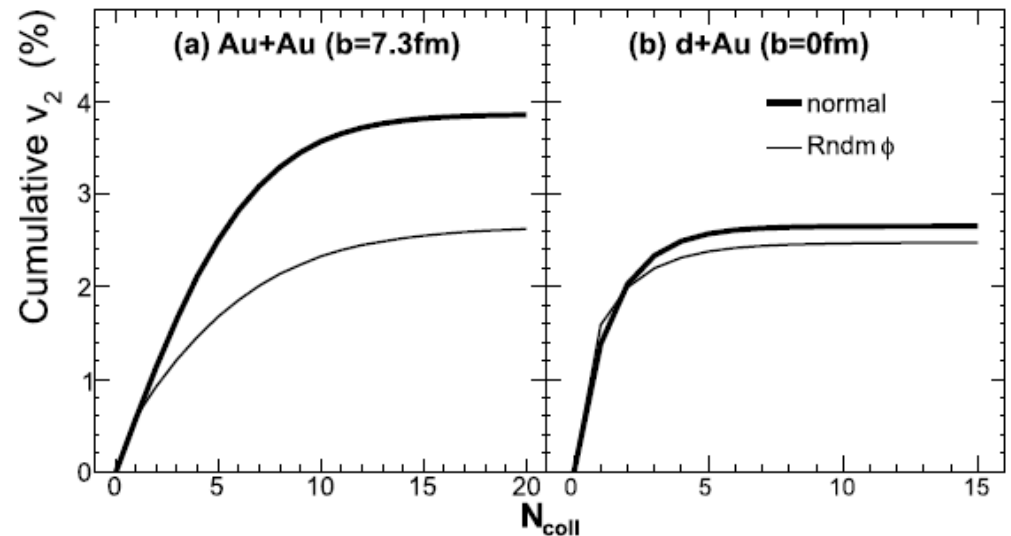
Heavy ion collision:

$dN/dy \sim 1000$, $\rho \sim 1000/\pi R^2 \tau \sim 6 \text{ fm}^{-3}$

$\rho\sigma L \sim 7\text{fm}^{-3} * 3\text{mb} * 3\text{fm} \sim 5$

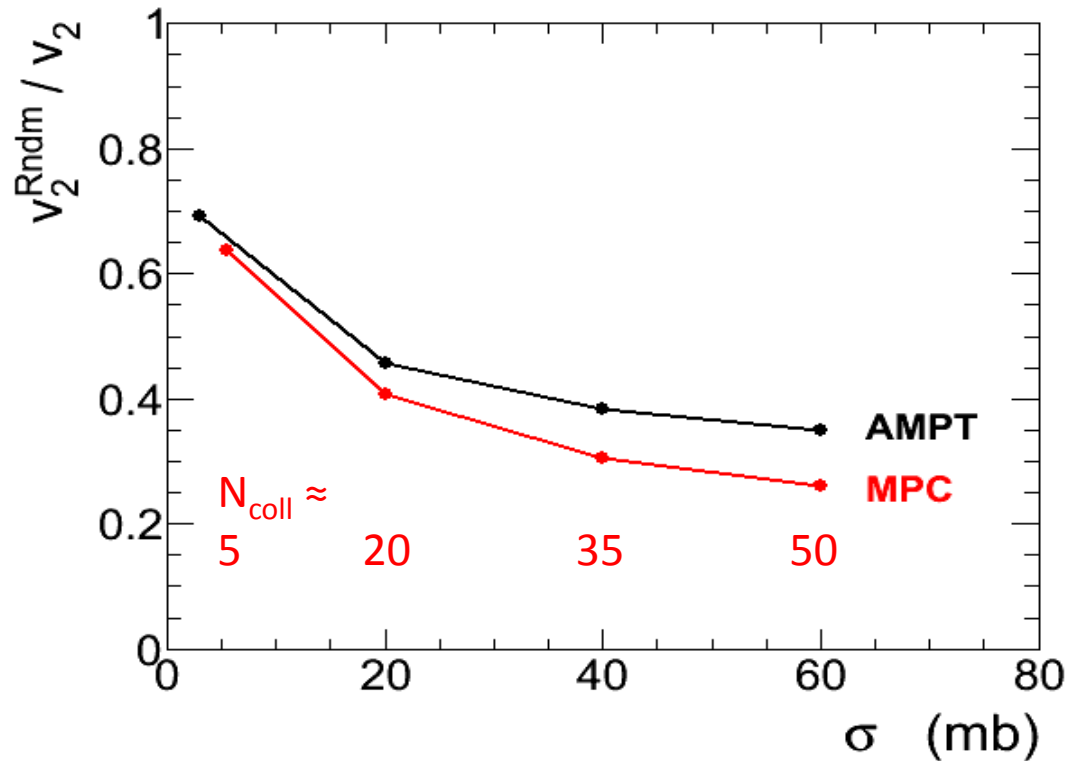


He, FW, et al. PLB753(16)506



Hydro questionable!

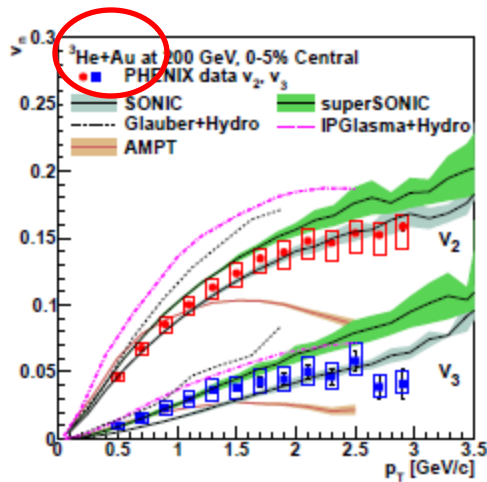
Relative escape contribution



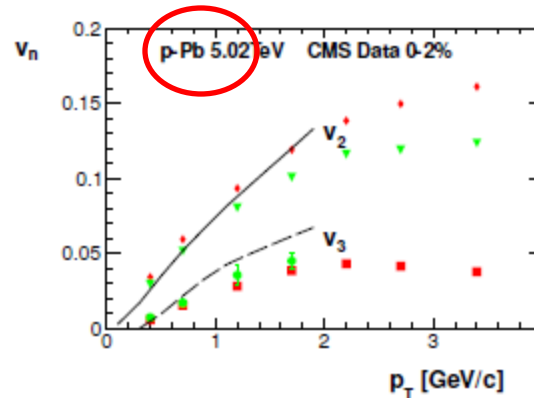
- Escape contribution still sizeable even at x10 larger x-sections.

Strong anisotropy, but no energy loss

Small system collisions



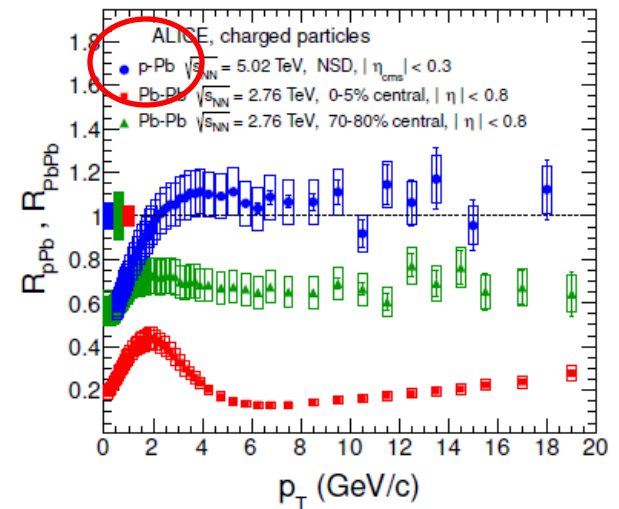
PHENIX, arXiv:1507.06273



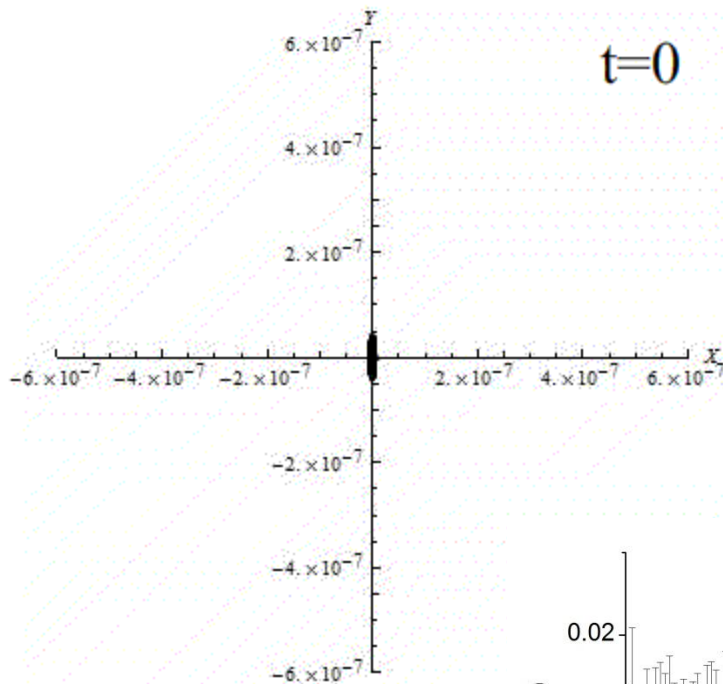
PB, W.Broniowski, G. Torrieri arXiv:1306.5442; G.Y. Qin, B.

Müller 1306.3439; I. Kozlov et al. 1405.3976; A. Bzdak et al.

1304.34003, K. Kawaguchi et al. Poster 206

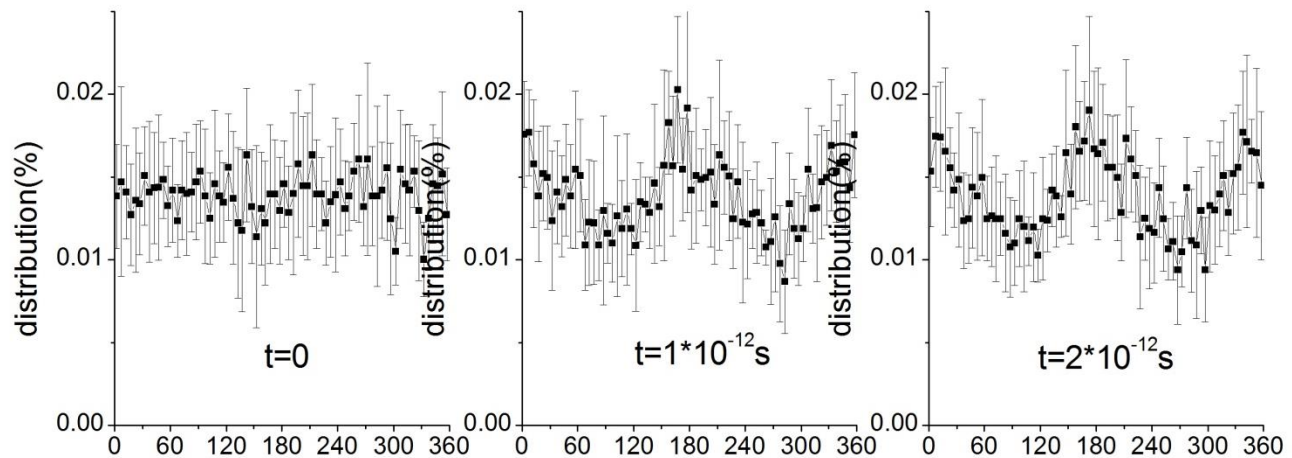


Flow from Coulomb interaction

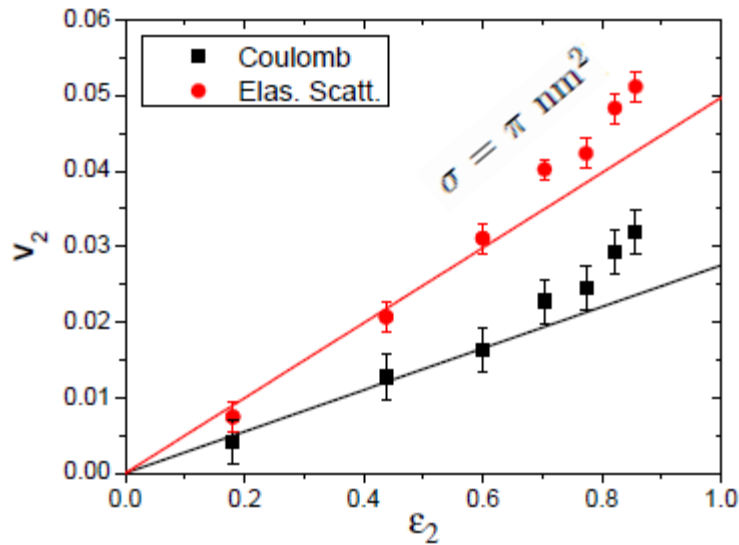


- Coulomb interactions
- Can change trap size and anisotropy, and ion density

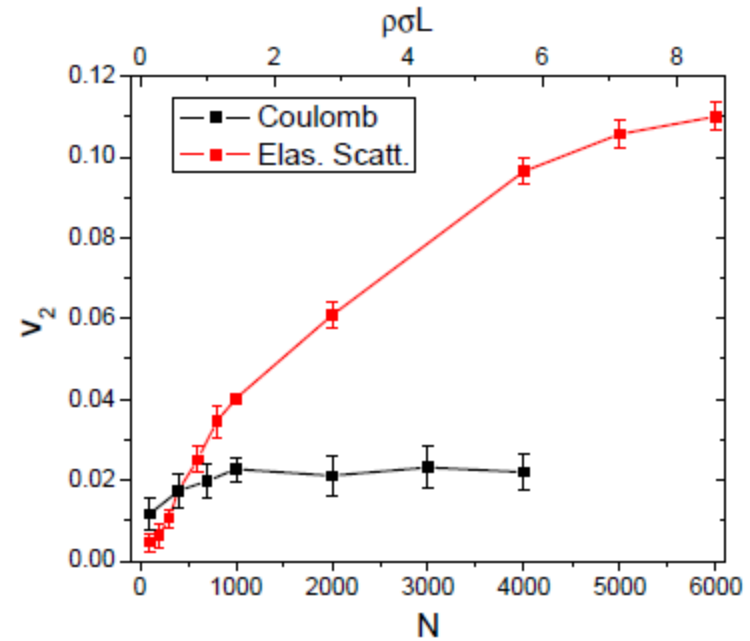
- $a = 10$ nm, $b = 80$ nm, $c = 100$ nm
- Number of ions = 1000
- Initial thermal velocity 2000 m/s



Flow from Coulomb and Elastic scattering

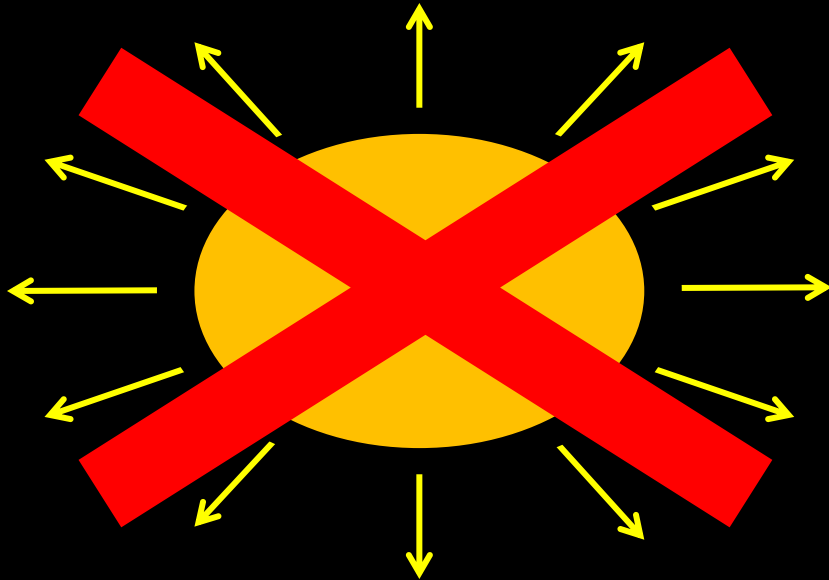


Trap size: $ab = 240 \text{ nm}^2$, $c = 50 \text{ nm}$.
Number of particles: 1000

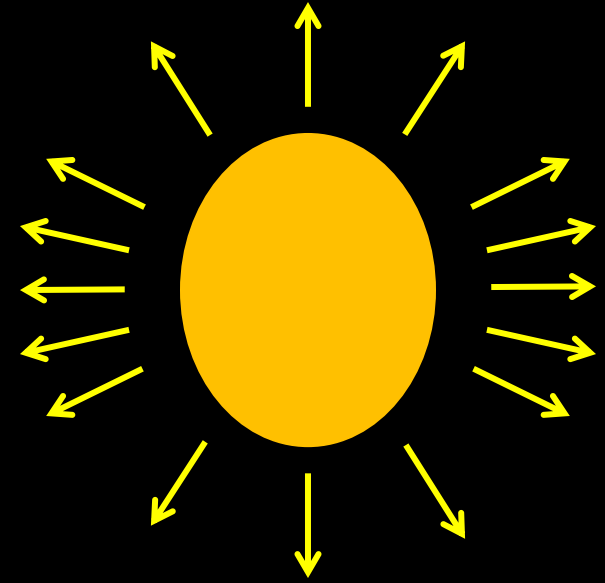


$a = 10 \text{ nm}$, $b = 24 \text{ nm}$, $c = 50 \text{ nm}$

Anisotropy mechanism



**Expansion, flow
Hydro paradigm**



**No expansion
Escape**

DATA

Heavy ion collisions

Low density/opacity
Mundane physics

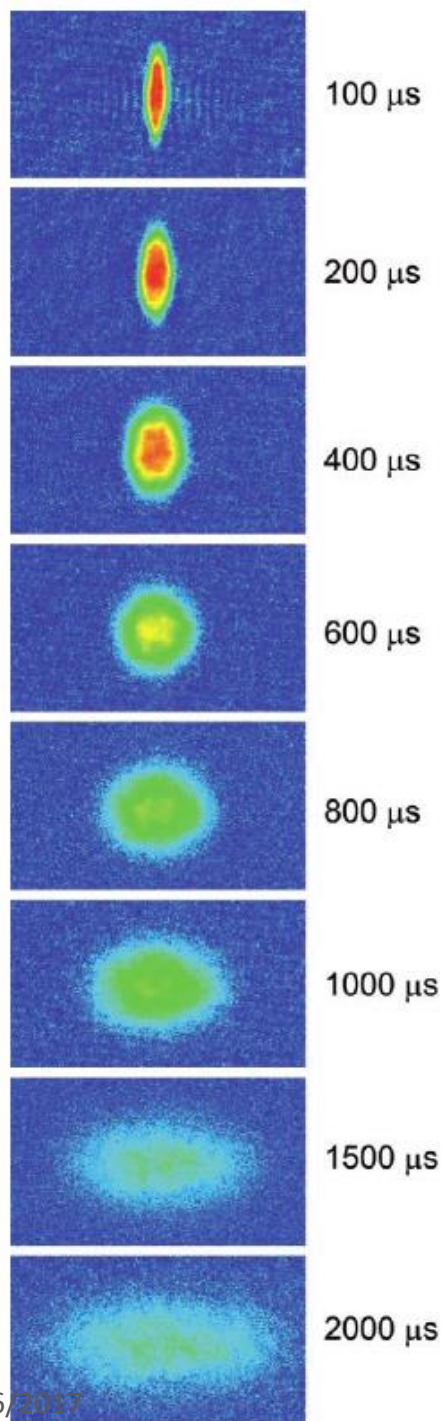
Perfect liquid
Hydrodynamics

Need experimental test!

Another system with large anisotropy

Large elliptic anisotropy
in cold atom systems
consistent with hydrodynamic flow

K. M. O'Hara *et al.*, Science 298, 2179 (2002)



Opacity

Mean free path: $L_{\text{mfp}} = 1/\rho\sigma$, $N_{\text{coll}} = \text{Opacity} = L/L_{\text{mfp}} = \rho\sigma L$

Cold atom system:

$$\begin{aligned}
 a &\approx 5 \times 10^{-5} \text{ cm} \\
 \sigma_{\text{int}} &\approx 10^{-8} \text{ cm}^2 \\
 \rho &\approx 5 \times 10^{13} / \text{cm}^3 \\
 L_{\text{mfp}} &\approx 2 \times 10^{-6} \text{ cm} \\
 L &\approx 2 \times 10^{-3} \text{ cm} \\
 \mathbf{L/L_{\text{mfp}} \approx 1000}
 \end{aligned}$$

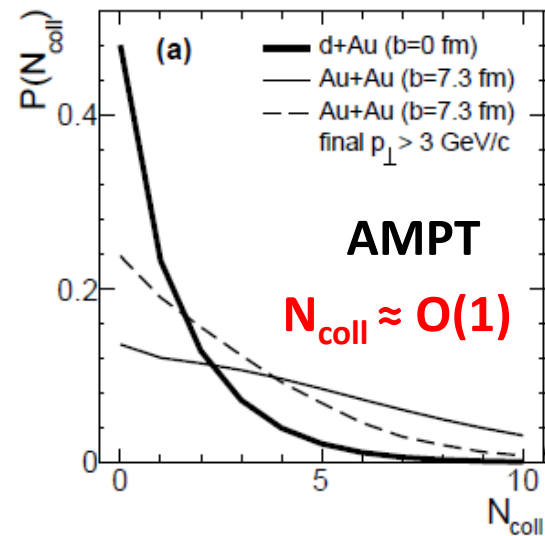
Very high opacity for the cold atom system

Indeed hydro!

Heavy ion collision:

$dN/dy \sim 1000$, $\rho \sim 1000/\pi R^2 \tau \sim 6 \text{ fm}^{-3}$

$\rho\sigma L \sim 7 \text{ fm}^{-3} * 3 \text{ mb} * 3 \text{ fm} \sim 5$

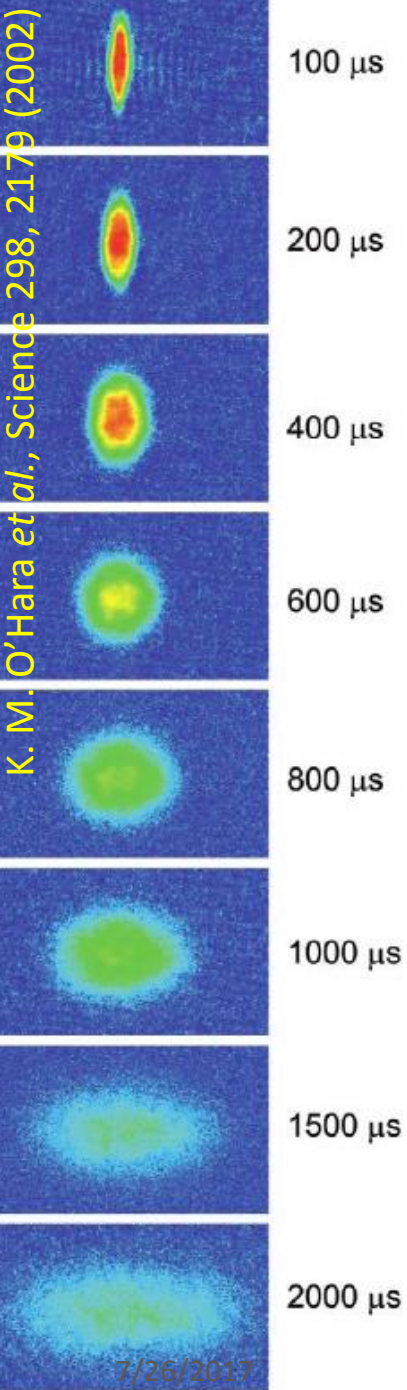


Low opacity in AMPT

Hydro??

K. M. O'Hara et al., Science 298, 2179 (2002)

To emulate QGP with cold atoms



$\rho\sigma_{\text{int}}L$: reduce opacity by 10^3
 $1000 \rightarrow 1$

Opacity: $\rho\sigma L$

$$a \approx 5 \times 10^{-5} \text{ cm}$$

$$\sigma_{\text{int}} \approx 10^{-8} \text{ cm}^2$$

$$\rho \approx 5 \times 10^{13} / \text{cm}^3$$

$$L_{\text{mfp}} \approx 2 \times 10^{-6} \text{ cm}$$

$$L \approx 2 \times 10^{-3} \text{ cm}$$

$$L/L_{\text{mfp}} \approx \text{1000}$$

$0-1$

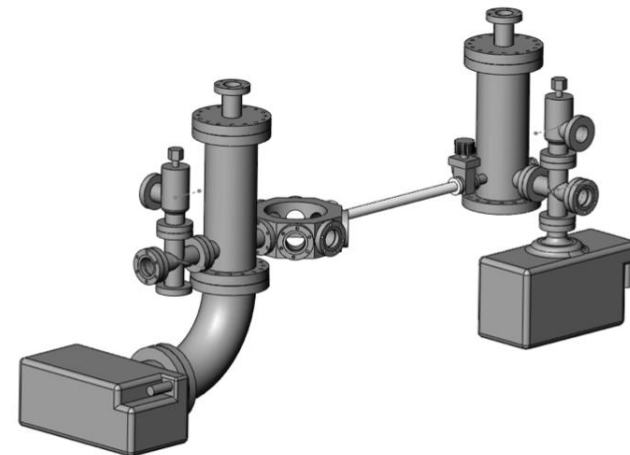
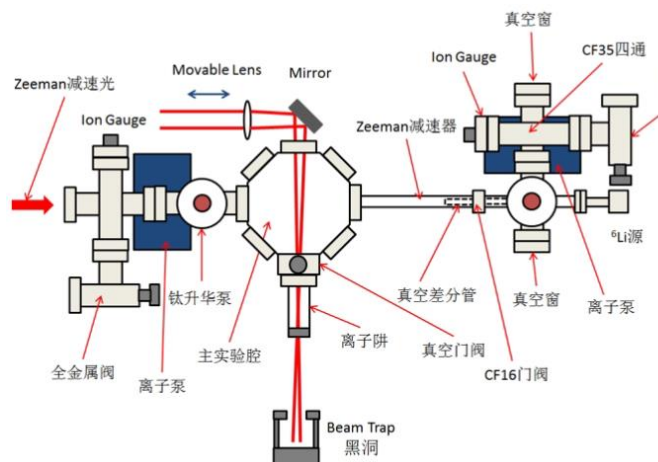
$\times 10^{-3}$

Very high opacity for
the cold atom system

Low opacity for the
cold atom system

A cold atom experiment

Setting up a cold atom experiment in Huzhou University



Summary

- Close connections between hot QGP and cold atoms.
- Cold atoms are hydrodynamical; QGP may not be.

Make it less interacting, more dilute, or smaller to mimic QGP.

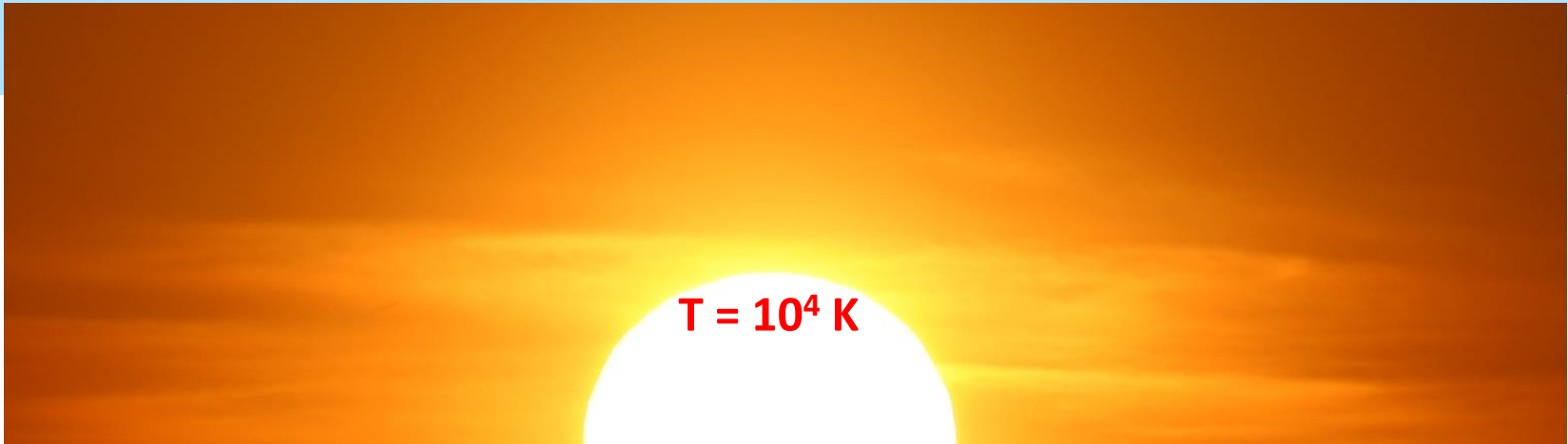
- QGP is quantum mechanical; cold atoms are “classical.”

Make it smaller, or colder to mimic QGP, and measure the uncertainty principle.

- Use controllability of cold atom system to address QGP questions

To verify the uncertainty principle*

Hot QGP vs Cold Atoms

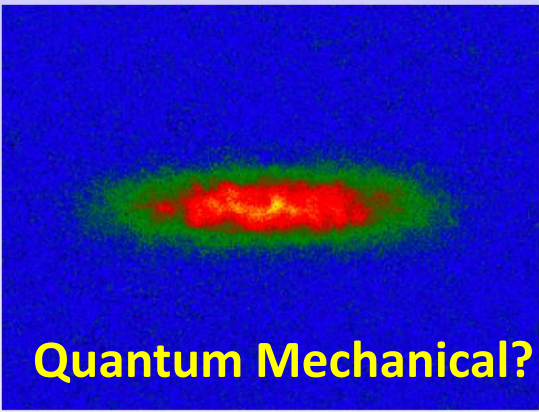


T = 10⁴ K



Classical?

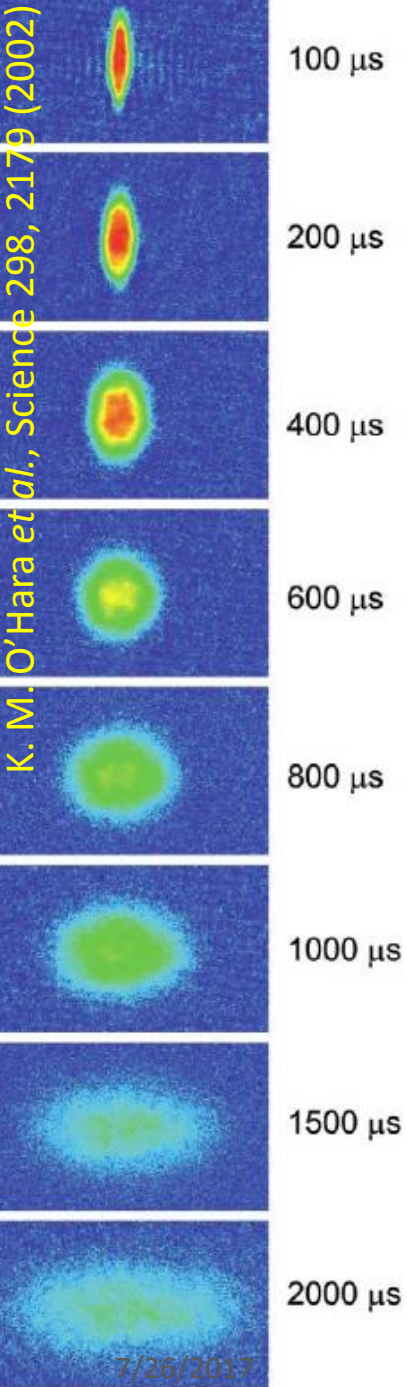
**Quark-gluon plasma T = 10¹² K BIG BANG
Computer simulation of RHIC collision**



Quantum Mechanical?

**Ultracold atomic gas
T = 10⁻⁷ K**

Is QGP classical?



K. M. O'Hara et al., Science 298, 2179 (2002)



Li: $M \sim 6000 \text{ MeV}$
 $T \sim 1 \mu\text{K} \sim 10^{-16} \text{ MeV}$
 $x \sim 20 \mu\text{m}, y \sim 100 \mu\text{m}$
 $p \sim (TM)^{1/2} \sim 10^{-6} \text{ MeV}$

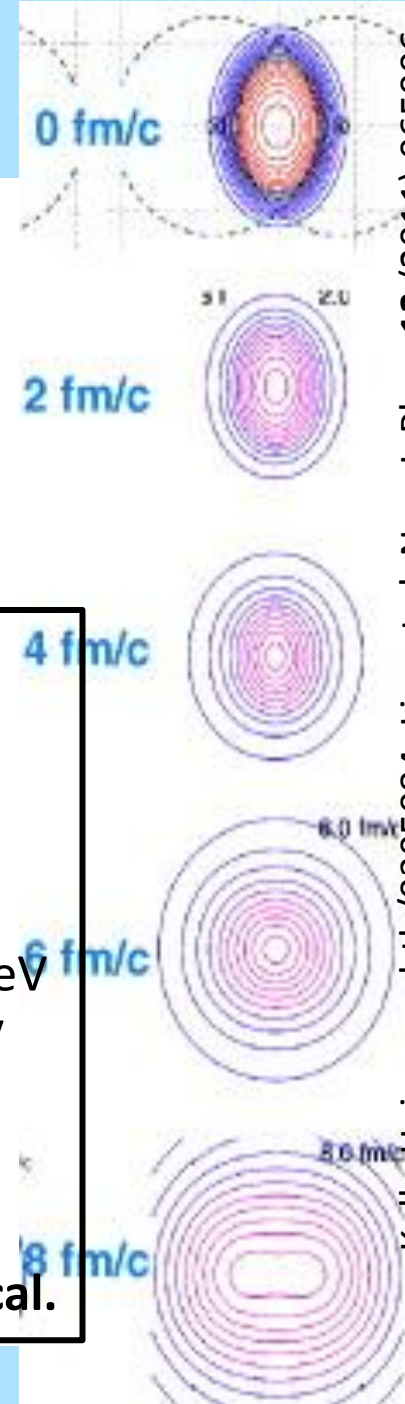
$p \text{ quan} \sim 1/r \sim 10^{-8} \text{ MeV}$
 $E \text{ quan} \sim 1/(mr^2) \sim 10^{-20} \text{ MeV}$
 Negligible!

Cold atoms are **hot**,
 “classical” w.r.t. trap size.

q,g: $M \sim 0 \text{ MeV}$
 $T \sim 200 \text{ MeV}$
 $x \sim 3 \text{ fm}, y \sim 4 \text{ fm}$
 $p \sim 200 \text{ MeV}$

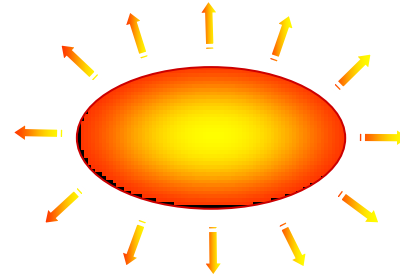
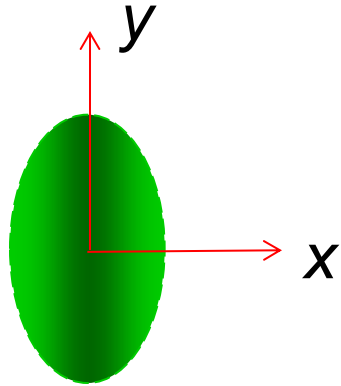
$p \text{ quan} \sim 1/r \sim 200 \text{ MeV}$
 $E \text{ quan} \sim 200 \text{ MeV}$
 Comparable!

QGP is **cold**,
 quantum mechanical.



Kolb, Heinz, nucl-th/0305084; Lisa et al. New J. Phys. 13 (2011) 065006

QM uncertainty principle



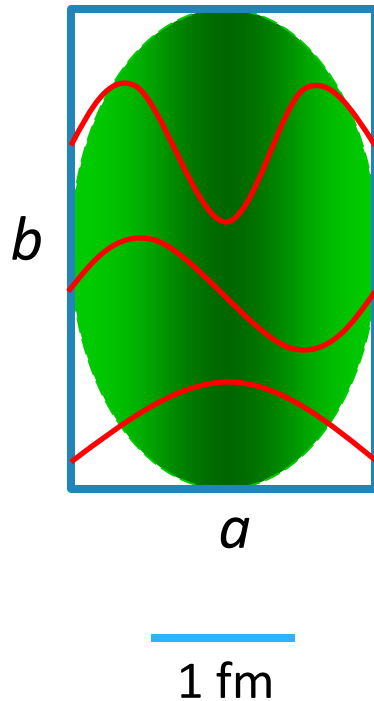
$$\Delta x \cdot \Delta p > \hbar / 2$$

$$p_x > p_y$$

$$\varepsilon = \frac{\langle y^2 \rangle - \langle x^2 \rangle}{\langle y^2 \rangle + \langle x^2 \rangle}$$

$$v_2 = \langle \cos 2\varphi \rangle = \frac{\langle p_x^2 \rangle - \langle p_y^2 \rangle}{\langle p_x^2 \rangle + \langle p_y^2 \rangle}$$

Infinite square well



$$-\frac{\hbar^2}{2m}\nabla^2\psi = E\psi \quad \Rightarrow \quad \psi \propto \begin{cases} \cos \frac{n_{\text{odd}}\pi}{a}x \\ \sin \frac{n_{\text{even}}\pi}{a}x \end{cases}$$

Take even mode for example:

$$\langle p_x^2 \rangle = \hbar^2 k^2 ; \quad \langle x^2 \rangle = \frac{a^2}{4} - \frac{2}{k^2} ; \quad k = \frac{n_{\text{odd}}\pi}{a}$$

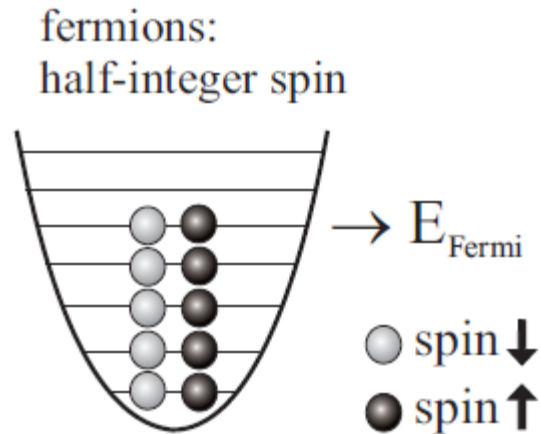
$$\sqrt{\langle p_x^2 \rangle \cdot \langle x^2 \rangle} = \hbar \sqrt{\frac{k^2 a^2}{4} - 2} = \hbar \sqrt{\frac{\pi^2}{4} n_{\text{odd}}^2 - 2} > \hbar / 2$$

$$v_2 = \frac{\langle p_x^2 \rangle - \langle p_y^2 \rangle}{\langle p_x^2 \rangle + \langle p_y^2 \rangle} = \frac{b^2 - a^2}{b^2 + a^2} = \varepsilon \quad \text{for all } n.$$

Single state anisotropy

Harmonic oscillator

$$\left(-\frac{\hbar^2}{2m} \nabla^2 + \frac{1}{2} m \omega^2 x^2 \right) \psi = E \psi ; \quad E = \left(n + \frac{1}{2} \right) \hbar \omega$$



$$\left\langle \frac{p_x^2}{2m} \right\rangle = \left\langle \frac{1}{2} m \omega^2 x^2 \right\rangle = \frac{E}{2} = \frac{1}{2} \left(n + \frac{1}{2} \right) \hbar \omega$$

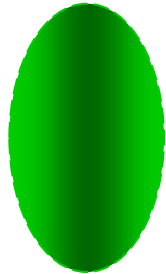
$$\sqrt{\langle p_x^2 \rangle \langle x^2 \rangle} = \left(n + \frac{1}{2} \right) \hbar$$

$$v_2 = \frac{\langle p_x^2 \rangle - \langle p_y^2 \rangle}{\langle p_x^2 \rangle + \langle p_y^2 \rangle} = \frac{\omega_x - \omega_y}{\omega_x + \omega_y}$$

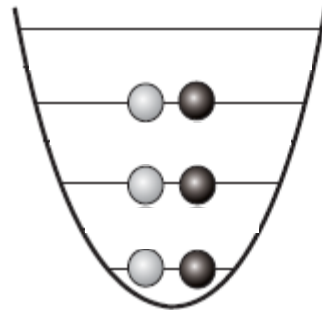
$$\varepsilon = \frac{\langle y^2 \rangle - \langle x^2 \rangle}{\langle y^2 \rangle + \langle x^2 \rangle} = \frac{\omega_x - \omega_y}{\omega_x + \omega_y}$$

$$v_2 = \varepsilon \quad \text{for each and all } n$$

Thermal probability



fermions:
half-integer spin

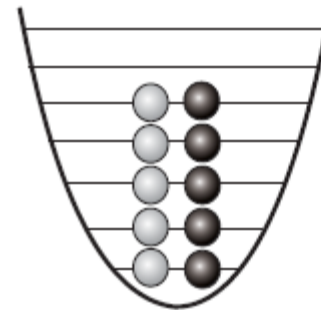


→ E_{Fermi}

○ spin ↓

● spin ↑

fermions:
half-integer spin



→ E_{Fermi}

○ spin ↓

● spin ↑

x, y at same Fermi energy, so different number of filled energy levels.

At high temperature, classical limit, sum is approximated by integral:

$$\frac{dN}{d\mathbf{p}} = N \frac{\int d\mathbf{r} e^{-H_1(\mathbf{p}, \mathbf{r})/T}}{\int d\mathbf{r} d\mathbf{p} e^{-H_1(\mathbf{p}, \mathbf{r})/T}} = N \frac{e^{-K(\mathbf{p})/T}}{\int d\mathbf{p} e^{-K(\mathbf{p})/T}}$$

then it's independent of potential.

It's isotropic at all temperature because $K=(p_x^2+p_y^2)/2m$ is isotropic.

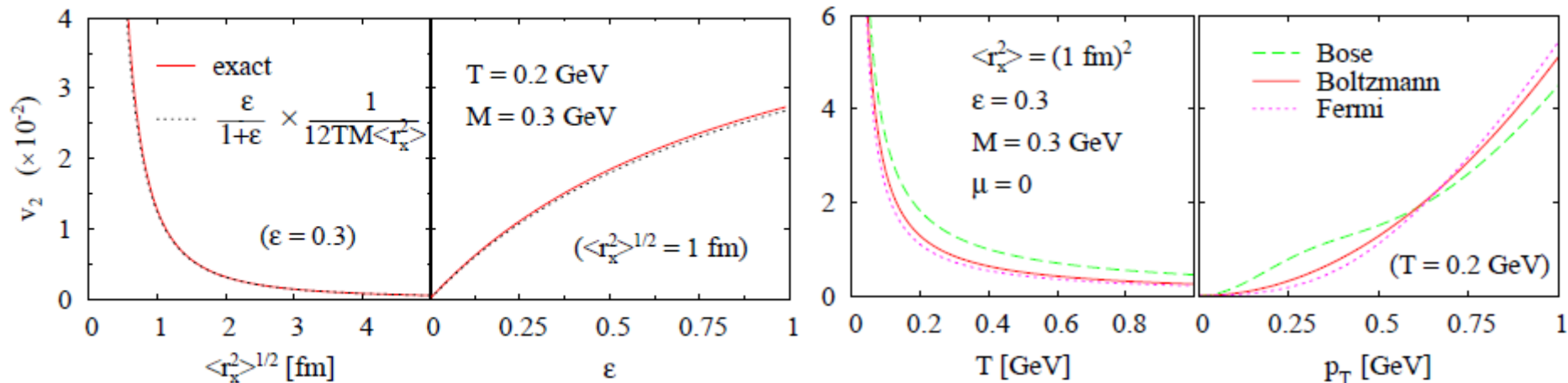
Thermal probability weight

$$\rho(\mathbf{r}) \equiv \frac{dN}{d\mathbf{r}} = \frac{1}{Z} \sum_j |\psi_j(\mathbf{r})|^2 e^{-E_j/T} \quad f(\mathbf{p}) \equiv \frac{dN}{d\mathbf{p}} = \frac{1}{Z} \sum_j |\psi_j(\mathbf{p})|^2 e^{-E_j/T}$$

$$Z \equiv \sum_j e^{-E_j/T}$$

$$\langle p_i^2 \rangle = \frac{M\omega_i}{2} \coth \frac{\omega_i}{2T}, \quad \langle r_i^2 \rangle = \frac{1}{2M\omega_i} \coth \frac{\omega_i}{2T}.$$

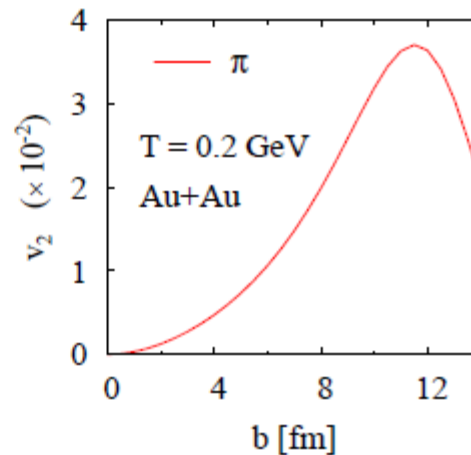
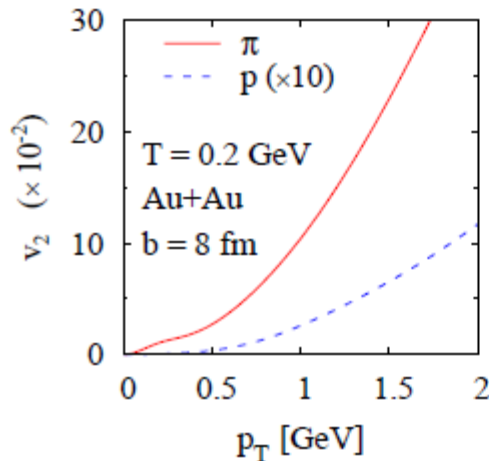
$$\bar{v}_2 \approx \frac{\hbar^2}{12k_B T M \langle r_x^2 \rangle} \cdot \frac{\varepsilon}{1 + \varepsilon}$$



Quantum physics anisotropy

D. Molnar, FW, and C.H. Greene, arXiv:1404.4119

$$\bar{v}_2 \approx \frac{\hbar^2}{12k_B T M \langle r_x^2 \rangle} \cdot \frac{\epsilon}{1 + \epsilon}$$



$b = 8$ fm: $\langle r_x^2 \rangle^{1/2} = 1.5$ fm and $\langle r_y^2 \rangle^{1/2} = 2.2$ fm.

$$\rho(\mathbf{r}) \propto \exp\left(-\sum_i \frac{r_i^2}{2\langle r_i^2 \rangle}\right), \quad f(\mathbf{p}) \propto \exp\left(-\sum_i \frac{p_i^2}{2\langle p_i^2 \rangle}\right)$$

Cold atoms

Strong elliptic anisotropy

K. M. O'Hara *et al.*, Science 298, 2179 (2002).

Lithium atoms $M \sim 6000 \text{ MeV}$

Temperature $T \sim 1 \text{ } \mu\text{K} \sim 10^{-16} \text{ MeV}$

Trap size $x \sim 20 \text{ } \mu\text{m}$, $y \sim 100 \text{ } \mu\text{m}$

Typical momentum $(TM)^{1/2} \sim 10^{-6} \text{ MeV}$

Intrinsic momentum quantum $\sim 1/r \sim 10^{-8} \text{ MeV}$, negligible.

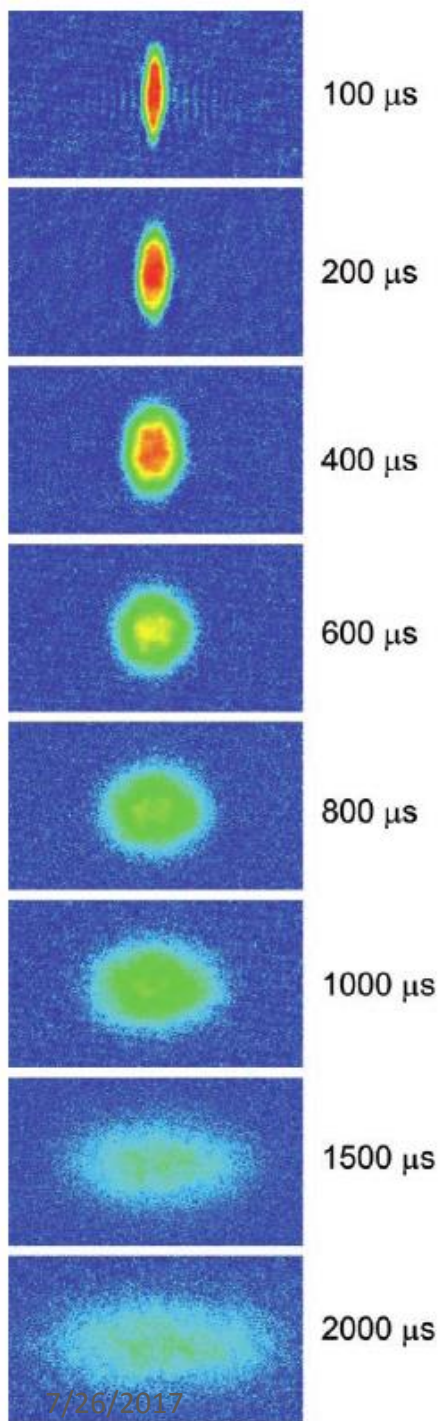
Typical energy $\sim T \sim 10^{-16} \text{ MeV}$

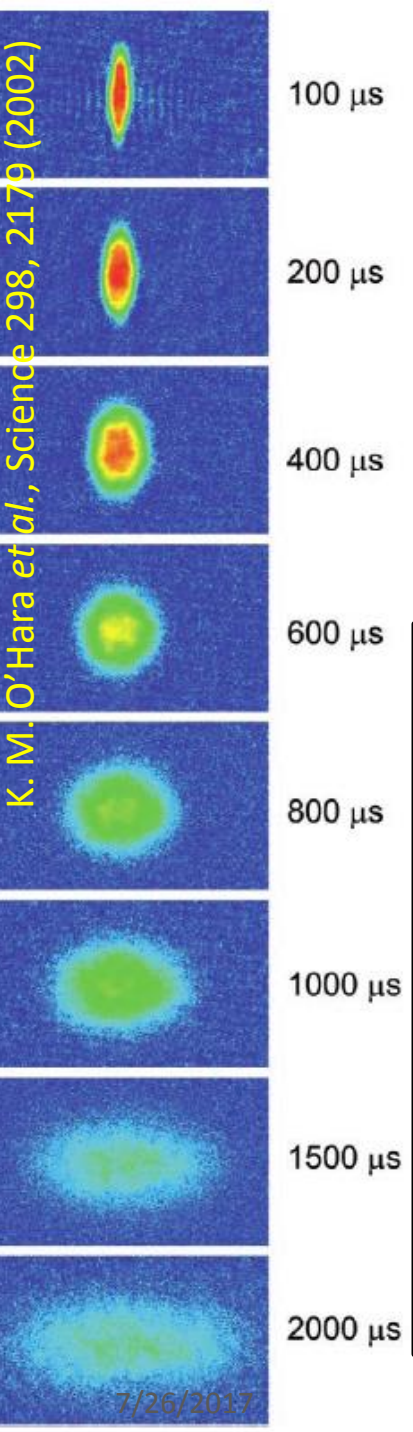
Intrinsic energy quantum $1/(mr^2) \sim 10^{-20} \text{ MeV}$, negligible.

Cold Lithium atoms are actually “hotter” than the hot QGP.

$$\bar{v}_2 \approx \frac{\hbar^2}{12k_B T M \langle r_x^2 \rangle} \cdot \frac{\epsilon}{1 + \epsilon} \sim 10^{-5}$$

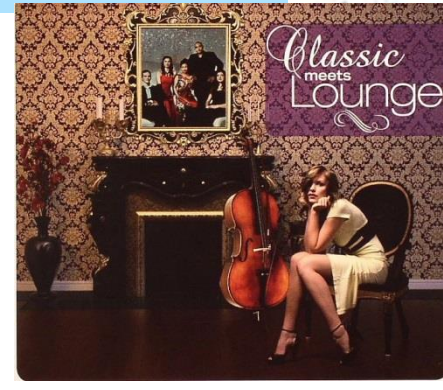
The observed large v_2 is indeed due to strong interactions.





Is quantum v_2 real in QGP?

- It should be... but need experiment to verify (cold atom experiment)
- **Cold atoms are "classical."**
Make it Quantum Mechanical.
- Would be neat to verify QM and uncertainty principle



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 Negligible!

Cold atoms are hot, classical.

q,g: $M \sim 0 \text{ MeV}$
 $T \sim 200 \text{ MeV}$
 $x \sim 3 \text{ fm}, y \sim 4 \text{ fm}$
 $p \sim 200 \text{ MeV}$

$p \text{ quan} \sim 1/r \sim 200 \text{ MeV}$
 $E \text{ quan} \sim 200 \text{ MeV}$
 Comparable!

QGP is cold, quantum mechanical.

$\times 10^{-4}$
 $\times 10^{-2}$

Summary

- Close connections between hot QGP and cold atoms.
- Cold atoms are hydrodynamical; QGP may not be.

Make it less interacting, more dilute, or smaller to mimic QGP.

- QGP is quantum mechanical; cold atoms are “classical.”

Make it smaller, or colder to mimic QGP, and measure the uncertainty principle.

- Use controllability of cold atom system to address QGP questions