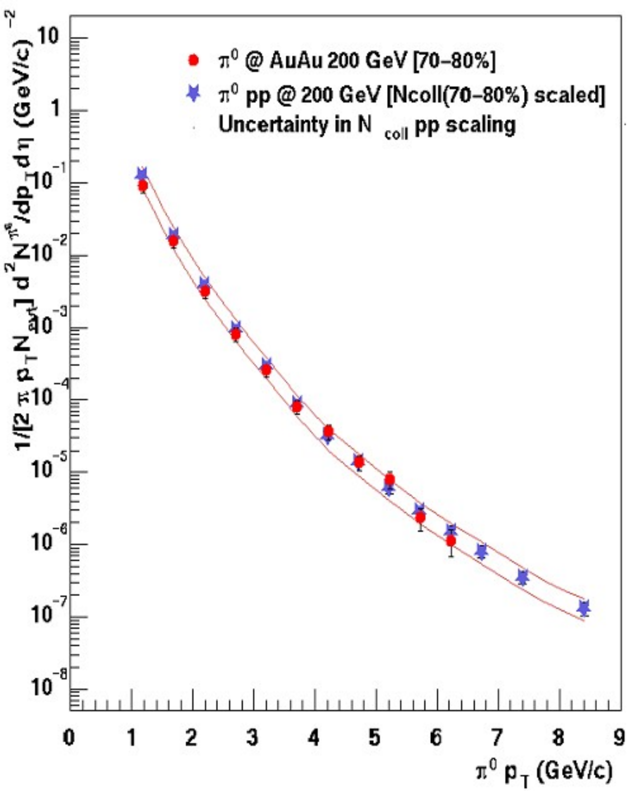
Comments on parton (quark) coalescence and its application in heavy ion collisions

Péter Lévai
MTA Wigner RCP, Budapest

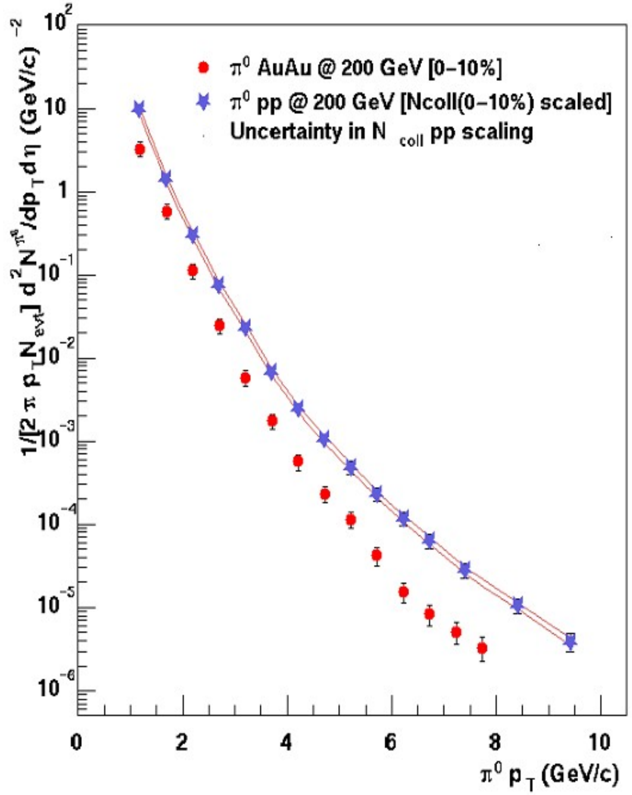
Sichuan State Univ., Chengdu,
25 July 2017

Beginning: Exciting results from RHIC at √s = 130 and 200 A GeV -- π^0

PHENIX Coll., D. d'Enterria, hep-ex/0209051, QM02 Conf.



Peripheral coll. N(bin)=12.3±4
binary scaling is working

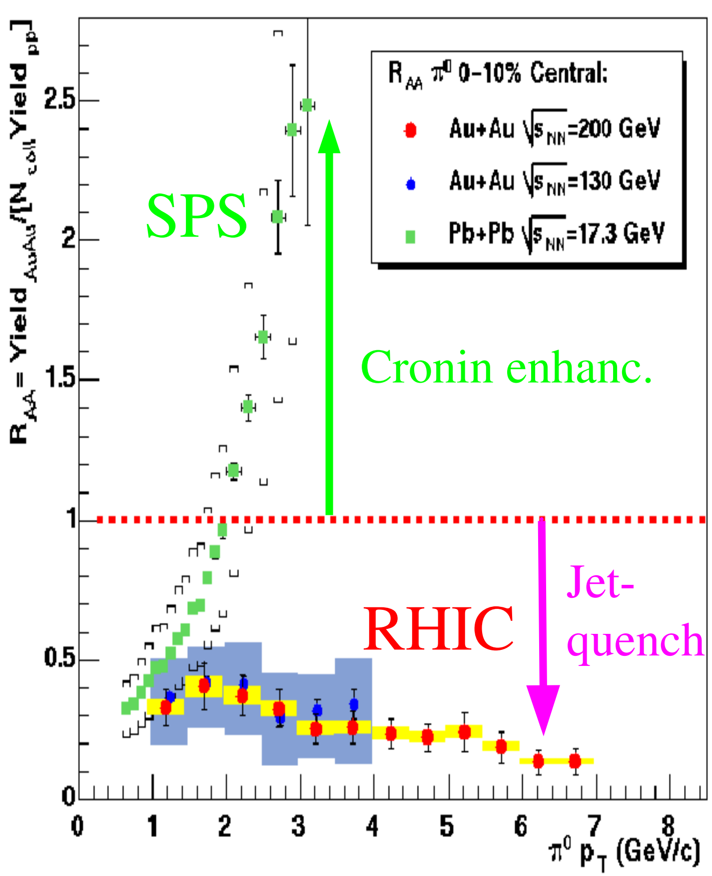


Central coll. N(bin)=975±94
binary scaling is violated

Nuclear modification factor:

$$R_{AA}(p_T) = \frac{(dN/dp_T)_{AA}}{\langle N_{bin}\rangle (dN/dp_T)_{pp}}$$

PRL 88, 022301 (2002)



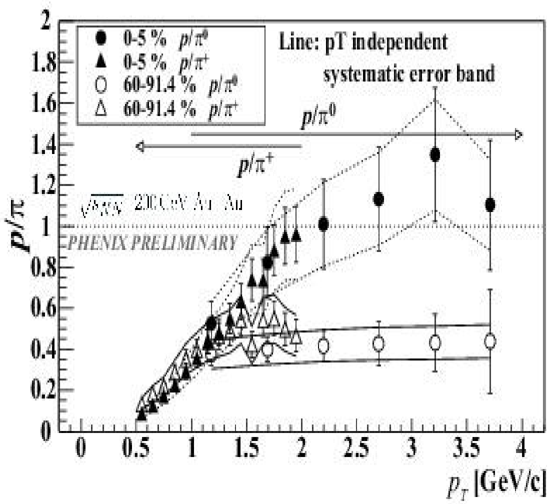
$$p_T > 6-8\,GeV : Hard\ physics - pQCD$$

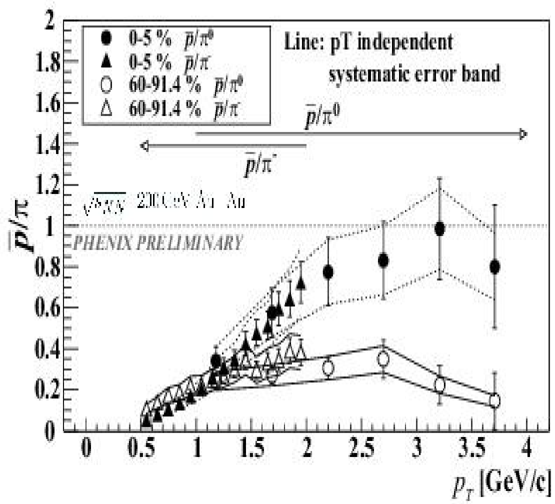
$$1.5\,GeV < p_T < 6\,GeV : Soft-hard\ overlap\ ???$$

Exciting results from RHIC at √s = 130 and 200 A GeV -- $p/\pi^{+}, \bar{p}/\pi^{-}$

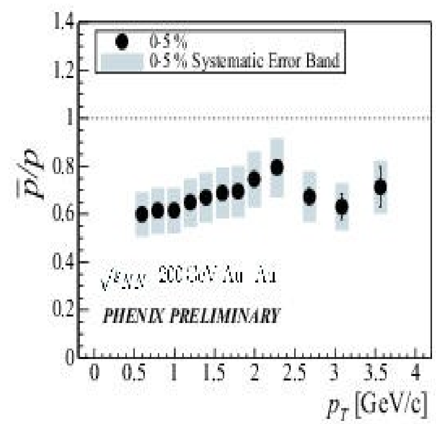
PHENIX Coll., T. Sakaguchi, nucl-ex/0209030, QM02 Conf.

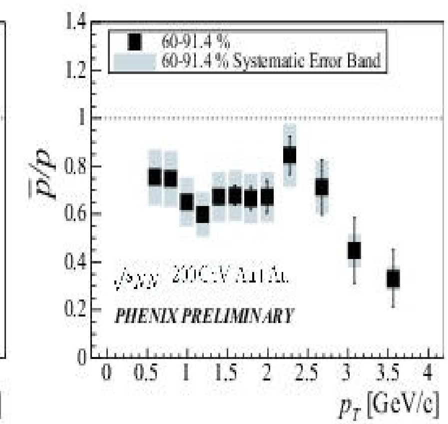
$$N(\bar{p}) > N(\pi^{-})!!!$$

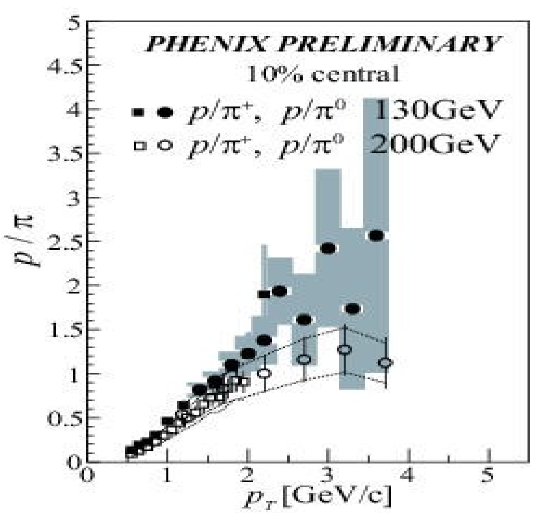


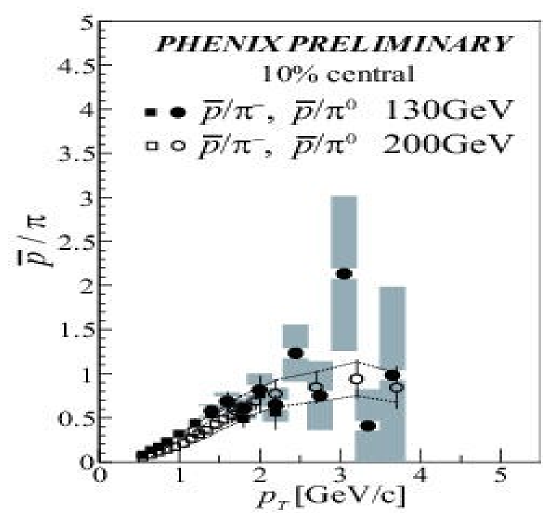


Anomalous antiproton (proton) production ??







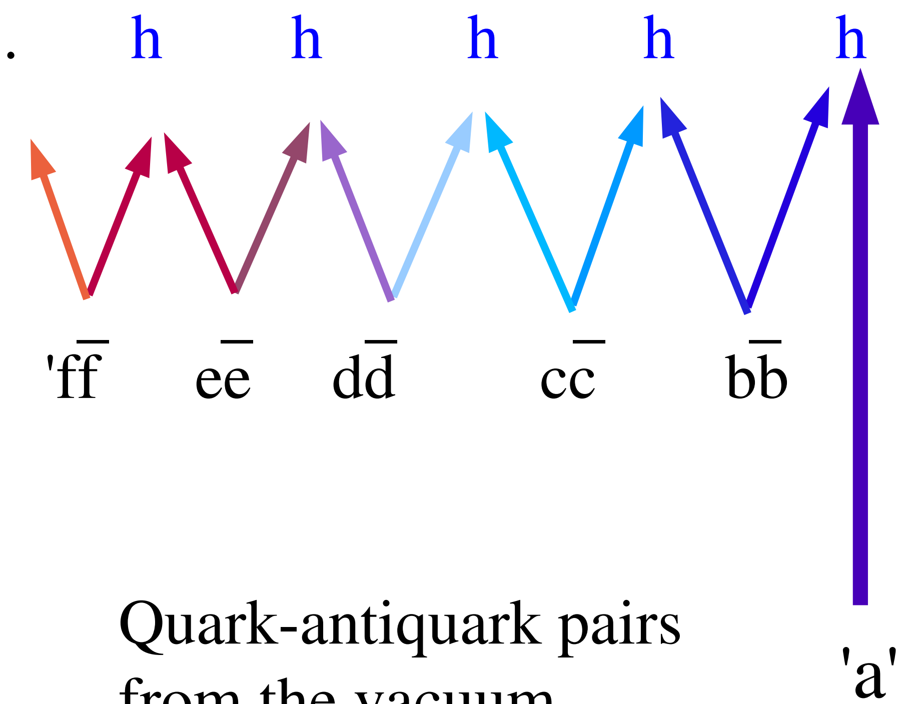


How are hadrons produced in AA?
What is the pQCD result?
What is the 'soft' result ?
Where is the soft-hard limit ?
Do we see QGP-like matter at RHIC ?

'Hard' physics: independent jet-fragmentation (FF)

Fragmentation function: $D_c^h(z)\,dz$

the probability to produce a hadron h with momentum z p from a jet c with momentum p



Quark-antiquark pairs from the vacuum

Original quark jet (flavour 'a')

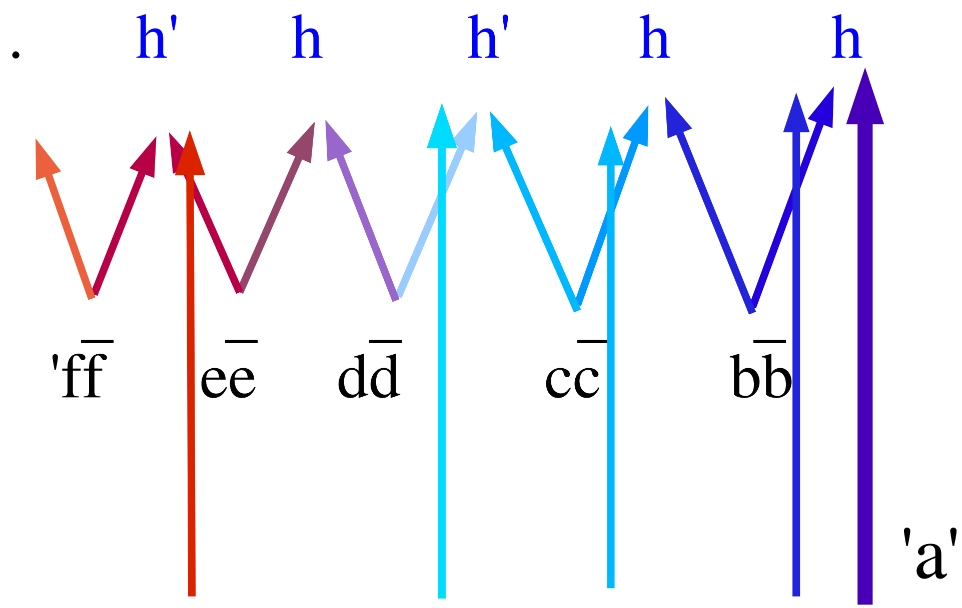
FF – parameterization of the pair creation : $D_c^h(z) = A\, z^{\alpha}(1-z)^{\beta}$
(selfsimilar system, PDF)

'Intermediate'-pT physics: jet-fragmentation in dense matter ??

Modified fragmentation function (??) : $\overline{D}_c^h(z)\,dz$ ←

the probaility to produce a hadron h with momentum z p from a jet c with momentum p

in the presence of a dense matter (ρ)



← parton density

d_i * f(p_T)

Dense parton matter ➢➢➢➢➢ comoving partons are favoured

jet fragmentation is screened

Parton coalescence becomes dominant at intermediate p_T

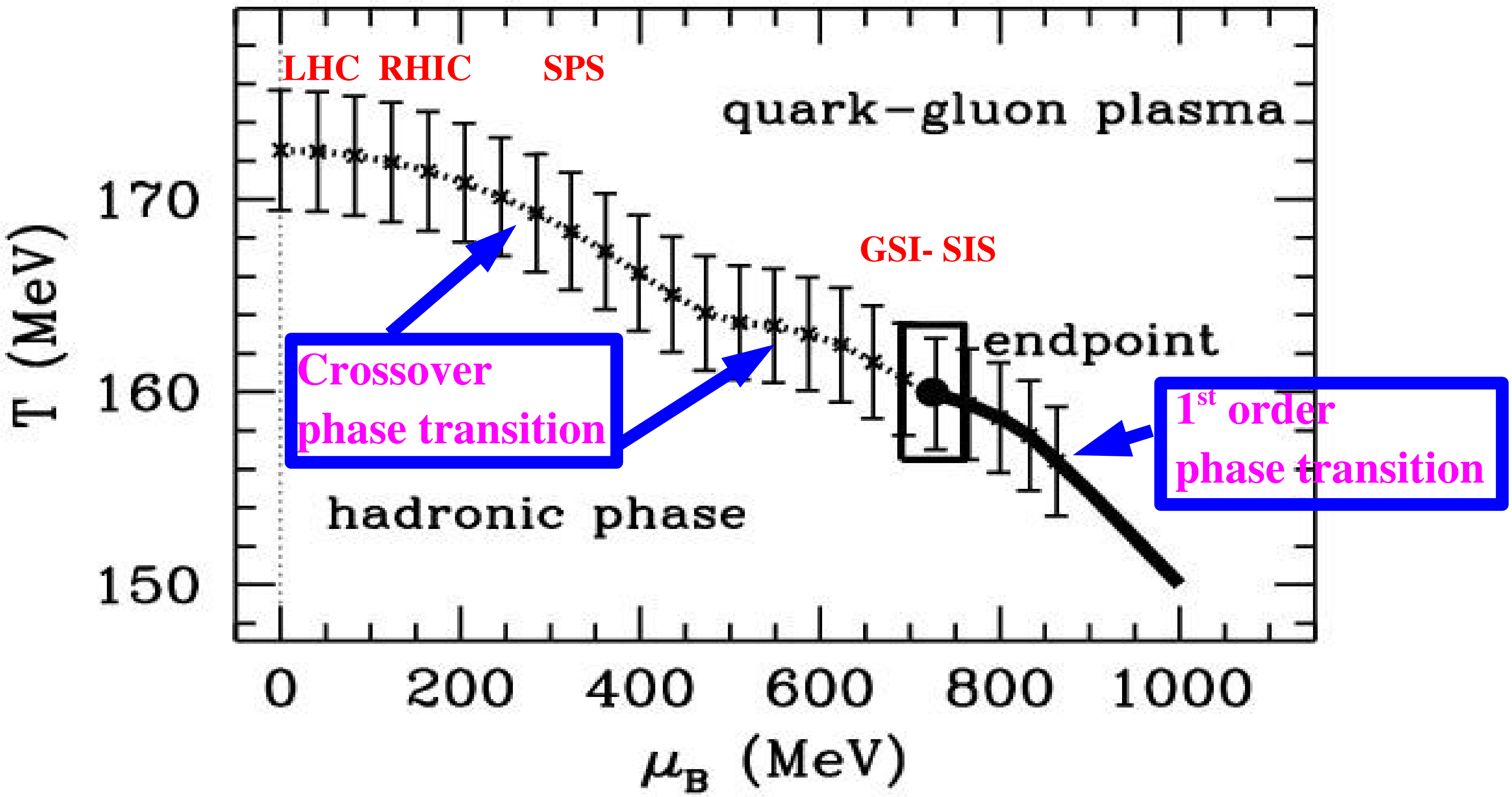
and overwhelm at low p_T

How to describe it microscopically ?? (quantitative results!?)

What are the relevant degrees of freedom ?

Continuation: Quark matter formation in heavy ion collisions

Lattice-QCD results at finite density, SU(3), Nf=2 μ>0 (Fodor et al.,2002)



Crossover phase transition at small and intermediate baryon densities:

What is the microscopical mechanism of the hadronization ????

⇒ **QUARK COALESCENCE**

COALESCENCE: deuteron production in heavy ion collisions

Statistical quantum mechanics: [Feynman '72] ⇒ Dover et al. PRC44(1991)1636.

Projecting the deuteron density matrix onto the two-nucleon density matrix:

[e.g. R. Scheibl, U. Heinz, PRC59(1999)1585.]

$$\frac{dN_d}{d^3P_d} \sim \frac{1}{2!} \int d^3x_1 d^3x_2 \; d^3x_1' d^3x_2' \; \varphi_d^*(x_1,x_2)\varphi_d(x_1',x_2') \; \langle \psi^+(x_2',t_f)\psi^+(x_1',t_f)\psi(x_1,t_f)\psi(x_2,t_f)\rangle$$

Deuteron wave-function: $\varphi_d(x_1,x_2)=(2\pi)^{-3/2}\exp[i P_d(x_1+x_2)/2] \; \phi_d(x_1-x_2)$

Internal wave-function: $\phi_d(r)=(\pi d^2)^{-3/4}\exp(-r^2/2d^2)$ ⇤ **inner structure !!**

Wigner transformation: $D(r,q)=\int d^3\xi \exp[-iq\xi]\phi_d(r+\xi/2)\phi_d^*(r-\xi/2)$

$$\Rightarrow 8\exp(-r^2/d^2-q^2\cdot d^2)$$

Two-nucleon density matrix ↣ one-particle density matrix:

(at freeze-out the nucleons are uncorrelated)

$$\langle \psi^+(x_2',t_f)\psi^+(x_1',t_f)\psi(x_1,t_f)\psi(x_2,t_f)\rangle = \langle \psi^+(x_2',t_f)\psi(x_2,t_f)\rangle\langle \psi^+(x_1',t_f)\psi(x_1,t_f)\rangle$$

One-body Wigner function from the one-particle density matrix:

$$\langle \psi^+(x',t_f)\psi(x,t_f)\rangle=\int \frac{d^3p}{(2\pi)^3} f^W(p;t_f,(x+x')/2)\exp[ip(x-x')]$$

The deuteron spectrum:

$$\frac{dN_d}{d^3P_d}=\frac{3}{(2\pi)^6}\int d^3r_d d^3q d^3r \, D(r,q) f_p^W(q_+,r_+) f_n^W(q_-,r_-)$$

Energy conservation: scattering on a third body before coalescence

QUARK COALESCENCE: meson production in bulk quark matter

Meson production: binding of a quark and an antiquark, $q + \bar{q} \Rightarrow M$

(constituent quark model, non-relativistic approx.)

--- (anti)quarks are inside a deconfined phase [QGP, QAP, CQM]

⇒ asymptotic wave functions do not exist inside deconf. phase !!!!

--- the interaction between quark and antiquark drives the meson production

⇒ non-relativistic $V(q\bar{q})$ potential (lattice-QCD results around T_c !)

--- direct calculation of coalescence matrix elements

$$M_{12} = \int d^3x_1 d^3x_2 \; \varphi_M(|x_1 - x_2|) \; e^{-iPX} \; V_{12}(|x_1 - x_2|) \; \phi_q(x_1)\phi_{\bar{q}}(x_2)$$

⇒ $V_{12}(r)$ is an effective coalescence potential: $V_{12} = -\alpha_{eff} \frac{\langle \lambda_1 \lambda_2 \rangle}{r}$

⇒ many coalescence channels exist (π, ρ, K, K*, ϕ, ...)

--- introducing $1+2 \rightarrow 3$ coalescence cross section [e.g. Biro et al, PLB347,1995,6]:

$$\sigma_{12}(k) = \frac{m_3^2}{4\pi^2} \sqrt{\frac{2 m_1 m_2}{(m_1 + m_2)^2}} \quad |M_{12}|^{12} = 16 m_3^2 \sqrt{\pi}\, \alpha_{eff}^2 \rho^3 \frac{a}{(1+(ka)^2)^2}$$

➡ a: Bohr radius

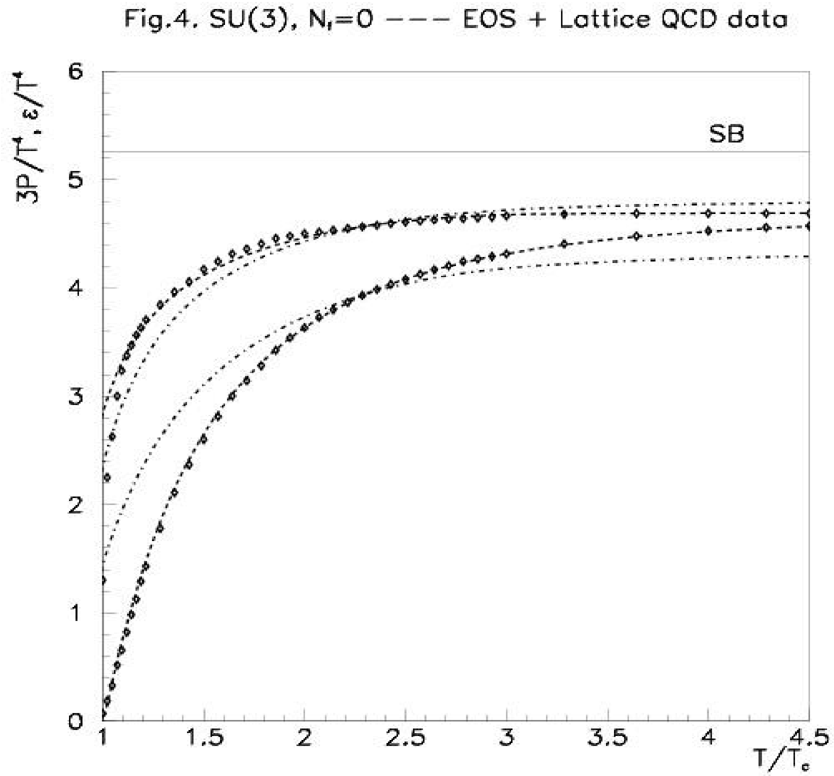
--- quark coalescence rate:

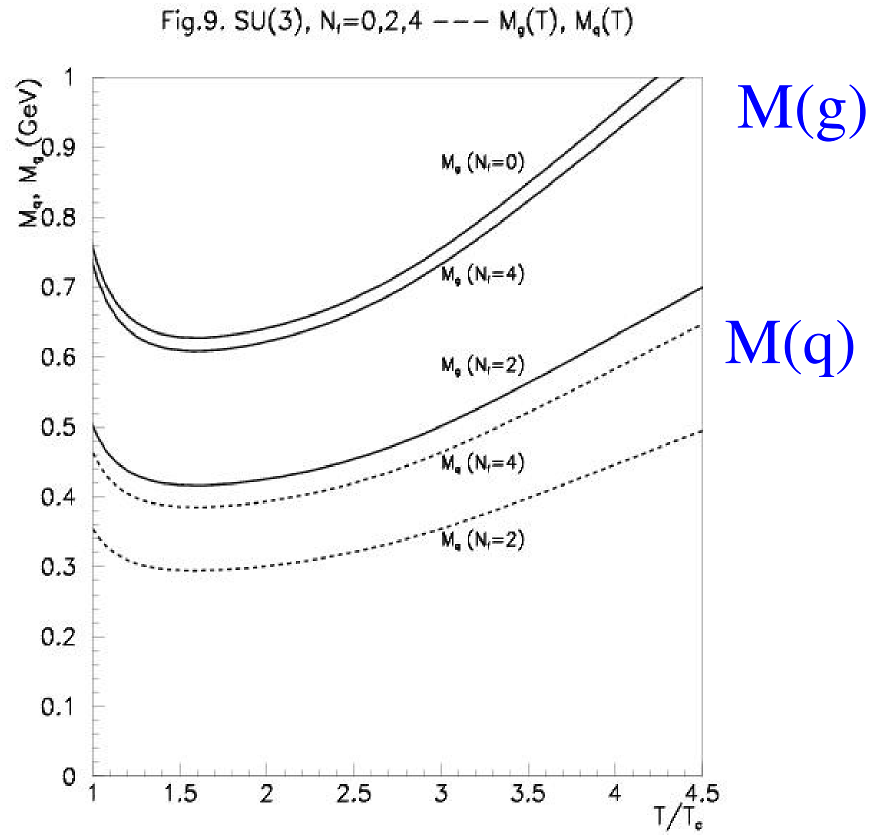
$$\langle \sigma_{12} v_{12} \rangle = \frac{\int d^3P_1 d^3P_2 f_1(P_1) f_2(P_2) \; \sigma_{12} v_{12}}{\int d^3P_1 d^3P_2 f_1(P_1) f_2(P_2)}$$

Can we use such a non-relativistic approximation ??? ➣ Quark mass !?!

Quark matter formation in heavy ion collisions

Lattice-QCD results around T_c, SU(3), Nf=0,2,4 μ=0   (1990 - ...)





Understanding in a quasiparticle picture: M(Q) ≃300 MeV, M(G)≃ 500-800 MeV

[L.P, Heinz U., 1998, PRC51,3326]

➔ Quark and antiquark dominated matter (QAP)

HADRONIZATION ⇔ QUARK COALESCENCE   (ALCOR '95)

('Cross-over' phase transition)   [T. Biró, P.L., J. Zimányi]

Quark matter formation in heavy ion collisions

Lattice-QCD results around T_c, SU(3), Nf=0,2,4 μ=0

Fig.10. SU(3), N_f=0,2,4 --- n_g/n_g^0, n_q/n_q^0

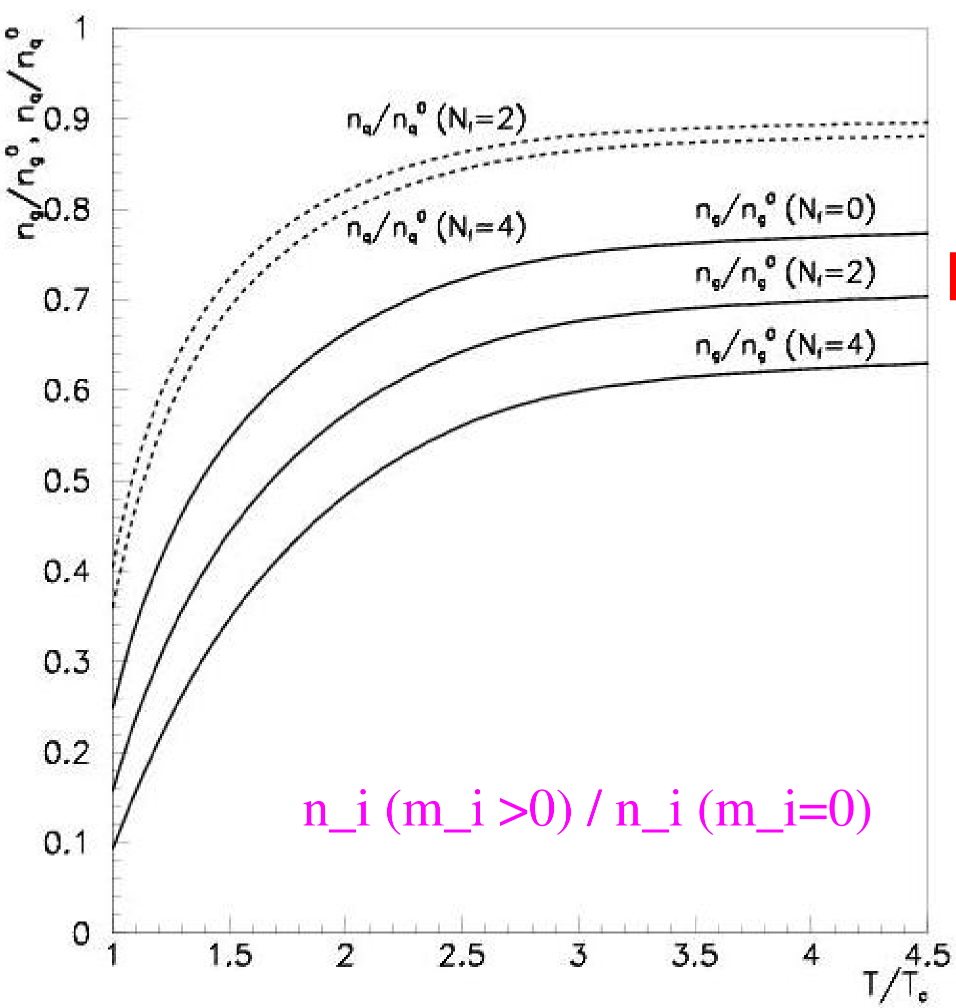
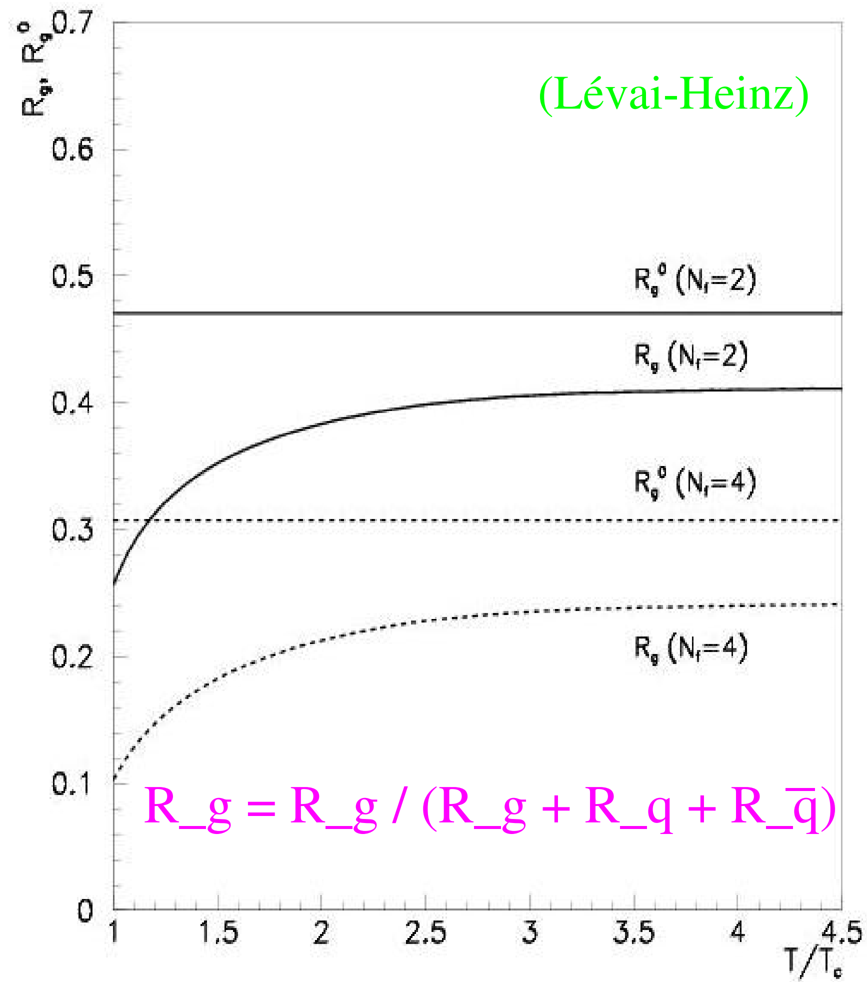
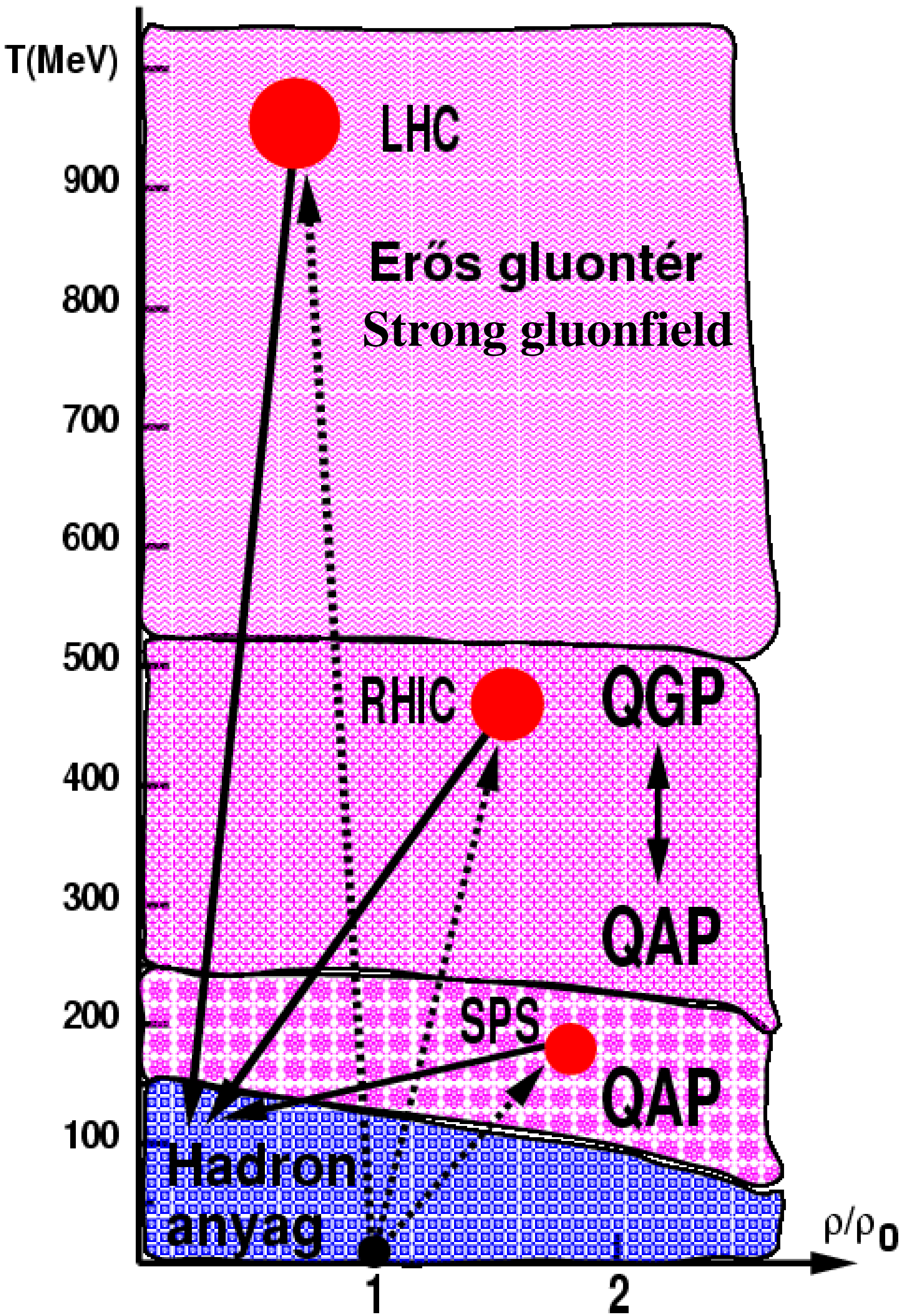


Fig.11. SU(3), N_f=2,4 --- $R_g(T)$, $R_g^0(T)$



➔ GLUON numbers are strongly suppressed at T_c and they will decay

QUARK-ANTIQUARK PLASMA



Summary

1982-2017: 35 years of HIC
s^1/2 = 0.05 - 5500 A GeV
(6 order of magnitude !!!!)

Most of these experiments end up in the same hadronization region!

We need to know the microscopical mechanisms working inside deconfined phase for

Basic candidate: quark coalescence

<u>Microscopical models could handle it</u>

QGP → → QAP → → HadronMatter

AMPT is an excellent candidate to follow the hadronisation

Quark matter formation in heavy ion collisions

ALCOR model for quark matter hadronization [Biró T.,L.P., Zimányi J. PLB347,6, 1995]

Massive quarks and antiquarks are the basic d.o.f. $u, \bar{u}, d, \bar{d}, s, \bar{s}$

Quarks from nucleus are melted (stopping)
Newly produced light quark-antiquark pairs
Newly produced strange quark-antiquark pairs ⇒

$$\frac{dN(u)}{dy} = P * N_u^{(total\,u)} + \frac{dN(\langle u\bar{u}\rangle)}{dy}$$

$$\frac{dN(s)}{dy} = \frac{dN(\langle s\bar{s}\rangle)}{dy}$$

Attractive potential between (anti-)quarks

$$V_{eff}(r) = -\alpha_{eff} \frac{\langle \lambda_i \lambda_j \rangle}{r}$$

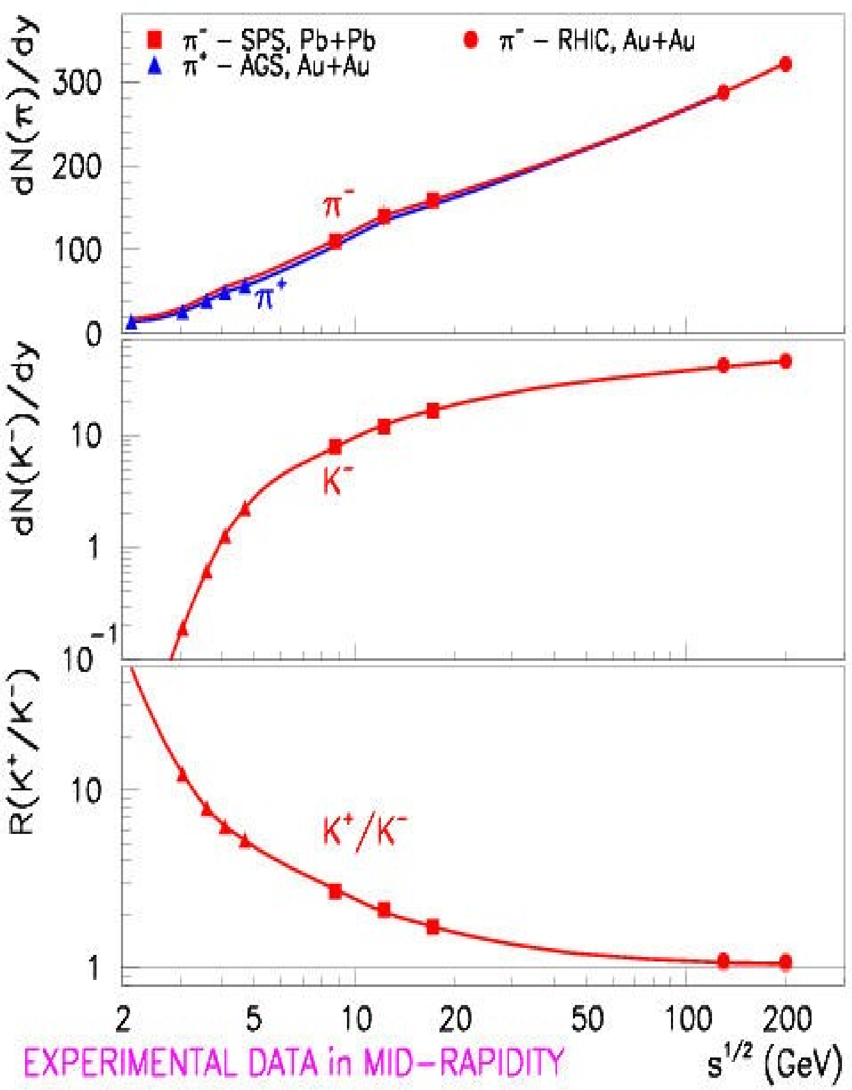
Heavy hadron resonances are produced -> decay

RESULT: analysis and understanding of the particle numbers and their ratios + energy dependence

Input parameters: $\underline{P}$; $\underline{\langle u\bar{u}\rangle} = \langle d\bar{d}\rangle$; $\langle s\bar{s}\rangle = \underline{f_s} * (\langle u\bar{u}\rangle + \langle d\bar{d}\rangle)$; $\underline{\alpha_{eff}}$

Quark matter formation between √s = 5 - 200 A GeV

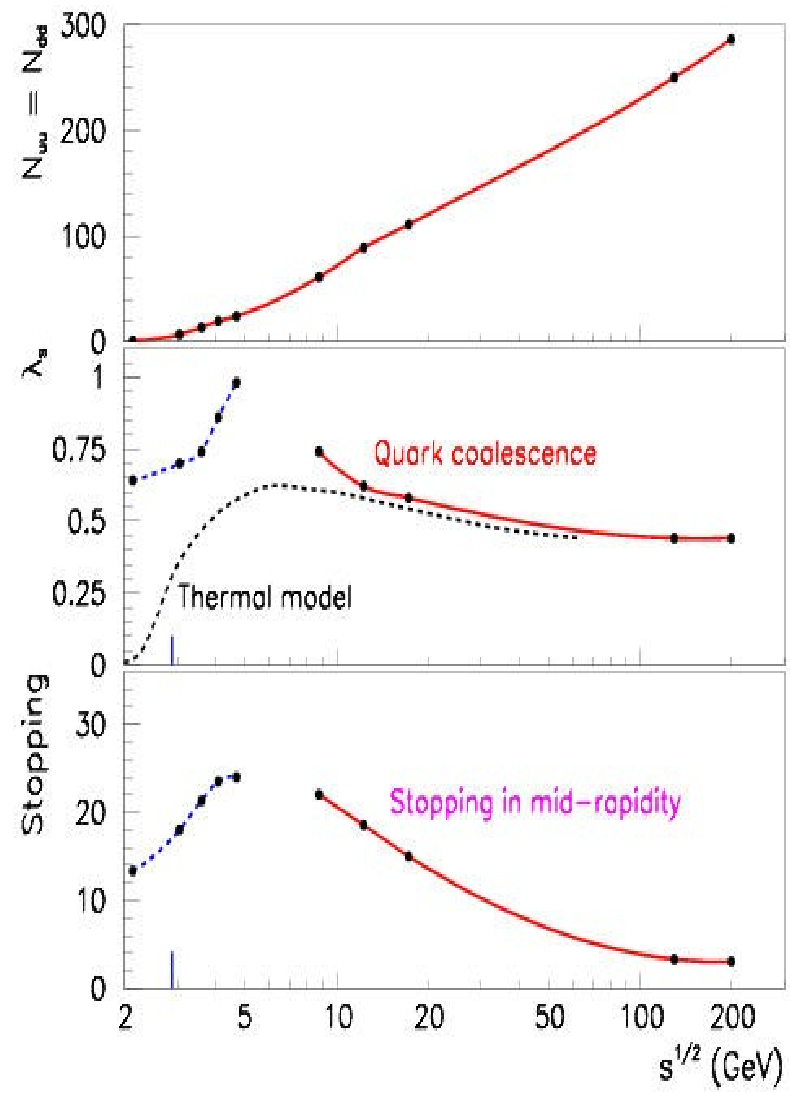
ALCOR model for quark matter hadronization [Zimányi J., Biró T.,L.P.]



N(q)

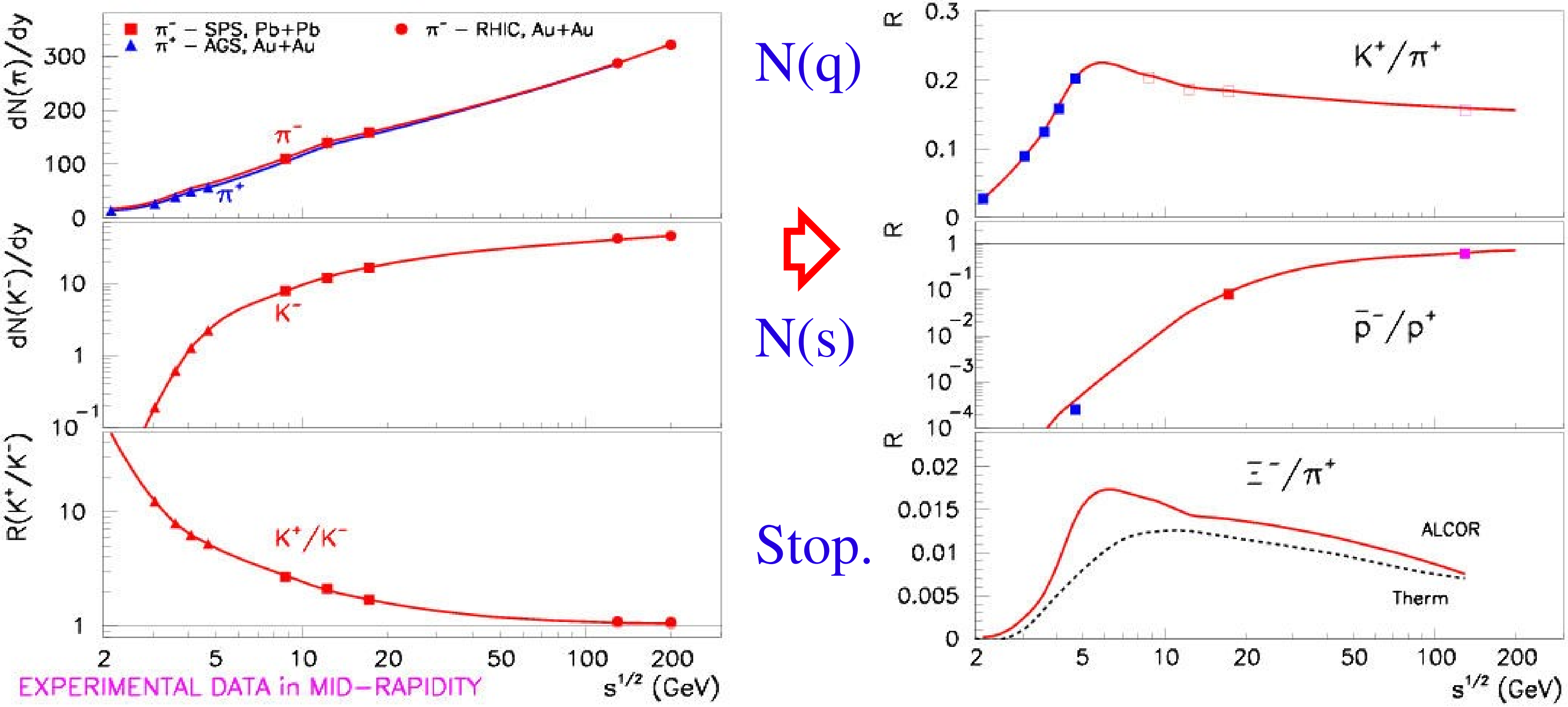
N(s)

Stop.



Quark matter formation between $\sqrt{s}$ = 5 - 200 A GeV

ALCOR model for quark matter hadronization [Zimányi J., Biró T.,L.P.]



Quark-coalescence reproduces most of the bulk properties

(particle numbers, ratios, their energy dependence)

What about gluons ? ⇔ QUARK ANTIQUARK PLASMA (QAP)

This description is valid for $p_T < 1.5$ GeV (99%)

It is valid at RHIC energy !

Quark matter formation at RHIC at $\sqrt{s}$ = 130 & 200 A GeV

ALCOR model for quark matter hadronization [Zimányi, Biró, L.P., 2002]

	ALCOR 130 AGeV fit	ALCOR 200 AGeV prediction
New pairs, $dN_{u\bar{u}}/dy$	250	286
Strangeness, f_s	0.22	0.22
Stopping, in %	3.3	3.0
Interaction, α_{eff}	0.55	0.55

Au+Au dN_i/dy	STAR 130 AGeV	ALCOR	STAR 200 AGeV	ALCOR
π^-	287 ± 20	287	327 ± 32	322
K^-	41.9 ± 5.5	40.4	49.5 ± 7.4	45.6
K^-/K^+	0.91 ± 0.11	0.93	0.92 ± 0.02	0.94
$\overline{\Xi}^+$	1.72 ± 0.1	1.76	1.81 ± 0.08	2.23
h^+		690	780	780
K^+	46.2 ± 6.1	43.1	51.3 ± 7.7	48.1
Ξ^-	2.05 ± 0.1	2.16	2.16 ± 0.09	2.59
$\langle\Omega^- + \overline{\Omega}^+\rangle$	0.55 ± 0.15	0.59	0.59 ± 0.14	0.72
$\overline{p}^-/p^+$	0.64 ± 0.07	0.70	0.77 ± 0.05	0.76
$\overline{\Lambda}/\Lambda$	0.71 ± 0.04	0.75	0.81 ± 0.07	0.810
$\overline{\Xi}^+/\Xi^-$	0.83 ± 0.05	0.81	0.84 ± 0.06	0.86
$\overline{\Omega}^+/\Omega^-$	0.95 ± 0.15	0.88	0.95 ± 0.15	0.92
K^+/π^+	0.161 ± 0.024	0.15	0.16 ± 0.02	0.150
K^-/π^-	0.146 ± 0.022	0.14	0.15 ± 0.02	0.142
Λ/h^-	0.054 ± 0.001	0.047		0.050
$\overline{\Lambda}/h^-$	0.040 ± 0.001	0.037		0.042
Ξ^-/π^-	0.006 ± 0.001	0.007	0.007 ± 0.001	0.008
K^{*0}	36.7 ± 5.5	28.5		31.7
Φ/K^{*0}	0.49 ± 0.13	0.37		0.37
Φ/K^-		0.26	0.13 ± 0.03	0.26
ρ^0/π^0		0.22	0.20 ± 0.04	0.22

Quark-coalescence:

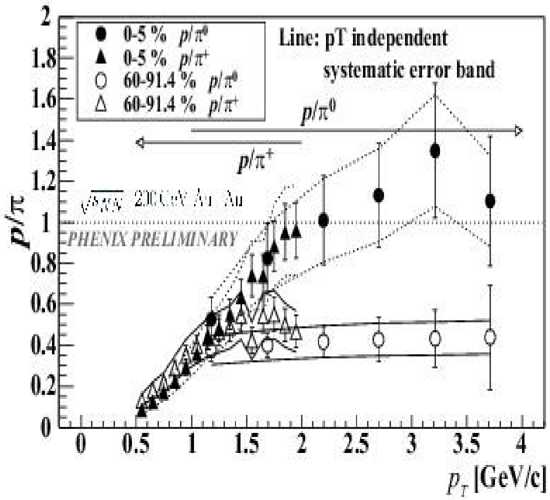
reproduces most of the bulk properties at RHIC energies (particle numbers, ratios, their energy dependence)

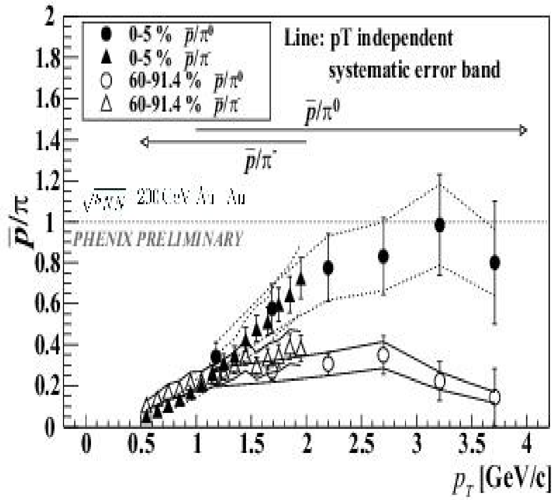
Exciting results from RHIC at √s = 130 and 200 A GeV -- $p/\pi^+, \bar{p}/\pi^-$

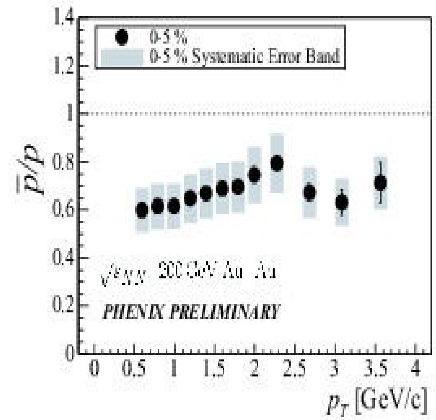
PHENIX Coll., T. Sakaguchi, nucl-ex/0209030, QM02 Conf.

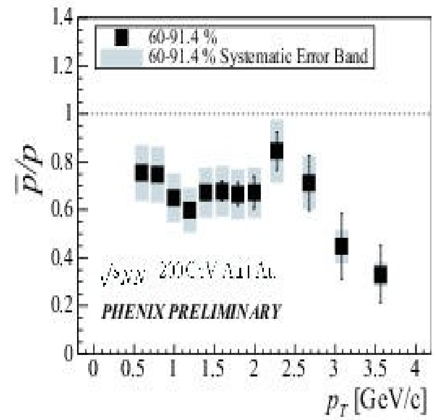
$$N(\bar{p}) > N(\pi^-)!!!$$

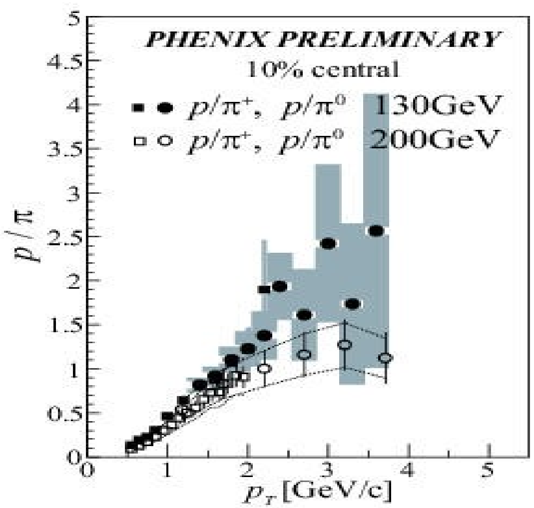
Anomalous antiproton (proton) production ??

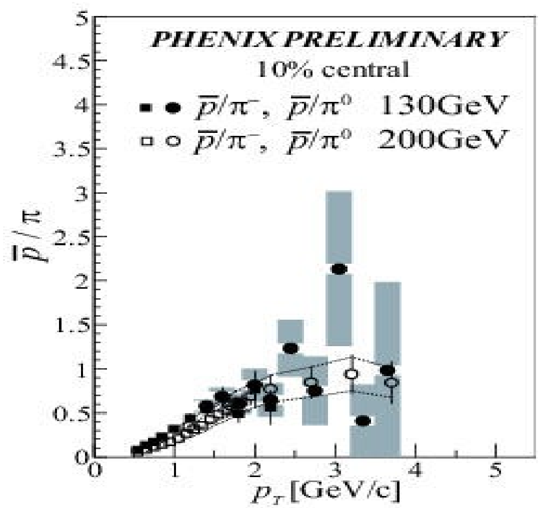












How are hadrons produced in AA?
What is the pQCD result?
What is the 'soft' result ?
Where is the soft-hard limit ?
Do we see QGP-like matter at RHIC ?

Parton coalescence: meson production

Greco, Ko, Levai, PRL90, 202302 (2003)
PRC68,034904 (2003)

Basic coalescence equation: $1 + 2 \to M$

$$\frac{dN_M}{d^3P_M} = g_M \int d^3r_a d^3r_b \frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3} f_1^W(\vec{p}_1,\vec{r}_a)\, f_2^W(\vec{p}_2,\vec{r}_b) \cdot \delta^3(\vec{P}_M - \vec{p}_1 - \vec{p}_2)\, \mathcal{F}_M^W(\vec{r}_a - \vec{r}_b, \vec{p}_1 - \vec{p}_2)$$

f_i^W: the Wigner function of parton i $(\to dN_i/d^3p)$

$\mathcal{F}_M^W$: the Wigner function of the produced meson M $(\to$ box-like$)$

$$\mathcal{F}_M(\vec{r}_a - \vec{r}_b, \vec{p}_1 - \vec{p}_2) = \frac{1}{\Delta_p^3\,\Gamma_r^3}\frac{9\pi}{2}\Theta(\Delta_p - |\vec{p}_1 - \vec{p}_2|)\cdot\Theta(\Gamma_r - |\vec{r}_a - \vec{r}_b|)\,,$$

Δ_p: a sharp cutoff in the relative momenta

Γ_r: a correlation length in space (the size of the meson)

Longitudinally invariant coalescence rate:

$$\frac{dN_M}{d^2P_M} = \frac{g_M}{V}\frac{6\pi^2}{\Delta_p^3}\int d^2p_1\, d^2p_2\, \frac{dN_1}{d^2p_1}\frac{dN_2}{d^2p_2}\,\delta^2(\vec{P}_{M,\perp} - \vec{p}_{1,\perp} - \vec{p}_{2,\perp})\,\Theta(\Delta_p - |\vec{p}_1 - \vec{p}_2|)\,.$$

Transverse explosion: comoving partons are able to coalesce, $\Phi_1 = \Phi_2$

$$\frac{dN_M}{2\pi P_{M,\perp}dP_{M,\perp}} = \frac{g_M}{V}\frac{6\pi^2}{\Delta_M^3}\int p_{1,\perp}dp_{1,\perp}\, p_{2,\perp}dp_{2,\perp}\,\frac{dN_1}{2\pi p_{1,\perp}dp_{1,\perp}}\,\frac{dN_2}{2\pi p_{2,\perp}dp_{2,\perp}} \cdot \frac{1}{P_{M,\perp}^2}\,\delta\left(1 - \frac{p_{1,\perp}+p_{2,\perp}}{P_{d,\perp}}\right)\,\Theta(\Delta_M - |p_{1,\perp} - p_{2,\perp}|)$$

R.C. Hwa, C.B. Yang, (nucl-th/0211010)

R.J. Fries, B. Muller, C. Nonaka, S.A. Bass, PRL90, 202303 (2003)

Parton coalescence: baryon production

Greco, Ko, Levai, PRL90, 202302 (2003) [567 cit.]
PRC68,034904 (2003) [427 cit.]

Basic coalescence equation: $1+2+3 \longrightarrow B$

$$\frac{dN_B}{d^3P_B} = g_B \int d^3r_1\, d^3r_2\, d^3r_3 \frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3}\frac{d^3p_3}{(2\pi)^3}\; f_1^W(\vec{p}_1,\vec{r}_1)\; f_2^W(\vec{p}_2,\vec{r}_2)\; f_3^W(\vec{p}_3,\vec{r}_3) \cdot \delta^3(\vec{P}_B-\vec{p}_1-\vec{p}_2-\vec{p}_3)\; \mathcal{F}_B^W(\vec{\rho},\vec{\lambda};\vec{q}_\rho,\vec{q}_\lambda)$$

f_i^W: the Wigner function of parton i $\quad(\to dN_i/d^3p)$

$\mathcal{F}_B^W$: the Wigner function of the produced baryon B $\quad(\to$ box-like)

$$\mathcal{F}_B(\vec{\rho},\vec{\lambda};\vec{q}_\rho,\vec{q}_\lambda) = \frac{1}{\Delta_\rho^3\,\Gamma_\rho^3}\frac{9\pi}{2}\,\Theta(\Delta_\rho-|\vec{q}_\rho|)\cdot\Theta(\Gamma_\rho-|\vec{\rho}|)\cdot \frac{1}{\Delta_\lambda^3\,\Gamma_\lambda^3}\frac{9\pi}{2}\,\Theta(\Delta_\lambda-|\vec{q}_\lambda|)\cdot\Theta(\Gamma_\lambda-|\vec{\lambda}|)\,.$$

$\Delta_\rho, \Delta_\lambda$: sharp cutoffs in the relative momenta

$\Gamma_\rho, \Gamma_\lambda$: correlation lengths in space ($\sim$ the size of the meson)

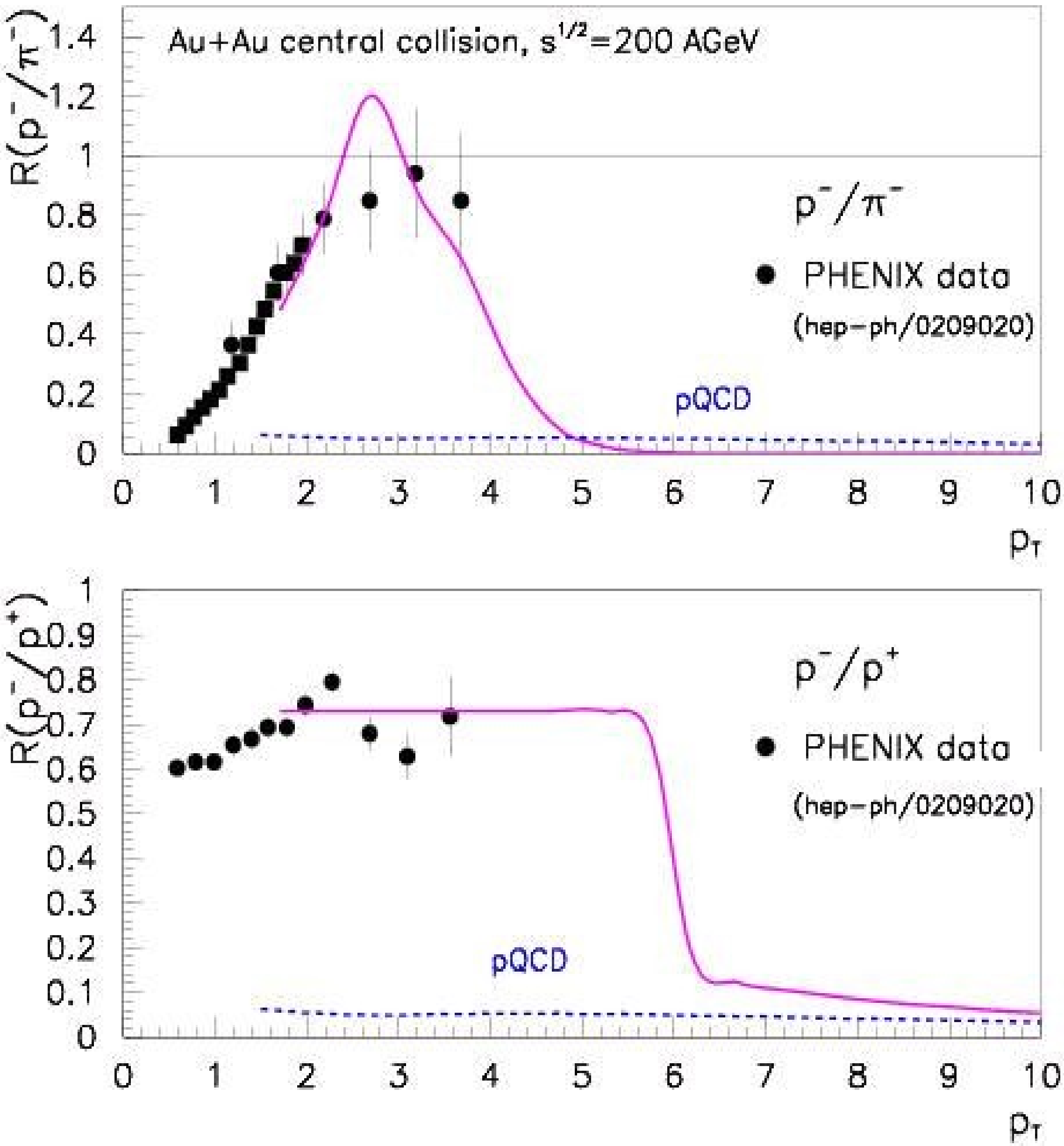
Longitudinally invariant coalescence rate:

$$\frac{dN_B}{d^2P_B} = \frac{g_B}{V^2}\frac{36\pi^4}{\Delta_\rho^3\,\Delta_\lambda^3}\int d^2p_1\, d^2p_2\, d^2p_3 \frac{dN_1}{d^2p_1}\frac{dN_2}{d^2p_2}\frac{dN_3}{d^2p_3} \cdot \delta^2(\vec{P}_{d,\perp}-\vec{p}_{1,\perp}-\vec{p}_{2,\perp}-\vec{p}_{3,\perp})\cdot\Theta(\Delta_\rho-|\vec{q}_{\rho,\perp}|)\cdot\Theta(\Delta_\lambda-|\vec{q}_{\lambda,\perp}|)\,.$$

Transverse explosion: comoving partons are able to coalesce, $\Phi_1=\Phi_2=\Phi_3=\Phi_B$

$$\frac{dN_B}{2\pi P_{B,\perp}dP_{B,\perp}} = \frac{g_B}{V^2}\frac{36\pi^4}{\Delta_B^6}\int p_{1,\perp}dp_{1,\perp}\; p_{2,\perp}dp_{2,\perp}\; p_{3,\perp}dp_{3,\perp}\prod_{i=1,2,3}\frac{dN_i}{2\pi p_{i,\perp}dp_{i,\perp}} \cdot\frac{1}{P_{B,\perp}^2}\,\delta\left(1-\frac{p_{1,\perp}+p_{2,\perp}+p_{3,\perp}}{P_{B,\perp}}\right)\prod_{i=1,2,3}\Theta_i(\Delta_B-|p_{i,\perp}-p_{i+1,\perp}|)$$

Parton colescence: antiproton/pion and antiproton/proton ratio



Intermediate (coalescence) region:

Location of the drop in
the antiproton/pion ratio:

- ☞ end of the intermediate region
- ☞ information about the partonic thermal bath (T, flow)
- ☞ shape analysis (details)

The edge of the drop in
the antiproton/proton ratio:

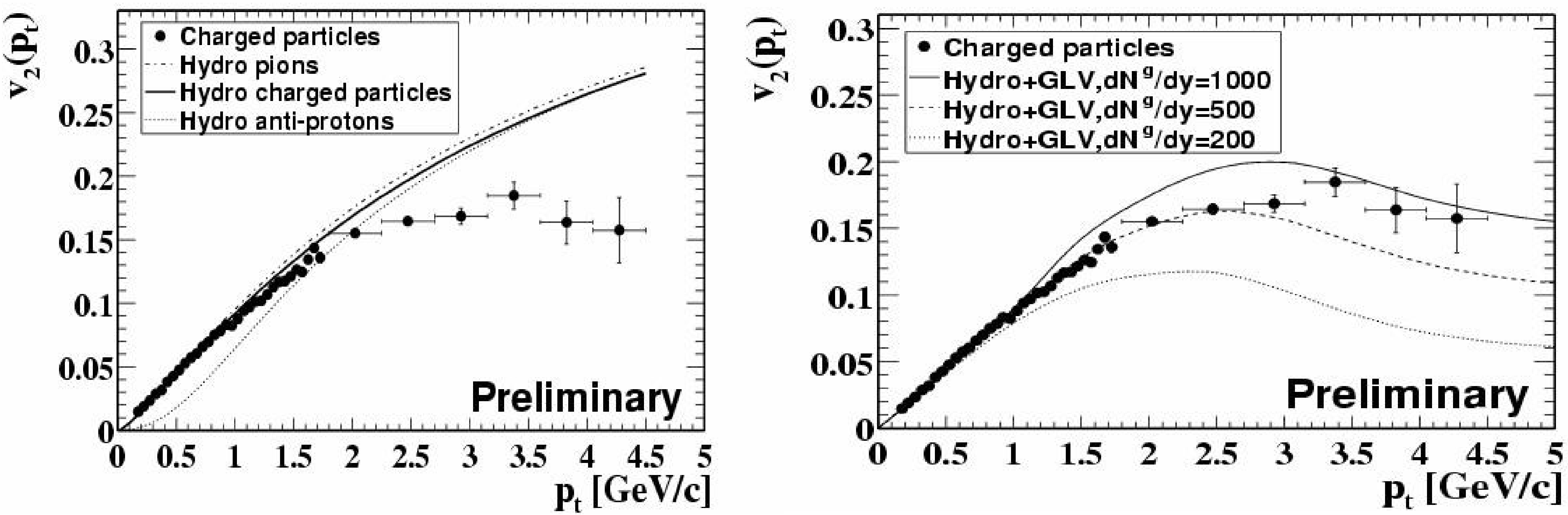
- ☞ p_c partonic cut-off
- ☞ Δ_p momentum window in the coalescence process

Exciting results from RHIC at √s = 130 and 200 AGeV -- v_2

Elliptic flow (anisotropy in momentum space):

$$v_2(p_T) = \langle \cos(2\phi) \rangle_{p_T} \equiv \frac{\int_{-\pi}^{\pi} d\phi \, \cos(2\phi) \, \frac{d^3N}{dy\, p_t\, dp_t\, d\phi}}{\int_{-\pi}^{\pi} d\phi \, \frac{d^3N}{dy\, p_t\, dp_t\, d\phi}}$$

STAR data at 130 A GeV (Nucl.Phys. A698 (2002) 193-198)



Parton colescence: elliptic flow

D.Molnar, S.A. Voloshin, (nucl-th/0302014)

Quark coalescence:

$$\frac{dN_B}{d^2p_\perp}(\vec{p}_\perp) = C_B\left[\frac{dN_q}{d^2p_\perp}(\vec{p}_\perp/3)\right]^3$$

$$\frac{dN_M}{d^2p_\perp}(\vec{p}_\perp) = C_M\left[\frac{dN_q}{d^2p_\perp}(\vec{p}_\perp/2)\right]^2$$

where the coefficients C_M and C_B are the probabilities for $q\bar{q} \to meson$ and $qqq \to baryon$ coalescence.

Anisotropic flow.
In the coalescence region, meson and baryon elliptic flow

$$v_{2,M}(p_\perp) \approx 2v_{2,q}(\frac{p_\perp}{2}), \qquad v_{2,B}(p_\perp) \approx 3v_{2,q}(\frac{p_\perp}{3}) \; ,$$

If partons have only elliptical anisotropy,
i.e., $dN_q/p_\perp dp_\perp d\Phi = (1/2\pi)dN_q/p_\perp dp_\perp[1 + 2v_{2,q}\cos(2\Phi)]$, then

$$v_{2,B}(p_\perp) = \frac{3v_{2,q}(p_\perp/3)}{1+6v_{2,q}^2(p_\perp/3)}$$

$$v_{2,M}(p_\perp) = \frac{2v_{2,q}(p_\perp/2)}{1+2v_{2,q}^2(p_\perp/2)} \, .$$

Coalescence region:

Starting from a mutual p_T spectra for quarks to produce mesons and/or baryons

- ☞ if elliptic flow exist for quarks then it will be herited for in different ways for mesons and for baryons

$$v_{2,B}(p_T) > v_{2,M}(p_T)$$

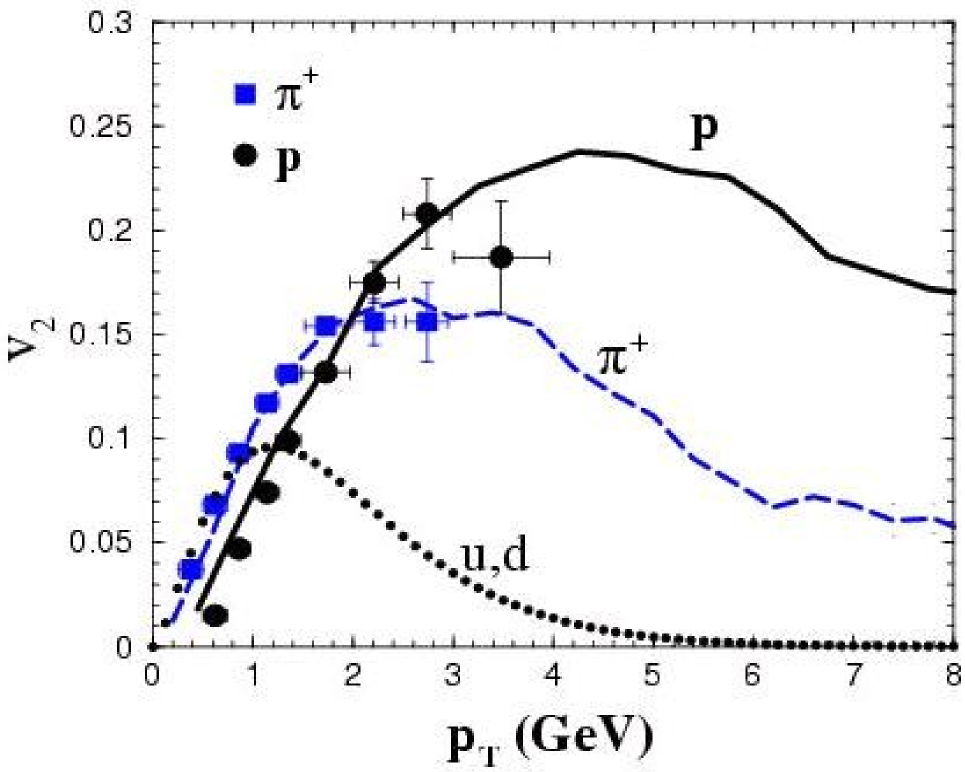
- ☞ different flows for different quark flavours : splitting in hadronic v_2

$$v_{2,p} > v_{2,\lambda}$$

Parton colescence: elliptic flow

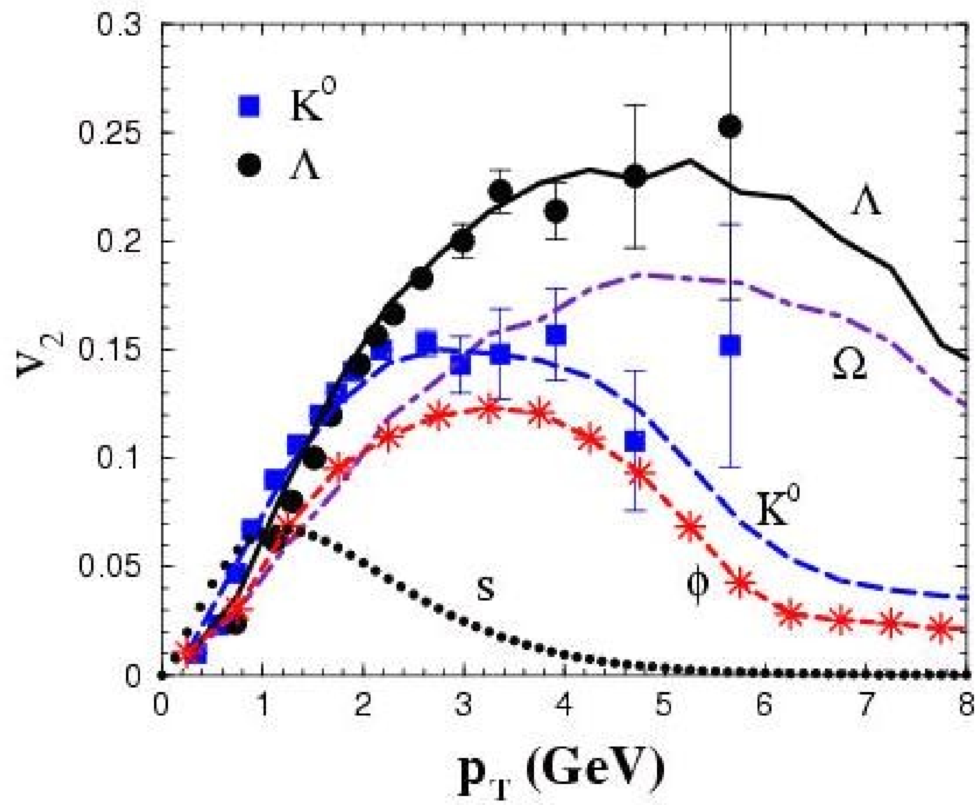
V. Greco, C.M. Ko, P. Levai , PRC 68 (2003) 034904
[427 citation]

Data: S. Esumi (PHENIX), nucl-ex/0210012



$$v_{2,p}(p_T) > v_{2,\pi}(p_T)$$

Data: R. Snellings (STAR), nucl-ex/0305001



$$v_{2,\lambda}(p_T) > v_{2,K}(p_T)$$

SUMMARY-1:

1. Three different region in particle production:

   I. Soft region ($p_T < 1$ GeV)

      Thermodynamics, hydrodynamics, ...

      ALCOR-type quark coalescence (mass, T and V(r) are important)

   II. Intermediate region ($1 < p_T < 5\text{-}6$ GeV)

      Parton coalescence driven by quantum mechanics is important (Gribov)

      Jet partons participate in recombination with neighbour comovers

   III. Hard region ($p_T > 5\text{-}6$ GeV)

      Perturbative QCD can be applied (PDF, FF, jet-quenching, ...)

      Independent fragmentation is dominant

2. This picture is supported by

   a, Pion suppression pattern

   b, Antiproton/pion enhancement

   c, Antiproton/proton ratio and its p_T dependence (have to be measured)

   d, Elliptic flow phenomena

3. Further studies are needed (resonances, details in coalescence process,...)

   Last 15 years in microscopic models → success of the AMPT

4. Further data are needed at RHIC and LHC energies – they were reproduced, mostly !!

Quark-coalescence is working in bulk matter !!

Where is the end of the coalescence region ?
Will it die out ?
How "particles" behave at high energy densities?

Where does "classical distribution functions" become invalid ?

Could we use quantum distributions in descriptions?

↣↣↣↣ Strong Field Physics (Wigner-distr.) for QCD calculations
hadron production in pp, dAu, AuAu collision
at RHIC, LHC and FCC energies

Where is the bottom energy ?

Chiral Magnetic Effect in the Dirac-Heisenberg-Wigner formalism

Dániel Berényi [1], Péter Lévai [1], Vladimir Skokov [2]

1, Wigner RCP, Budapest, Hungary
2, RIKEN/BNL, Upton, USA

07. 07. 2017.
EPS-HEP 2017, Venice

Chiral Magnetic Effect

What is the Chiral Magnetic Effect?

- Given a background EM magnetic field, and the QCD gauge fields.
- An initially vanishing chiral imbalance could obtain non-zero value due to the interaction with the gauge fields with non-zero Q_w winding number.

$$Q_w = \frac{g^2}{32\pi^2} \int \mathrm{d}^4 x F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a \in \mathbb{Z} \tag{1}$$

Axial charge:

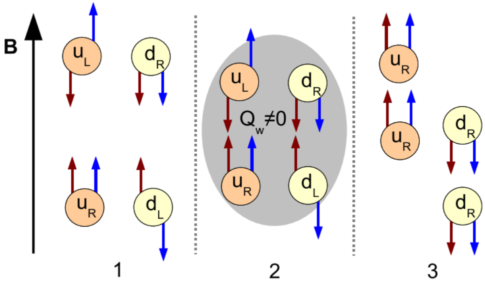
$$(N_L - N_R)_{t=\infty} = 2 N_f Q_w \tag{2}$$

Axial current (on the background field):

$$j^5_\mu = \langle \bar{\psi} \gamma_\mu \gamma_5 \psi \rangle_A \tag{3}$$

D. E. Kharzeev, L. D. McLerran and H. J. Warringa, Nucl. Phys. A 803, 227 (2008).

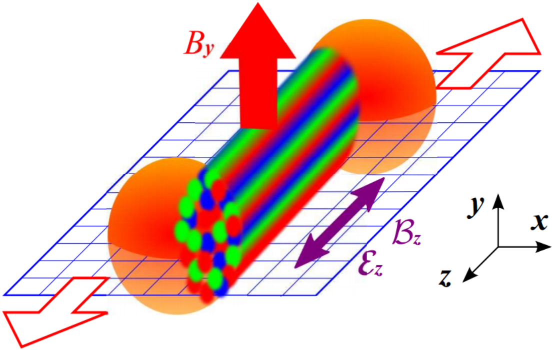
Chiral Magnetic Effect



1. Chirally neutral mixture in very strong B field: particles constrained to Lowest Landau Level.
2. Gauge interaction with non-zero Q_w fields change chirality.
3. Chirality separation leads to charge separation, that leads to current.

Chiral Magnetic Effect

Possible realisation in heavy-ion collisions:



- Background: very strong B field due to highly charged nuclei passing near each other.
- Gauge: QCD gluons

Chiral Magnetic Effect

- Transition between different topologies can happen via tunneling.
- The simplest configuration is a flux-tube, where the gauge fields are $E||B$.
- This can be described by the Schwinger effect $\rightarrow$ connection to pair production.
- Already investigated for constant fields

  Kenji Fukushima, Dmitri E. Kharzeev, and Harmen J. Warringa Phys. Rev. Lett. 104, 212001 2010.
- Main idea: color diagonalisation leads to QED description with E_z, B_z from chromoelectric/magnetic fields and with B_y from EM.

Chiral Magnetic Effect

Main charachteristics of the CME (electric) current j_μ:

- $E_z = B_z = B_y = 0$, nothing happens, everything is zero
- $E_z = 0, B_z \neq 0, B_y \neq 0$, B fields alone does not create any current
- $E_z \neq 0, B_z = 0, B_y = 0$, E field alone only drives current in its direction
- $E_z \neq 0, B_z \neq 0, B_y = 0$, still nothing...
- $E_z \neq 0, B_z = 0, B_y \neq 0$, still nothing...
- Only in the case, when none of the three is zero, is there a CME current!

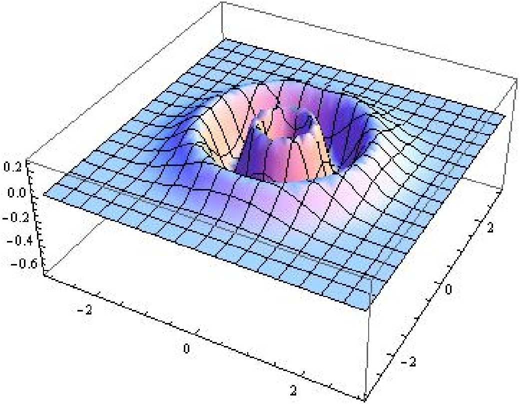
Chiral Magnetic Effect

- Q: How can we investigate the time dependence of this process?
- A: Generalizing the Schwinger description as usual:
  Wigner functions in the real time formalism.

Wigner function

Tool of description: the Wigner function:

- Quantum analogue of the classical phase space distribution.



Wigner function of an n=3 Fock state.

Wigner function

How it is defined?

- Take the equal time density matrix in terms of 'center of mass' coordinates:

$$\hat{\rho}(\vec{x},\vec{s},t) = e^{-ig\int_{-1/2}^{1/2}\vec{A}(\vec{x}+\lambda\vec{s},t)\vec{s}\mathrm{d}\lambda}\left[\Psi(\vec{x}+\frac{\vec{s}}{2},t),\bar{\Psi}(\vec{x}-\frac{\vec{s}}{2},t)\right] \tag{4}$$

- Take the expectation value.
- Fourier transform it w.r.t the coordinate difference:

$$W(\vec{x},\vec{p},t) = -\frac{1}{2}\int e^{-i\vec{p}\vec{s}}\langle\Omega|\hat{\rho}(\vec{x},\vec{s},t)|\Omega\rangle\mathrm{d}^3s \tag{5}$$

Wigner function

The evolution equation:

$$D_t W = -\frac{1}{2}\vec{D}_{\vec{x}}[\gamma^0\vec{\gamma}, W] - im[\gamma^0, W] - i\vec{P}\{\gamma^0\vec{\gamma}, W\} \tag{6}$$

The equation has the following non-local differential operators:

$$D_t = \partial_t + g\vec{\mathcal{E}}(\vec{x},t)\vec{\nabla}_{\vec{p}} - \frac{g\hbar^2}{12}(\vec{\nabla}_{\vec{x}}\vec{\nabla}_{\vec{p}})^2\vec{\mathcal{E}}(\vec{x},t)\vec{\nabla}_{\vec{p}} + \dots \tag{7}$$

$$\vec{D}_{\vec{x}} = \vec{\nabla}_{\vec{x}} + g\vec{\mathcal{B}}(\vec{x},t)\times\vec{\nabla}_{\vec{p}} - \frac{g\hbar^2}{12}(\vec{\nabla}_{\vec{x}}\vec{\nabla}_{\vec{p}})^2\vec{\mathcal{B}}(\vec{x},t)\times\vec{\nabla}_{\vec{p}} + \dots \tag{8}$$

$$\vec{P} = \vec{p} + \frac{g\hbar}{12}(\vec{\nabla}_{\vec{x}}\vec{\nabla}_{\vec{p}})\vec{\mathcal{B}}(\vec{x},t)\times\vec{\nabla}_{\vec{p}} + \dots \tag{9}$$

For spin-1/2, the 4x4 gamma matrix basis is used:

$$W(x,p,t) = \frac{1}{4}\left[\mathbb{1}\mathbb{s} + i\gamma_5\mathbb{p} + \gamma^\mu\mathbb{v}_\mu + \gamma^\mu\gamma_5\mathbb{a}_\mu + \sigma^{\mu\nu}\mathbb{t}_{\mu\nu}\right] \tag{10}$$

Wigner function

The equation has the following non-local differential operators
for homogeneous E and B:

$$D_t = \partial_t + g\vec{\mathcal{E}}(t)\vec{\nabla}_{\vec{p}} \tag{11}$$

$$\vec{D}_{\vec{x}} = g\vec{\mathcal{B}}(t) \times \vec{\nabla}_{\vec{p}} \tag{12}$$

$$\vec{P} = \vec{p} \tag{13}$$

Equations of motion for the spin-1/2 Wigner function

We arrive at a system for 16 unknown real functions:

$$
\begin{aligned}
&D_t\mathbb{s} & & & - & 2\vec{P}\cdot\vec{\mathbb{t}}_1 &= 0 & \quad (14)\\
&D_t\mathbb{p} & & & + & 2\vec{P}\cdot\vec{\mathbb{t}}_2 &= 2m\mathbb{a}_0 & \quad (15)\\
&D_t\mathbb{v}_0 & + & \vec{D}_{\vec{x}}\cdot\vec{\mathbb{v}} & & &= 0 & \quad (16)\\
&D_t\mathbb{a}_0 & + & \vec{D}_{\vec{x}}\cdot\vec{\mathbb{a}} & & &= 2m\mathbb{p} & \quad (17)\\
&D_t\vec{\mathbb{v}} & + & \vec{D}_{\vec{x}}\mathbb{v}_0 & + & 2\vec{P}\times\vec{\mathbb{a}} &= -2m\vec{\mathbb{t}}_1 & \quad (18)\\
&D_t\vec{\mathbb{a}} & + & \vec{D}_{\vec{x}}\mathbb{a}_0 & + & 2\vec{P}\times\vec{\mathbb{v}} &= 0 & \quad (19)\\
&D_t\vec{\mathbb{t}}_1 & + & \vec{D}_{\vec{x}}\times\vec{\mathbb{t}}_2 & + & 2\vec{P}\mathbb{s} &= 2m\vec{\mathbb{v}} & \quad (20)\\
&D_t\vec{\mathbb{t}}_2 & - & \vec{D}_{\vec{x}}\times\vec{\mathbb{t}}_1 & - & 2\vec{P}\mathbb{p} &= 0 & \quad (21)
\end{aligned}
$$

Equations of motion for the $m = 0$ spin-1/2 Wigner function

Simplification: $m = 0$.

Only the vector current / charge and the axial current / charge remains in the equations:

A system for 8 unknown real functions remains:

$$D_t \mathbb{v}_0 + \vec{D}_{\vec{x}} \cdot \vec{\mathbb{v}} = 0 \quad (22)$$

$$D_t \mathbb{a}_0 + \vec{D}_{\vec{x}} \cdot \vec{\mathbb{a}} = 0 \quad (23)$$

$$D_t \vec{\mathbb{v}} + \vec{D}_{\vec{x}} \mathbb{v}_0 + 2\vec{P} \times \vec{\mathbb{a}} = 0 \quad (24)$$

$$D_t \vec{\mathbb{a}} + \vec{D}_{\vec{x}} \mathbb{a}_0 + 2\vec{P} \times \vec{\mathbb{v}} = 0 \quad (25)$$

Numerical Solution

Ingredients of the 3+1 D numerical solver:

- Pseudospectral collocation
- Rational Chebyshev polynomial basis
- 4th order Runge-Kutta
- GPU Acceleration (30x speed up)

wigner

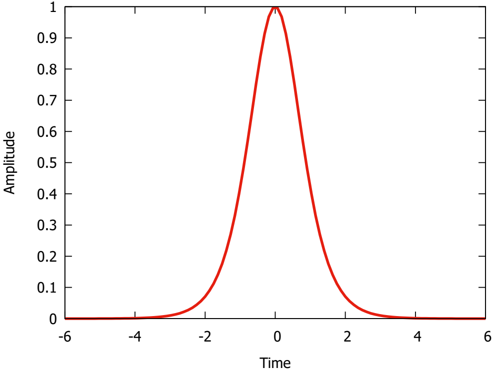
Sauter field

Start with a toy field that we know very well:

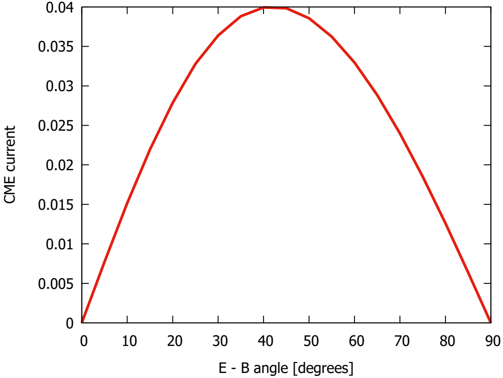
$$S(t) = \cosh^{-2}(t/\tau) \qquad (26)$$

Let the fields be:

$$E_z = A \cdot S(t),$$
$$B_z = A \cdot \cos(\alpha) S(t),$$
$$B_y = A \cdot \sin(\alpha) S(t)$$



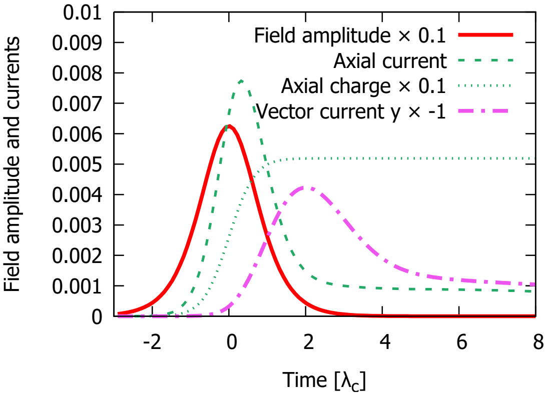
Sauter field



Only in the case, when none of the three is zero, is there a CME current!

Results

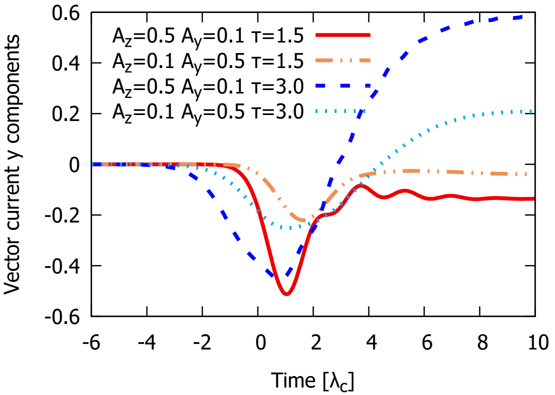
Chiral Magnetic Current formation during the interaction:



Field comes first, then axial current, axial charge separation and finally the electric current!

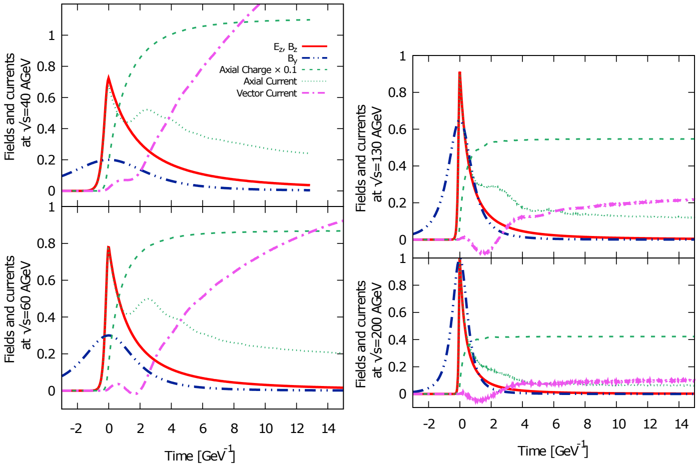
Results

Interplay of Amplitude and time extent:



Longer time scales lead to larger effect together with a sign change.

CEM effect at different collision energies:



Sign change at 40-60 AGeV, disappearance above 200 AGeV

Summary

- Heavy-ion collisions at RHIC energies indicated the appearance of a specific phenomena, the Chiral Magnetic Effect, which is generated by a strong gluon field modifying the chirality of the plasma.
- The CME effect can be successfully modelled by the Dirac-Heisenberg-Wigner description of time dependent strong fields.
- The detailed calculations indicate the expected disappearance of CME at energies above 200 AGeV.
- Between 40-60 AGeV we have found an interesting sign change in the CME current.

Supporters: OTKA Grants No. NK-106119, K16-120660, and Wigner GPU Laboratory