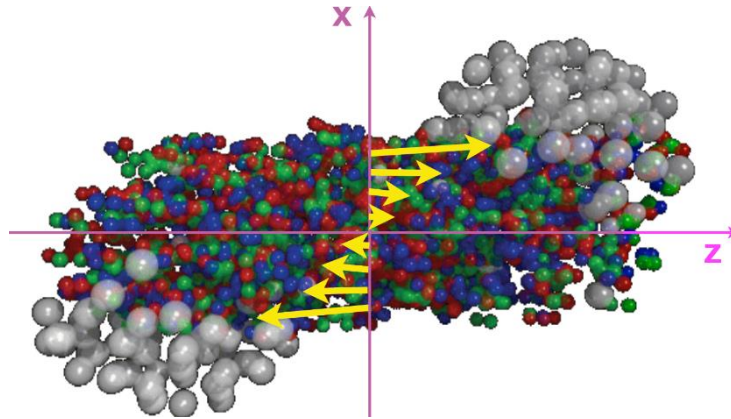


Workshop on AMPT for Relativistic Heavy Ion Collisions

The Global Λ Polarization in HIC



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arXiv: 1704.01507

Outline

- Introduction
- Lambda polarization from vorticity
- Numerical results and discussion
- Summary

Introduction

- Strong magnetic field

Skokov, et.al. 2009, 2011

Deng, Huang 2012

Tuchin 2013

Zhong, Yang, Cai, Feng 2014

Gursoy, Kharzeev, Rajagopal 2014

Li, Sheng, Wang 2016 ...

- Large angular momentum

&& vorticity

Jiang, Lin, Liao 2016

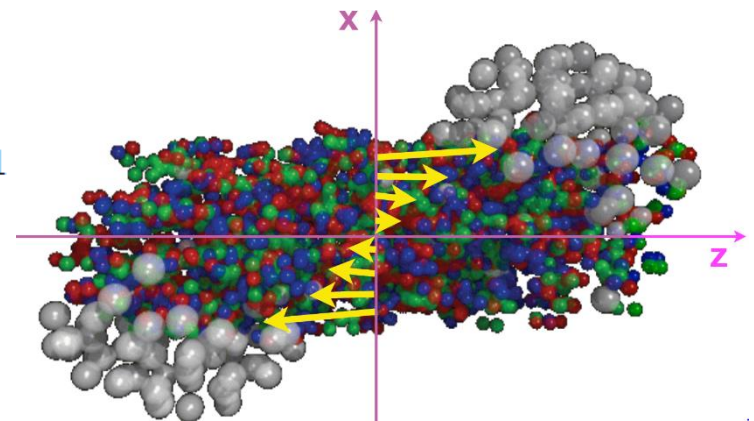
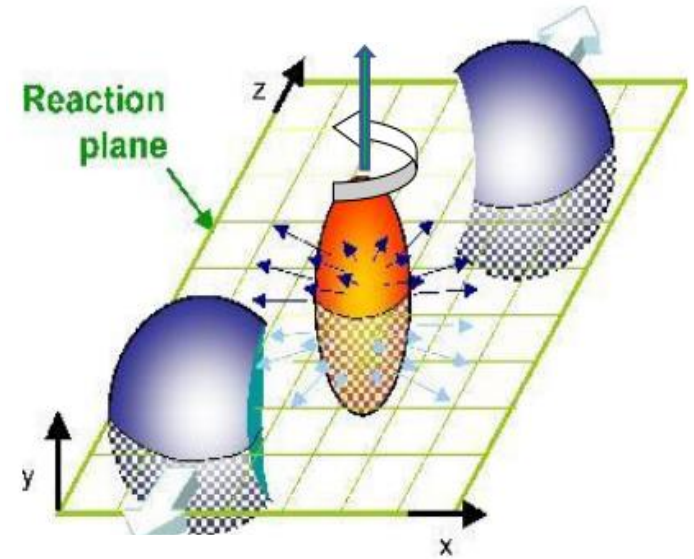
Deng, Huang 2016

$$eB_y \sim m_\pi^2$$

$$J_y \sim 10^5 \hbar$$

$$\langle \omega_y \rangle \sim 0.01 \text{fm}^{-1}$$

- CME, CMW, CVE, CVW,
Polarization ...



Introduction

EM vector Field

\mathbf{A}

Magnetic Field

$$\mathbf{B} = \nabla \times \mathbf{A}$$

Lorentz Force

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$$

Spin-magnetic coupling

$$\mathbf{S} \cdot \mathbf{B}$$

Magnetic induced polarization

\mathbf{V}

Fluid velocity field

Vortical Field

$$\boldsymbol{\omega} = \nabla \times \mathbf{V}$$

Coriolis Force

$$\mathbf{F}_{col} = 2m\mathbf{v} \times \boldsymbol{\omega}$$

Spin-vorticity coupling

$$\mathbf{S} \cdot \boldsymbol{\omega}$$

Vorticity induced polarization

Spin-orbit coupling

Liang, Wang, PRL 94,102301(2005)

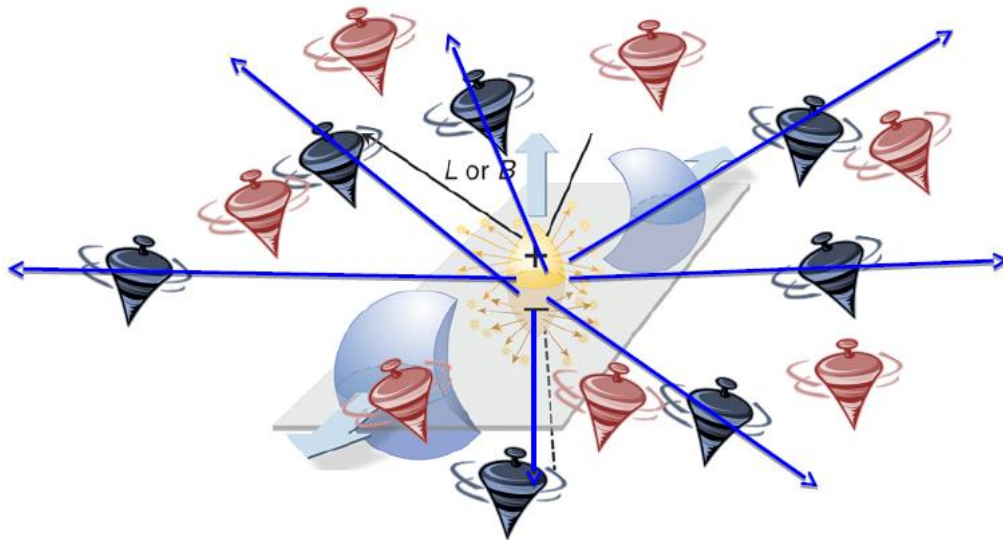
Voloshin, nucl-th/0410089

Betz, Gyulassy, Torrieri, PRC 76, 044901(2007)

Probe the vorticity and magnetic field

Global polarization along angular momentum

Global Λ polarization probes B and ω



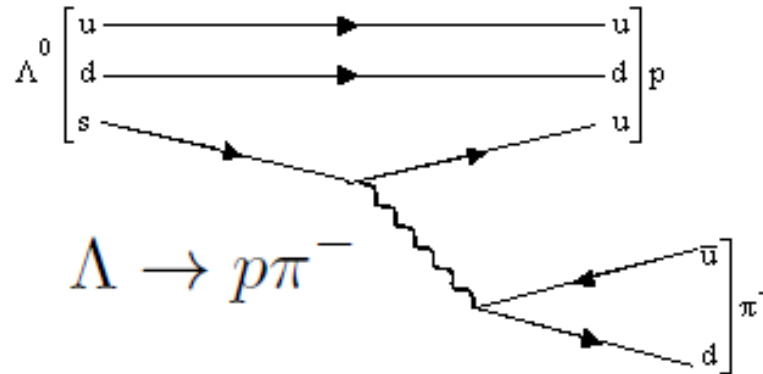
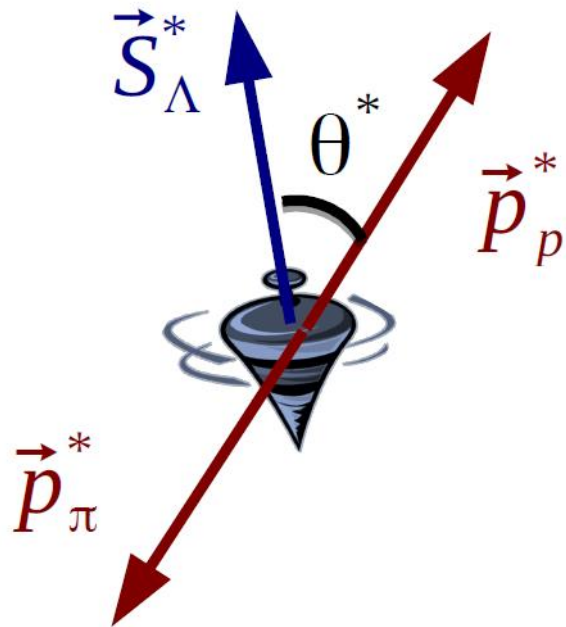
- Spin-orbit coupling:
 Λ and anti- Λ spins aligned with L
Liang, Wang 2004; Voloshin 2004; ...
- Λ spin anti-aligned along B
- anti- Λ spin aligned along B

$$P_{\Lambda} \simeq \frac{1}{2} \frac{\omega}{T} + \frac{\mu_{\Lambda} B}{T}$$
$$P_{\bar{\Lambda}} \simeq \frac{1}{2} \frac{\omega}{T} - \frac{\mu_{\Lambda} B}{T}$$

Becattini, Karpenko, Lisa,
Upsal, Voloshin 2016

How to measure it?

- Λ is “self-analyzing”

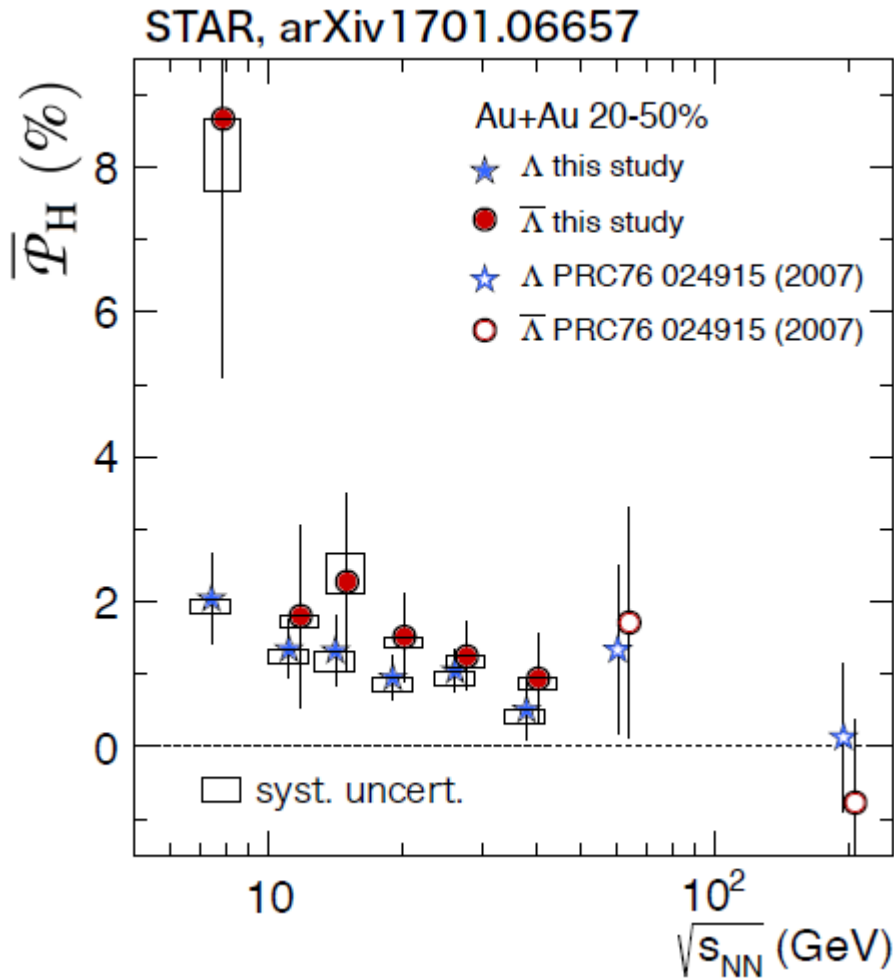


$$\frac{dN}{d \cos \theta^*} = \frac{1}{2} \left(1 + \alpha_H |\vec{\mathcal{P}}_H| \cos \theta^* \right)$$

$$\alpha_\Lambda = -\alpha_{\bar{\Lambda}} = 0.642 \pm 0.013$$

$$\bar{\mathcal{P}}_H \equiv \langle \vec{\mathcal{P}}_H \cdot \hat{j}_{\text{sys}} \rangle = \frac{8}{\pi \alpha_H} \frac{\langle \cos(\phi_p^* - \phi_{\hat{j}_{\text{sys}}}^*) \rangle}{R_{\text{EP}}^{(1)}}$$

STAR measurement



$$|\eta| < 1$$

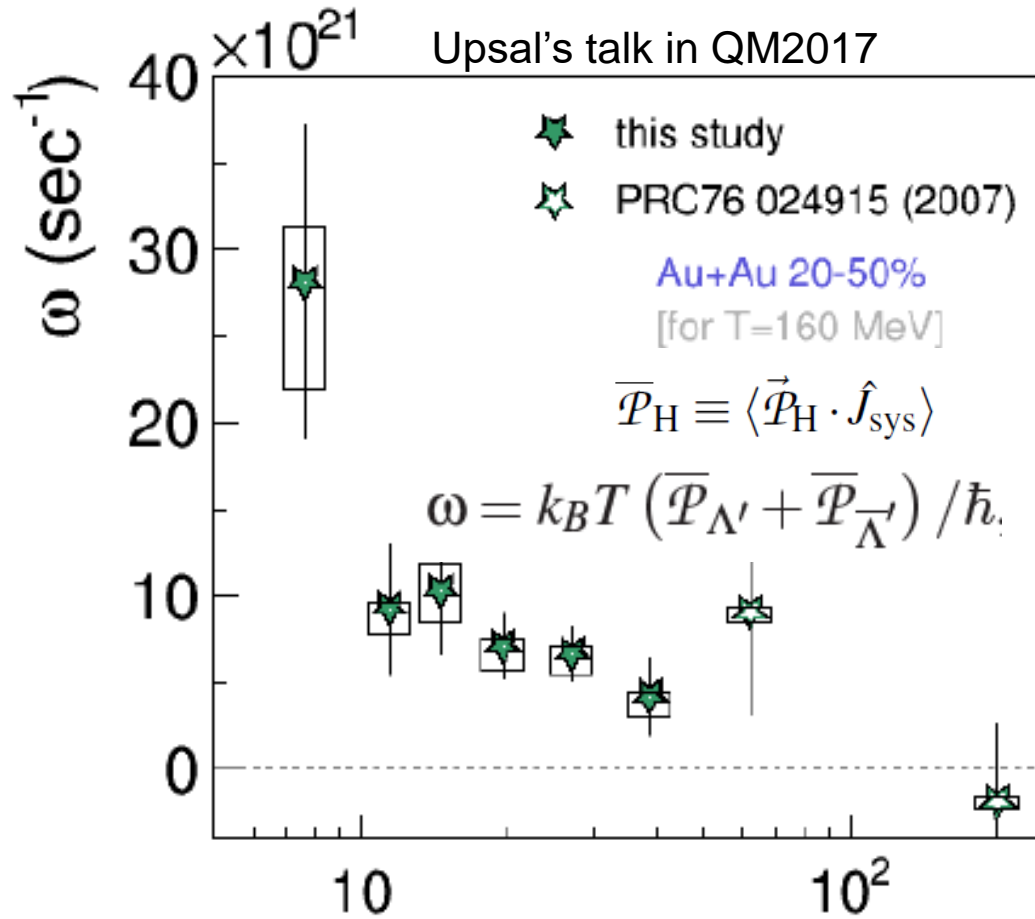
- Positive signals: vorticity
- $P(\text{anti-}\Lambda) > P(\Lambda)$: B-field

$$P_{\Lambda} \simeq \frac{1}{2} \frac{\omega}{T} + \frac{\mu_{\Lambda} B}{T}$$

$$P_{\bar{\Lambda}} \simeq \frac{1}{2} \frac{\omega}{T} - \frac{\mu_{\Lambda} B}{T}$$

- Λ spin anti-aligned along B
- anti- Λ spin aligned along B

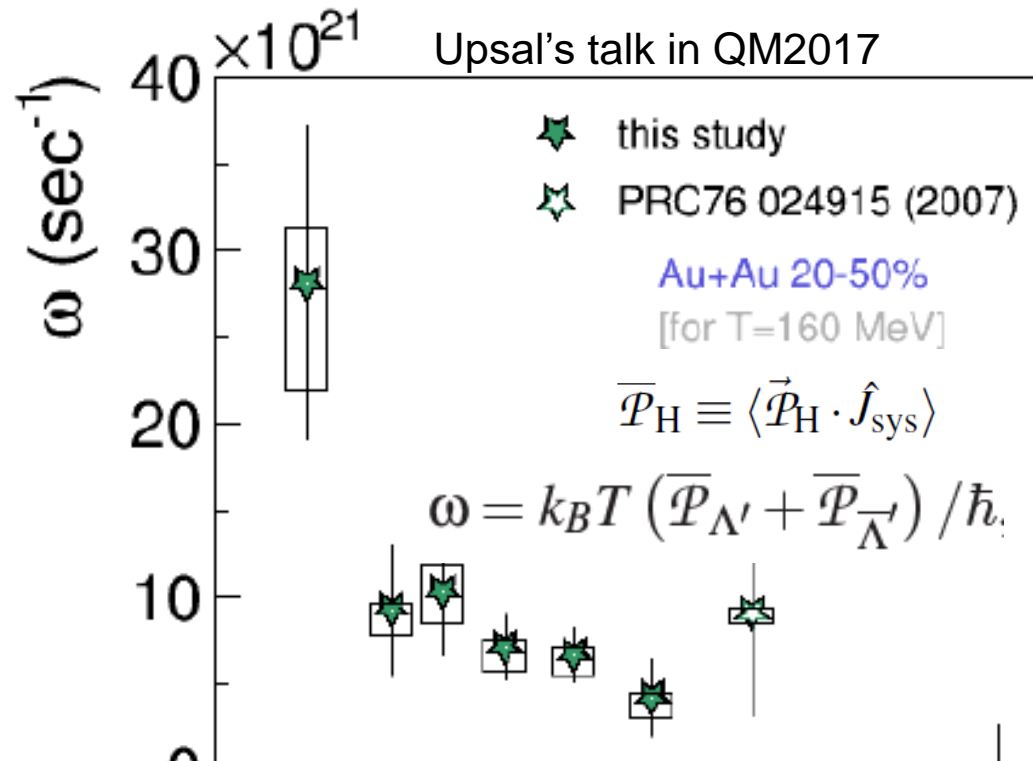
Introduction



$$\omega_y = (9 \pm 1) 10^{21} \text{ s}^{-1} \quad \sqrt{s_{NN}} \text{ (GeV)}$$

RHIC produces the fluid with highest vorticity!

Introduction



Solar subsurface flow	10^{-7} s^{-1}
Supercell tornado cores	10^{-1} s^{-1}
Rotating soap bubbles	10^2 s^{-1}
Super Fluid	10^7 s^{-1}

RHIC produces the fluid with highest vorticity!

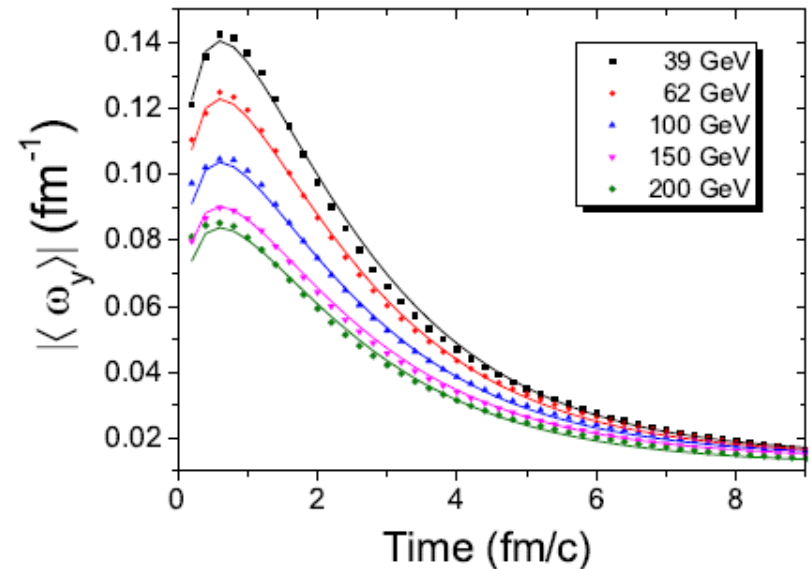
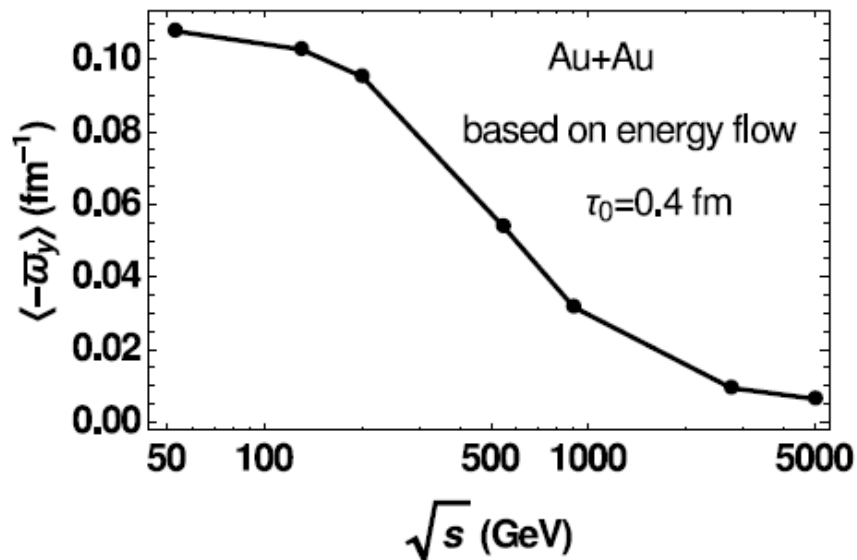
Introduction

Properties of vorticity field

$$\bar{\omega} \equiv \frac{\int d^2 \mathbf{x}_\perp \varepsilon(\mathbf{x}_\perp) \omega(\mathbf{x}_\perp)}{\int d^2 \mathbf{x}_\perp \varepsilon(\mathbf{x}_\perp)}$$

$$\langle \omega_y \rangle = \frac{\int d^3 \vec{r} [\mathcal{W}(\vec{r})] \omega_y(\vec{r})}{\int d^3 \vec{r} [\mathcal{W}(\vec{r})]}$$

$$\mathcal{W}(\vec{r}) = \varepsilon(\vec{r})$$



Jiang, Lin, Liao, Phys.Rev. C94 (2016)

Deng, Huang, Phys.Rev. C93 (2016)

Spin and polarization in relativistic case

Classical Lagrangian formulation

$$L = \frac{1}{2} m (\mathbf{v}_r + \boldsymbol{\omega} \times \mathbf{r})^2 - U(r)$$

$$\mathbf{p} = \frac{\partial L}{\partial \mathbf{v}_r} = m (\mathbf{v}_r + \boldsymbol{\omega} \times \mathbf{r})$$

$$H = \mathbf{p} \cdot \mathbf{v}_r - L = \frac{\mathbf{p}^2}{2m} - \boldsymbol{\omega} \cdot \mathbf{L} + U(r)$$

Dirac Lagrangian in rotating frame

$$\mathcal{L}_D = \bar{\psi} (i\gamma^\mu (\partial_\mu + \Gamma_\mu) - m) \psi$$

$$\Gamma_\mu = \frac{1}{4} \times \frac{1}{2} [\gamma^a, \gamma^b] \Gamma_{abc}$$

$$H = \gamma^0 (\boldsymbol{\gamma} \cdot \mathbf{p} + m) - \boldsymbol{\omega} \cdot (\mathbf{x} \times \mathbf{p} + S)$$

$$= H_0 - \boldsymbol{\omega} \cdot \mathbf{J}$$

Density matrix

$$\hat{\rho} = \frac{1}{Z} \exp \left(-\beta \hat{H} + \beta \nu \hat{Q} + \beta \boldsymbol{\omega} \cdot \hat{\mathbf{J}} \right)$$

$$\beta = \frac{1}{T} \quad O = \text{tr} (\rho \hat{O})$$

$$\mathbf{S} = \text{tr} (\hat{\rho} \hat{S})$$

$$= \frac{1}{2} \tanh \left(\frac{\omega}{2T} \right)$$

$$\sim \frac{\omega}{4T}$$

Spin and polarization in relativistic case

Non-relativistic case: $\hat{\rho} = \frac{1}{Z} \exp \left(-\beta \hat{H} + \beta \nu \hat{Q} + \beta \boldsymbol{\omega} \cdot \hat{\mathbf{J}} \right) \quad \beta = \frac{1}{T} \quad O = \text{tr} \left(\rho \hat{O} \right)$

Spin vector $\mathbf{S} \sim \frac{\boldsymbol{\omega}}{4T}$

Relativistic case:

$$\hat{\rho} = \frac{1}{Z} \exp \left[-\beta u_\nu \hat{P} + \frac{1}{2} \beta \hat{J}^{\nu\rho} \omega_{\nu\rho} + \beta \xi \hat{Q} \right] \quad \hat{J}^{\nu\rho} = \hat{J}_L^{\nu\rho} + \hat{J}_S^{\nu\rho}$$

Spin vector (Pauli-Lubanski) $S^\mu = \text{Tr} \left(\hat{\rho} \hat{S}^\mu \right)$

Spin ½ particle

$$S^\mu(x, p) = -\frac{1}{8m} (1 - n_F) \epsilon^{\mu\nu\rho\sigma} p_\nu \varpi_{\rho\sigma}(x)$$

Statistical-hydro model

Becattini, et.al., 2008, 2013

Wigner function method

Fang, Pang, Wang, Wang, 2016

Chiral kinetic equation

Sun, Ko, 2017

Lambda polarization from vorticity

Spin vector of a spin $\frac{1}{2}$ particle

$$S^\mu(x, p) = -\frac{1}{8m} (1 - n_F) \epsilon^{\mu\nu\rho\sigma} p_\nu \varpi_{\rho\sigma}(x)$$

$$1 - n_F \simeq 1$$

- Boltzmann limit



Thermal vorticity

$$\varpi_{\mu\nu} = \frac{1}{2} (\partial_\nu \beta_\mu - \partial_\mu \beta_\nu)$$

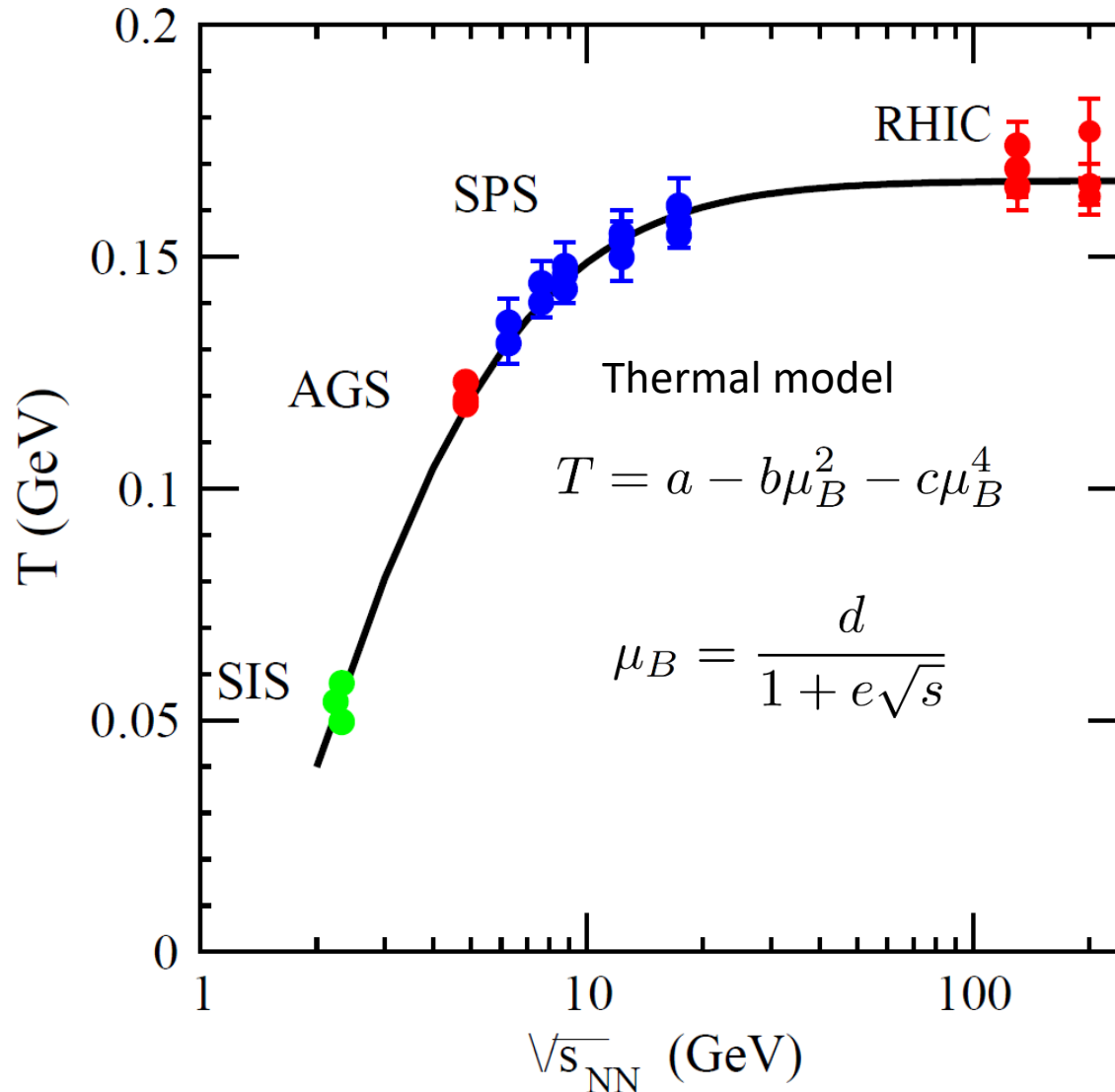
$$\beta^\mu = \frac{u^\mu}{T}$$

Relativistic vorticity

$$\Omega_{\mu\nu} = \frac{1}{2} (\partial_\nu u_\mu - \partial_\mu u_\nu)$$

$$S^\mu(x, p) = -\frac{1}{8mT} \epsilon^{\mu\nu\rho\sigma} p_\nu \Omega_{\rho\sigma}(x)$$

Lambda polarization from vorticity



STAR: J.Cleymans, et al PRC 73,034905

Lambda polarization from vorticity

$$S^\mu(x, p) = -\frac{1}{8mT} \epsilon^{\mu\nu\rho\sigma} p_\nu \Omega_{\rho\sigma}(x)$$

$$\Omega_S = (\Omega_{yz}, \Omega_{zx}, \Omega_{xy}) = \frac{1}{2} \nabla \times \mathbf{u},$$

$$\Omega_T = (\Omega_{0x}, \Omega_{0y}, \Omega_{0z}) = \frac{1}{2} (\nabla \gamma + \partial_t \mathbf{u})$$

C.M. Frame

$$S^0(x, p) = \frac{1}{4mT} \mathbf{p} \cdot \Omega_S,$$

$$\mathbf{S}(x, p) = \frac{1}{4mT} (E_p \Omega_S + \mathbf{p} \times \Omega_T)$$

In Lambda's rest frame

$$S_\Lambda^{*\mu} = (0, \mathbf{S}_\Lambda^*)$$

$$\mathbf{S}_\Lambda^* = \mathbf{S}_\Lambda - \frac{\mathbf{p} \cdot \mathbf{S}_\Lambda}{E_p (m + E_p)} \mathbf{p}$$

dominated by

$$\mathbf{S} \simeq \frac{\gamma_\Lambda \Omega_S}{4T}$$

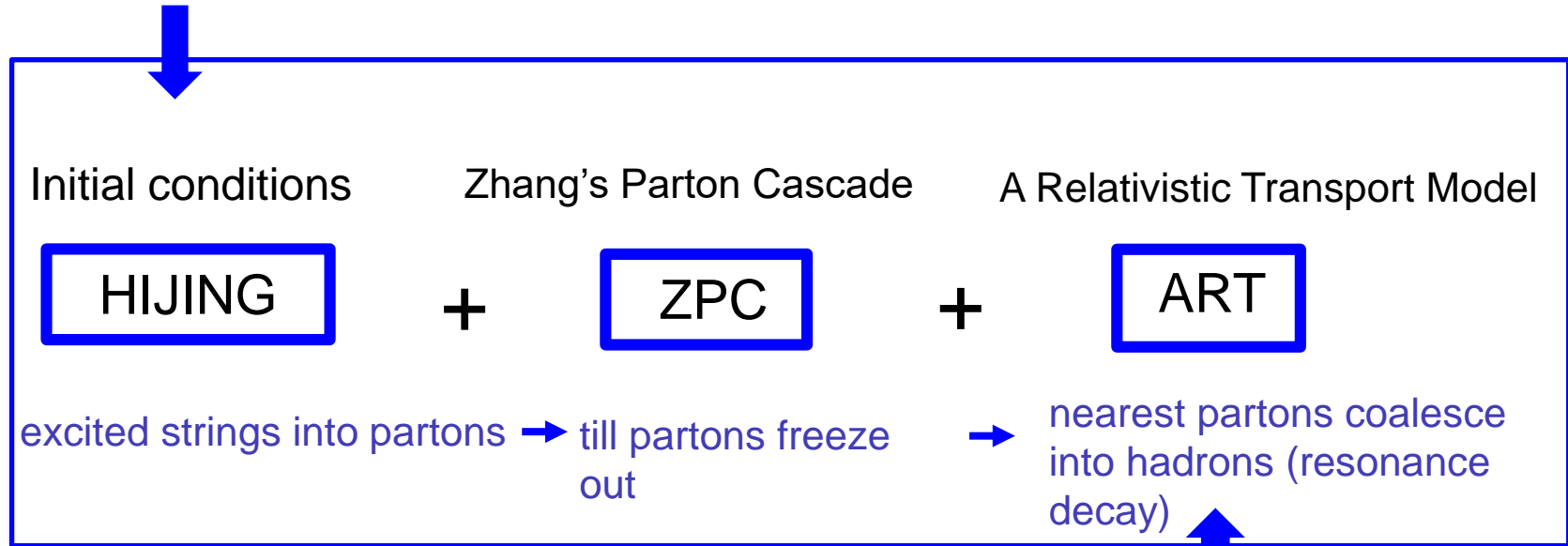
classic limit

$$\mathbf{S} \simeq \frac{\boldsymbol{\omega}}{4T}$$

Numerical method

Transport process:

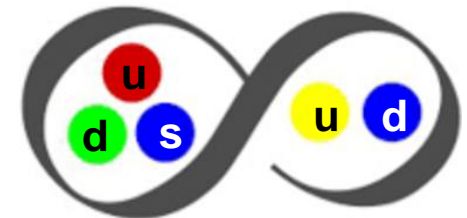
Participants



Spectators

- Track the partons' and hadrons' position and momentum information
- Find final-state particles (Λ hyperons)

Lin, Ko, Li, Zhang, Pal, Phys. ReV. C 72, 064901 (2005)



Numerical method

- **Velocity:**

- averaging in cell (size: d)

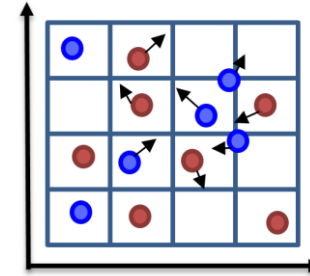
$$\mathbf{v}(t, \mathbf{x}) = \frac{\sum_i \mathbf{p}_i}{\sum_i E_i}$$

- Gaussian smearing (width: σ)

$$\mathbf{v}(t, \mathbf{x}) = \frac{\sum_i \mathbf{p}_i G(\mathbf{x}_i - \mathbf{x})}{\sum_i E_i G(\mathbf{x}_i - \mathbf{x})}$$

- sum is over both particles and events
- 10^5 events, totally 10TB disk used

- **Vorticity:** finite-difference method



comparable if $\frac{d^2}{12} = \sigma^2$

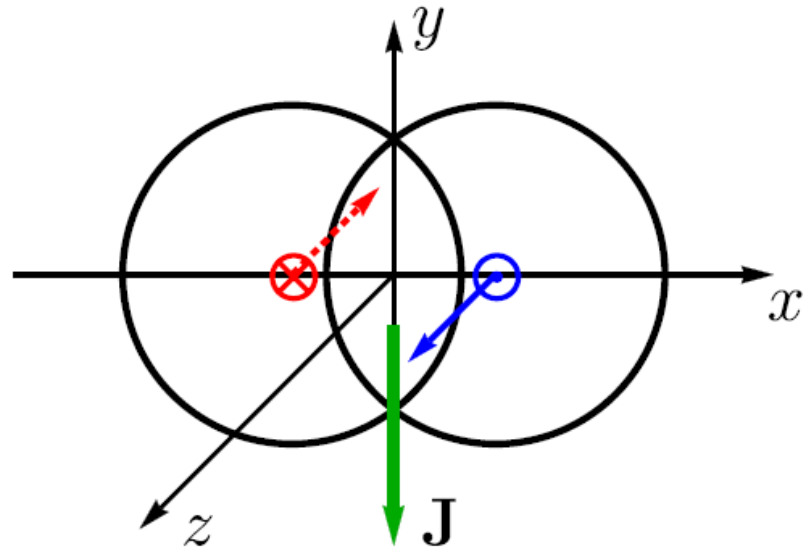
Compare different methods:
Oliinychenko, Petersen 2016
Deng, Huang 2016

Numerical method

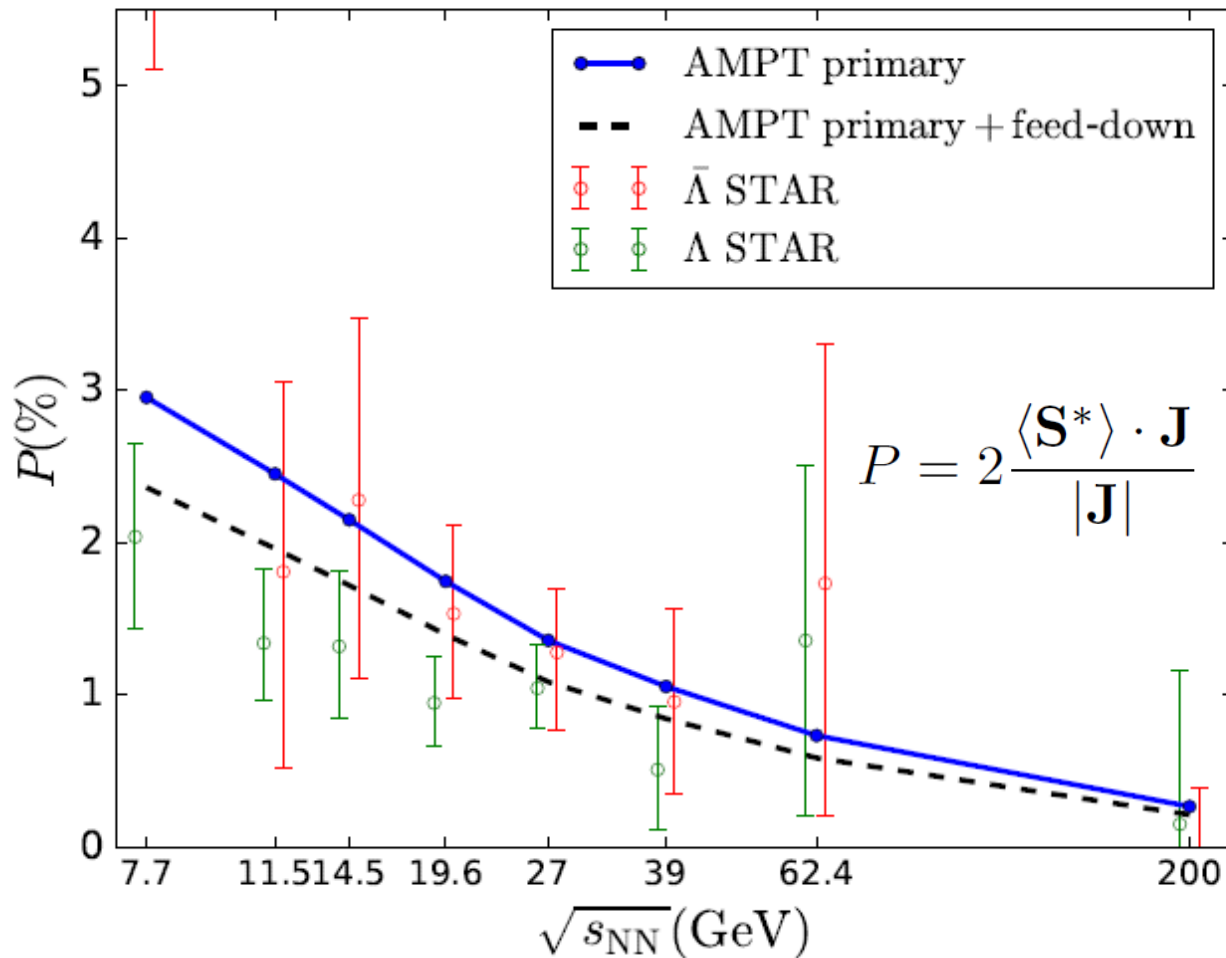
- Polarization: averaging over Lambdas $|\eta| < 1$

$$\langle \mathbf{S}^* \rangle = \frac{1}{N} \sum_{i=1}^N \mathbf{S}^*(x, p)$$

$$P = 2 \frac{\langle \mathbf{S}^* \rangle \cdot \mathbf{J}}{|\mathbf{J}|}$$

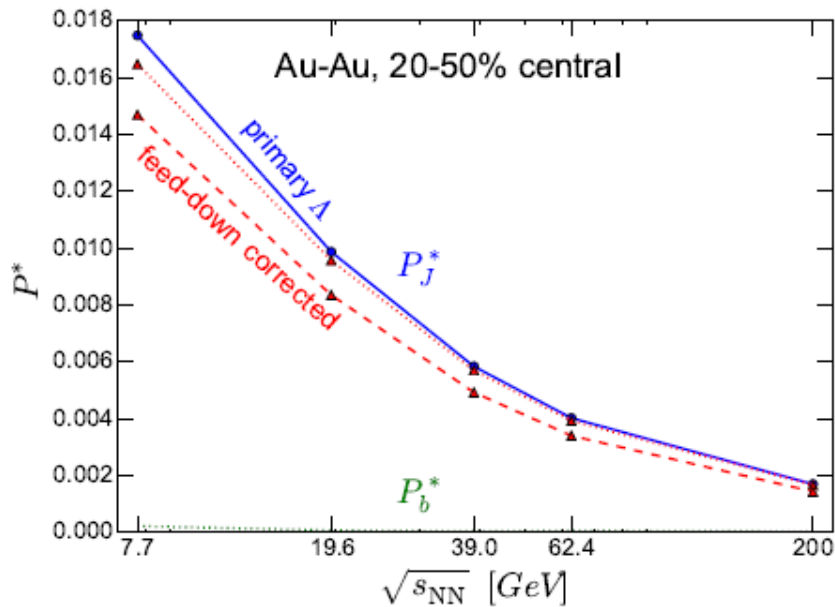


Global polarization from AMPT

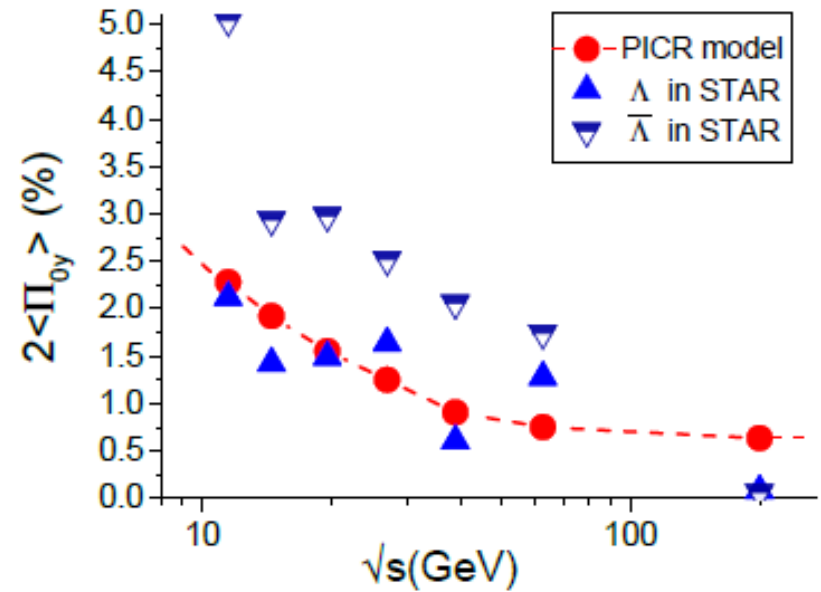


Au-Au 20%-50% with feed-down contribution

Global polarization in other models

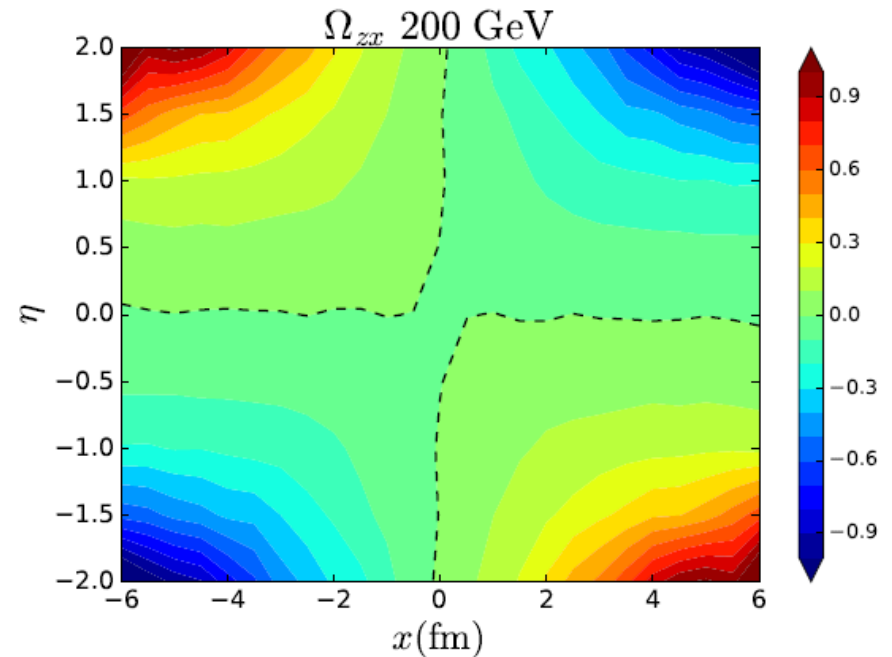
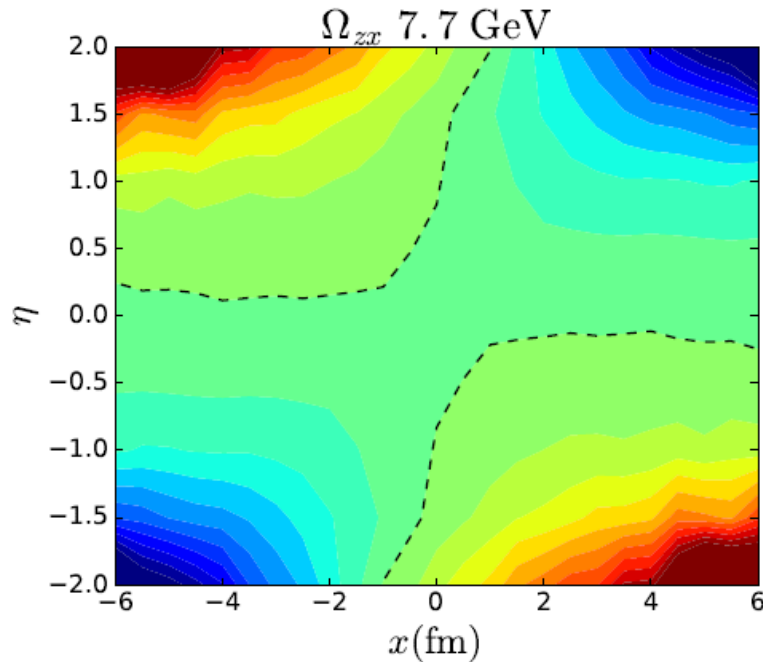


Karpenko, Becattini, EPJC 77, 2017
UrQMD + vHLE hydro



Xie, Wang, Csernai, PRC 95, 2017
PICR hydro

Vorticity



Vorticity on reaction plane

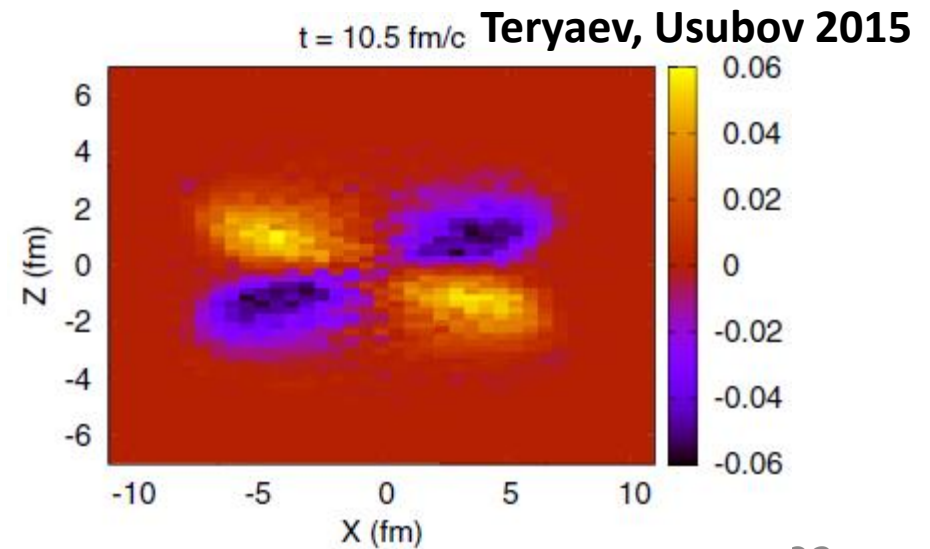
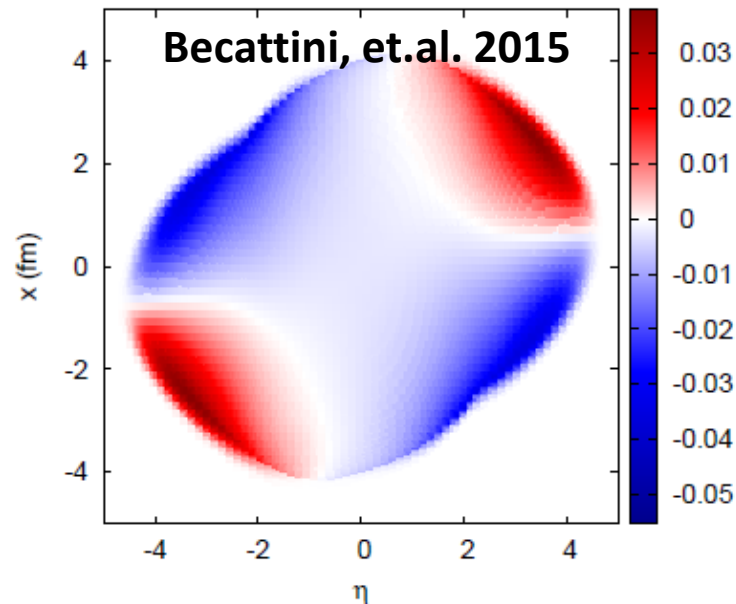
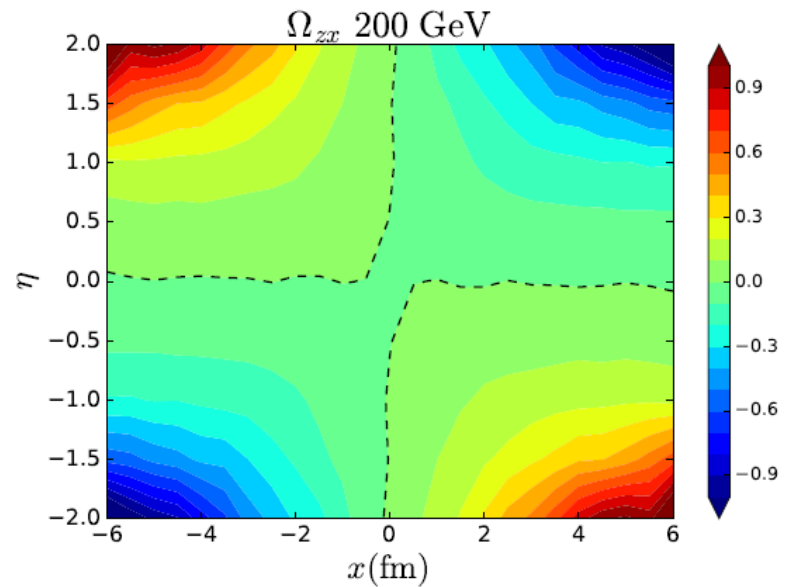
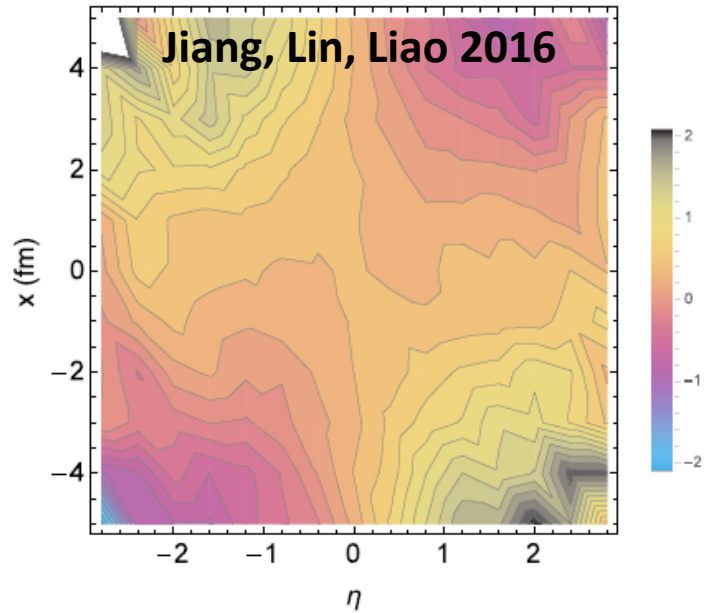
- Nearly odd function of x and η
- Less odd-symmetric at lower energy

$$\langle \mathbf{S}^* \rangle \sim \int d^4x f_\Lambda(x) \Omega_{zx}(x)$$

$$\Omega_{zx} = \partial_z u^x - \partial_x u^z$$

$$\omega_y = \partial_z v^x - \partial_x v^z$$

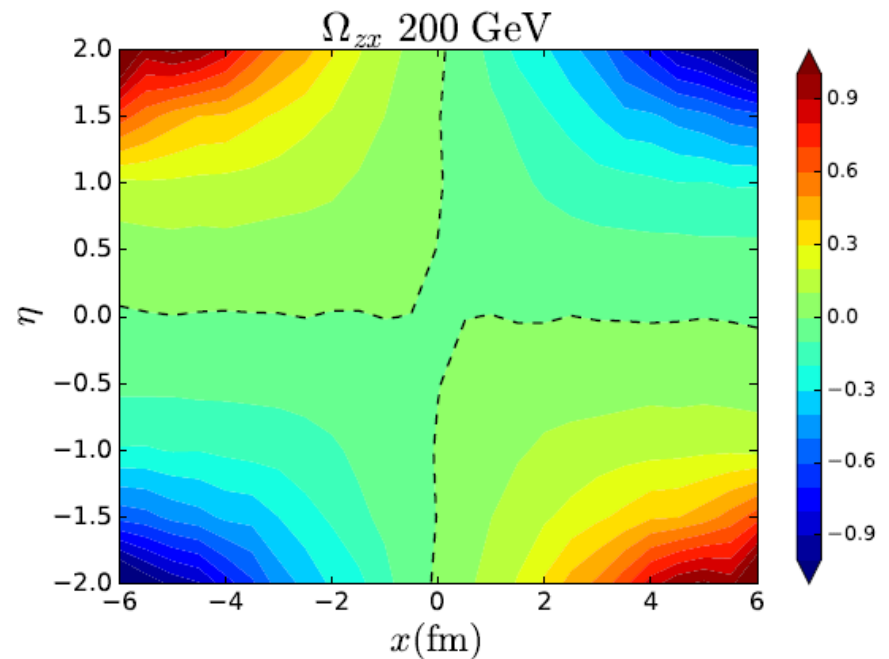
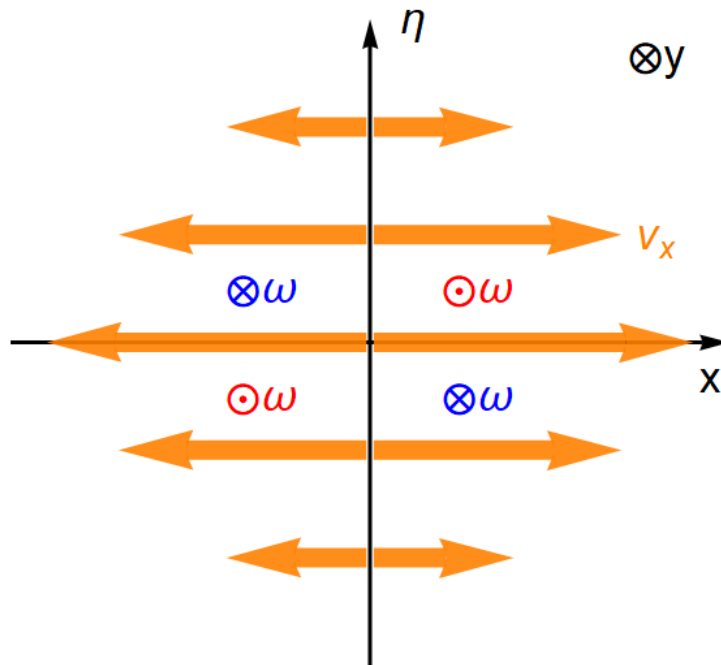
Vorticity



Vorticity and radial flow

- The odd-symmetry can be understood by the **radial flow**

Jiang, Lin, Liao 2016

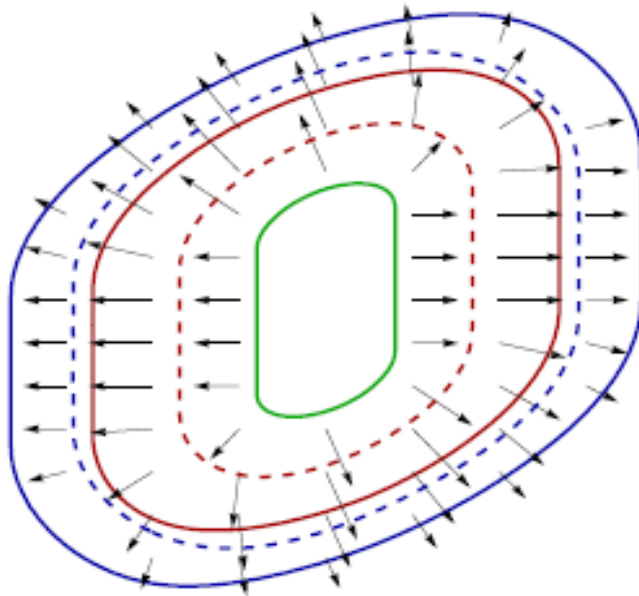


$$\Omega_{zx} = \partial_z u^x - \partial_x u^z$$

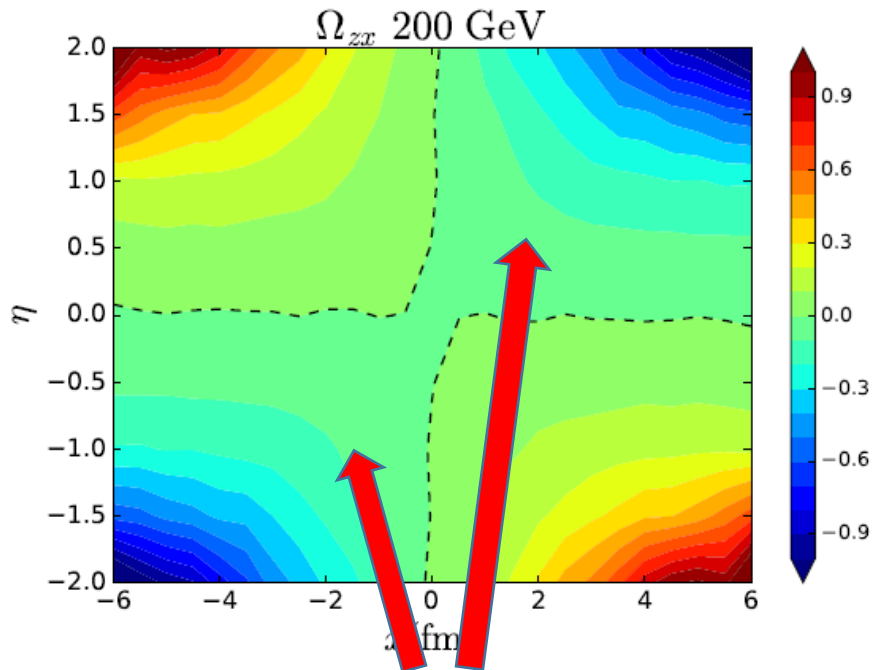
$$\langle \mathbf{S}^* \rangle \sim \int d^4x f_\Lambda(x) \Omega_{zx}(x)$$

Angular momentum

- Angular momentum is NOT zero because of fireball's tilt.



Bozek, Wyskiel 2010

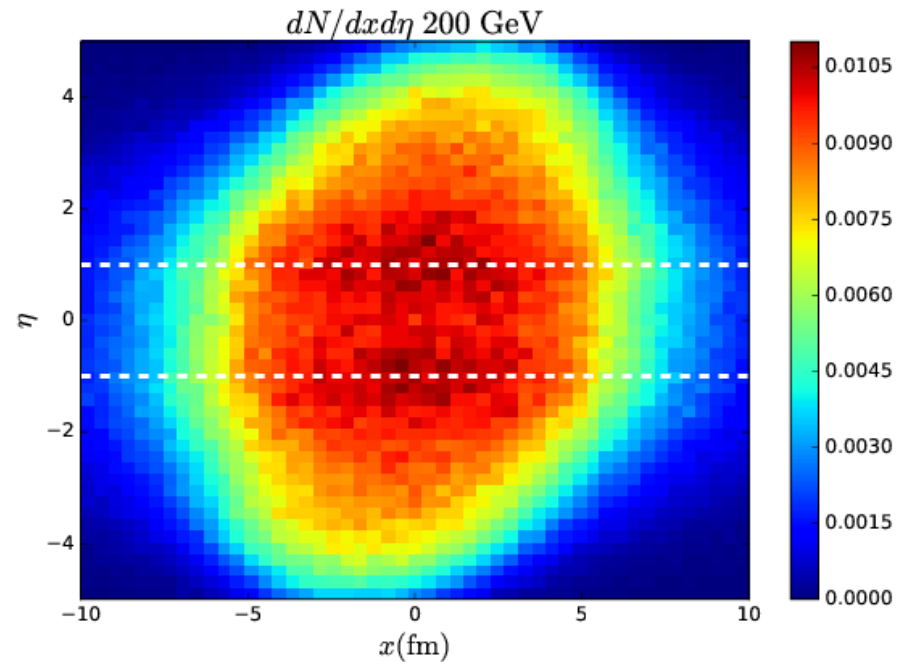
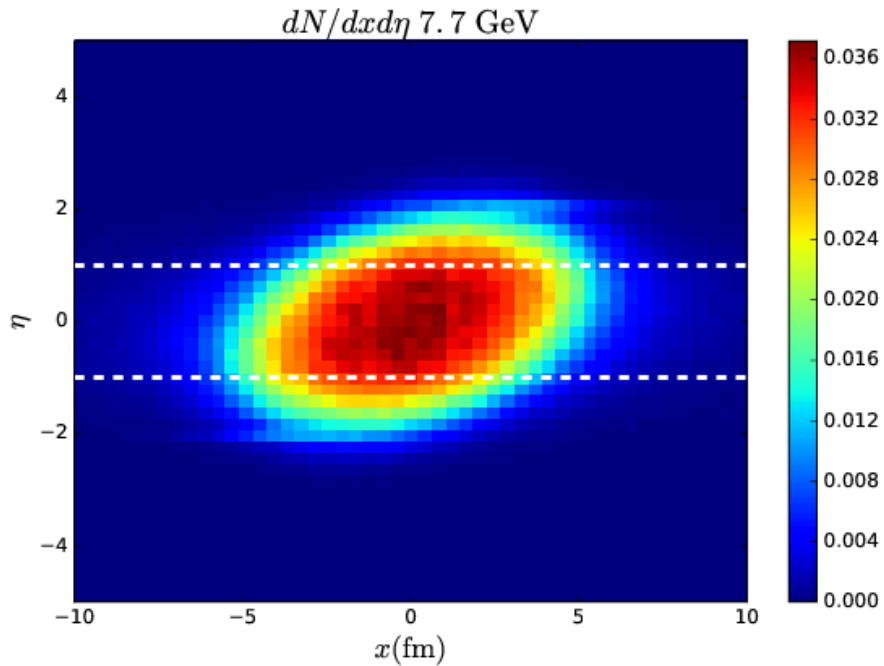


More matter here!

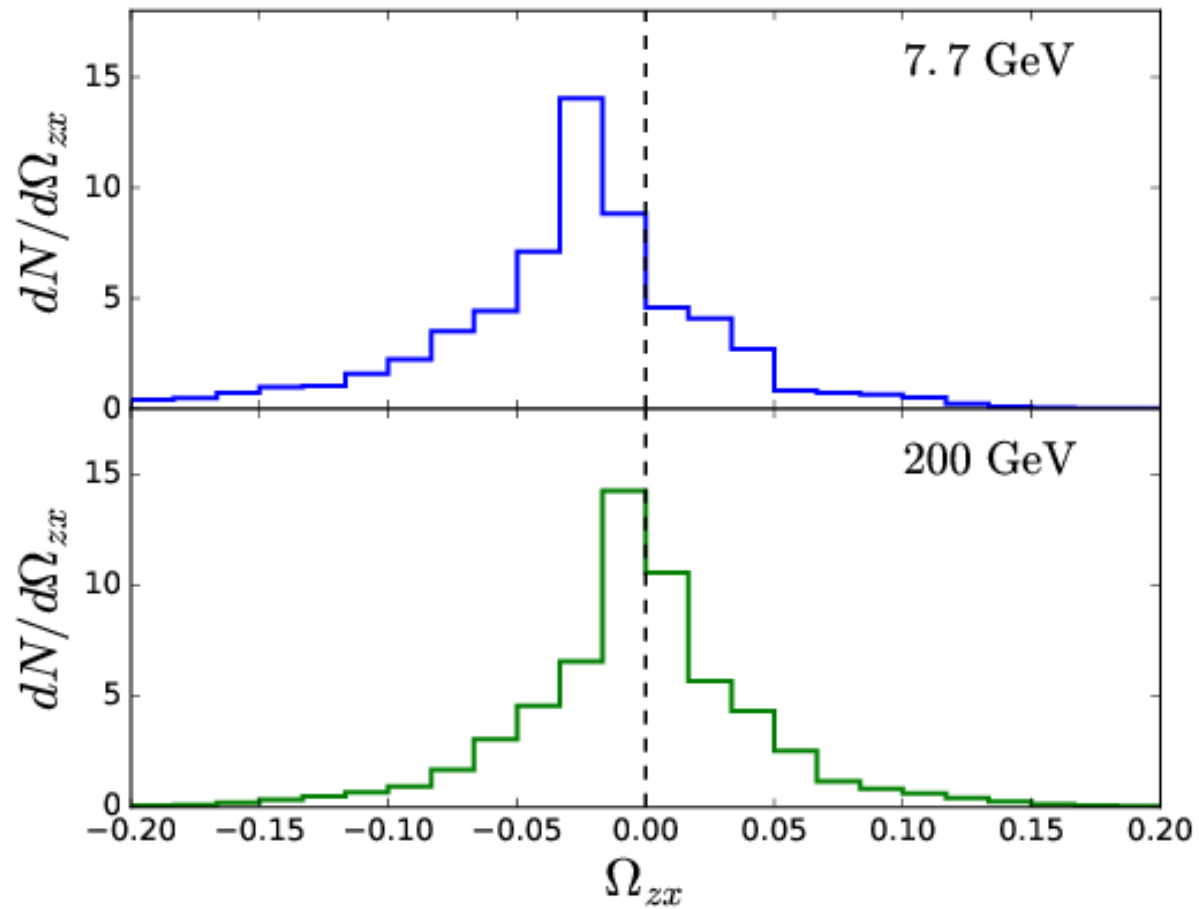
Λ distribution

$$\langle \mathbf{S}^* \rangle \sim \int d^4x f_\Lambda(x) \Omega_{zx}(x)$$

- Tilt at 7.7 GeV
- Symmetric at 200 GeV due to the rapid expansion



Λ distribution



Summary and outlook

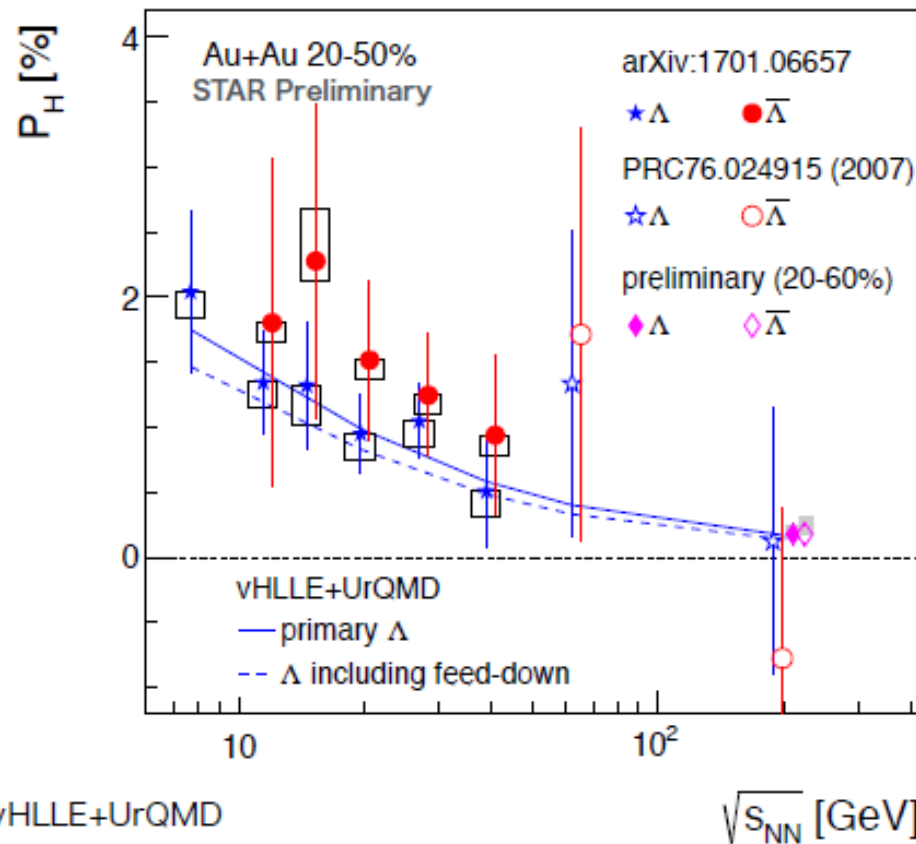
- We calculated the vorticity induced **global Λ polarization** with AMPT. The result agrees with the experimental data within error-bars.
- The polarization energy dependence can be understood by the asymmetry between the forward- and backward-going participants at low energy.
- ✓ Direct probe of **the vorticity** and magnetic field of the rotational fluid
- ✓ Study the **energy behavior** of the global polarization signal
- More studies on magnetic/vorticity field are called.

Thanks for your attention!

buckup

New data at 200 GeV

Niida's talk at UCLA QCD Chirality workshop 2017



vHLE+UrQMD
Karpenko and Becattini, arXiv:1610.0477

buckup

Vorticity distribution at freeze-out
Karpenko, Becattini 2017

$|\eta| < 0.3$

