





Light Nuclei Production in Relativistic Heavy-Ion Collisions and QCD Critical Endpoint

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- QCD Critical Endpoint (CEP)
- Baryon density fluctuation and coalescence production of light nuclei in HIC
- $O = \frac{N^3 H N_p}{N^2}$ as a probe of QCD CEP
- Summary and Outlook

Main References:

[1] Kai-Jia Sun (孙开佳) & LWC, PRC95, 044905 (2017) [arXiv:1701.01935] [2] Kai-Jia Sun (孙开佳), LWC, C. M. Ko, and Z. Xu, arXiv:1702.07620

Workshop on AMPT for Relativistic Heavy Ion Collisions (AMPT2017), Sichuan University, Chengdu, China, July 24-27, 2017





QCD Critical Endpoint (CEP) Baryon density fluctuation and coalescence production of light nuclei in HIC O = ^{N³H^Np}/_{N²d} as a probe of QCD CEP Summary and Outlook



QCD Phase Diagram

A selection of representations of the QCD phase diagram in the (μ_B, T) plane.



"All science is either physics or stamp collecting." --- Ernest Rutherford "The Way Forward – Closing Remarks at Quark Matter 2017", W.A. Zajc, [arXiv:1707.01993]



QCD Phase Diagram

V.E. Fortov, Extreme States of Matter – on Earth and in the Cosmos, Springer-Verlag, 2011



Small baryon chemical potential: Smooth Crossover Transition
 Large baryon chemical potential: First-order Phase Transition
 QCD Critical Endpoint: where the first-order phase transition ends

CEP from Lattice and effective field theories



TABLE I: Locations of the QCD critical point from Lattice QCD and DSE, respectively.

		Lattice		DSE		
(μ_B^E, T^E)	I [33]	II [34, 35]	III [36–39]	I [40]	II [41]	III [42]
MeV	(360, 162)	(285, 155)	$\mu_B^E/T^E > 2$	(372, 129)	(405, 127)	(504, 115)

X.F. Luo and N. Xu, Nucl. Sci. Tech. 28, 112 (2017).

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Strategy for locating CEP in experiments







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First-order phase transition and baryon density fluctuation



Taken from Feng Li's talk at Shanghai, 01/2017

Transport Model + NJLnsitionNo first-order phase transition



F. Li and C.M. Ko, Phys. Rev. C 93, 035205 (2016).



Baryon density fluctuation vs light nuclei production

Baryon density fluctuation is closely related to the correlation between nucleons.

The correlation between nucleons determines the production of light nuclei



Baryon density fluctuation in vicinity of first-order phase transition could be deciphered from the production of light nuclei



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Coalescence model provides a useful approach to describe light cluster production in HIC

- Coalescence model provides a useful tool to describe light nuclei production in HIC
- **Coalescence model** also provides a useful tool to describe hadron production from partonic matter (hadronization)

Butler, Pearson, Sato, Yazaki, Gyulassy, Frankel, Remler, Dove, Scheibl, Heinz, Schnedermann, Mattiello, Nagle, Polleri, ...
Biro, Zimanyi, Levai, Csizmadia, Hwa, Yang, Ko, Lin, Greco, Chen, Fries, Muller, Nonaka, Bass, Voloshin, Molnar, Xie, Shao, ...



Parton coalescence mechanism at RHIC



Lin/Ko PRL89, 202302 (2002); S.A. Voloshin, *NPA715*, *379*(2003); Greco/Ko/Levai, PRL90, 202302 (2003); Fries/Muller/Nonaka/Bass *PRL90*, *202303* (2003); Hwa/Yang *PRC67, 034902 (2003);* 064902 (2003); Molnar/Voloshin PRL91, 092301 (2003) $B(p_T) \Leftarrow 3q(p_T/3)$ $M(p_T) \Leftarrow 2q(p_T/2)$



Deuteron production in HIC

The correlation between neutron and proton with small relative momentum and deuteron formation both appear due to the final state interaction (S. Mrowczynsky, PLB248 (1990), P. Danielewicz et al., PLB274 (1992))

The n-p pair in a scattering state with small relative momentum and deuteron (n-p pair in a bound state) should provide the same space-time information about the size of an emission source

Using stiff symmetry energy will produce more deuterons than using soft symmetry energy?

Similarly to the n-p correlation function (HBT), deuteron yield in HIC's induced by neutron-rich nuclei is also a sensitive probe of the nuclear symmetry energy!!!

n-p HBT: Chen/Greco/Ko/Li, PRL90, 162701(2003); PRC68, 014605 (2003) Deuteron: Chen/Ko/Li, PRC68, 017601 (2003); NPA729, 809 (2003)



Coalescence model



- > Depends on constituents' space-time structure at freeze-out (Reaction Dynamics)
- > Neglecting the binding energy effect (T or E >> E_{binding}),

Coalescence probability: Semi-classical Wigner function. (Wave Function, Correlation)

Rare process has been assumed (the coalescence process can be treated perturbatively).
Higher energy collisions and higher energy cluster/particle production!

() 上海交通大学 Freeze-out: Blast-Wave-Like Parametrization

Anisotropic Blast-Wave Model



FIG. 1. (Color online) The source weighting function Ω as a function of the normalized elliptical radius \tilde{r} for several values of the surface diffuseness parameter a_s .

The freeze-out distribution is infinite in the beam (z) direction and elliptical in the transverse (x-y) plane. (The x-z plane is the reaction plane.) The transverse shape is controlled by the radii R_y and R_x , and the spatial weighting of source elements is given by

$$\Omega(r,\phi_s) = \Omega(\tilde{r}) = \frac{1}{1 + e^{(\tilde{r}-1)/a_s}},\tag{1}$$

where a fixed value of the "normalized elliptical radius,"

$$\tilde{r}(r,\phi_{s}) \equiv \sqrt{\frac{[r\cos(\phi_{s})]^{2}}{R_{x}^{2}} + \frac{[r\sin(\phi_{s})]^{2}}{R_{y}^{2}}}, \qquad (2$$

corresponds to a given elliptical subshell within the solid volume of the freeze-out distribution.

F. Retiere and M.A. Lisa, PRC70, 044907 (2004)



FIG. 2. Schematic illustration of an elliptical subshell of the source. Here, the source is extended out of the reaction plane $(R_y > R_x)$. Arrows represent the direction and magnitude of the flow boost. In this example, $\rho_2 > 0$ [see Eq. (4)].

Boosted by the transverse rapidity $\rho(r, \phi_s) = \tilde{r}[\rho_0 + \rho_2 \cos(2\phi_b)]. \quad (4)$

8 parameters: T, ρ_0 , ρ_2 , R, s_2 , $a_s \tau_0$, $\Delta \tau$

Light (Anti-)(Hyper-)Nuclei Production: Sun/Chen, PLB751, 272 (2015); PRC93, 064909 (2016); PRC94, 064908 (2016)

上海交通大学 Analytical Coalescence Formula

Analytical coalescence formula: COAL-SH

$$N_{c} = g_{\text{rel}}g_{\text{size}}g_{c}M^{3/2} \left[\prod_{i=1}^{A} \frac{N_{i}}{m_{i}^{3/2}}\right] \times \prod_{i=1}^{A-1} \frac{(4\pi/\omega)^{3/2}}{Vx(1+x^{2})} \left(\frac{x^{2}}{1+x^{2}}\right)^{l_{i}} G(l_{i},x).$$

$$Sun/LWC, PRC95, 044905 (2017) X^{2} = \frac{2T_{\text{eff}}}{w}$$

$$g_{\text{rel}} \approx 1, g_{\text{size}} \approx 1, l_{i} = 0$$

$$N_{\rm d} = g_{\rm d} \frac{(m_n + m_p)^{3/2}}{m_p^{3/2} m_n^{3/2}} \frac{N_p N_n}{V} \frac{(4\pi/\omega_{\rm d})^{3/2}}{x_{\rm d}(1 + x_{\rm d}^2)},$$

$$N_{^3\rm H} = g_{^3\rm H} \frac{(2m_n + m_p)^{3/2}}{m_p^{3/2} m_n^3} \frac{N_p N_n^2}{V^2} \frac{(4\pi/\omega_{^3\rm H})^3}{x_{^3\rm H}^2(1 + x_{^3\rm H}^2)^2}, \qquad w_d = 8.1 \text{ MeV}$$

$$w_{^3\rm H} = 13.4 \text{ MeV}$$

$$x_d \gg 1, x_{^{3}\mathrm{H}} \gg 1$$

$$N_{\rm d} = \frac{3}{2^{1/2}} \left(\frac{2\pi}{m_0 T_{\rm eff}}\right)^{3/2} \frac{N_p N_n}{V},$$

$$N_{^3\rm H} = \frac{3^{3/2}}{4} \left(\frac{2\pi}{m_0 T_{\rm eff}}\right)^3 \frac{N_p N_n^2}{V^2}.$$

$$m_n = m_p = m_0$$

K. J. Sun, LWC, C.M. Ko, and Z. Xu, arXiv:1702.07620 (2017).



上海交通大學 Cluster Yields w/o density fluctuation

$$N_{\rm d} = \frac{3}{2^{1/2}} \left(\frac{2\pi}{m_0 T_{\rm eff}}\right)^{3/2} \frac{N_p N_n}{V},$$
$$N_{^3\rm H} = \frac{3^{3/2}}{4} \left(\frac{2\pi}{m_0 T_{\rm eff}}\right)^3 \frac{N_p N_n^2}{V^2}.$$

The above equations are consistent with conventional thermal model:

$$N^{\text{th}} = \frac{gV}{(2\pi)^3} 4\pi T m^2 K_2(\frac{m}{T}) e^{\frac{\mu}{T}} \qquad T = T_{\text{eff}}$$

$$N^{\text{th}}_p = \frac{g_p V}{(2\pi)^3} (2\pi m_0 T)^{\frac{3}{2}} e^{\frac{\mu_p - m_0}{T}},$$

$$N^{\text{th}}_d = \frac{g_d V}{(2\pi)^3} (4\pi m_0 T)^{\frac{3}{2}} e^{\frac{2\mu_p - 2m_0}{T}},$$

$$N^{\text{th}}_{3\text{H}\epsilon} = \frac{g_{3\text{H}\epsilon} V}{(2\pi)^3} (6\pi m_0 T)^{\frac{3}{2}} e^{\frac{3\mu_p - 3m_0}{T}},$$

K. J. Sun, LWC, C.M. Ko, and Z. Xu, arXiv:1702.07620 (2017).

上海交通大学 Cluster Yields with density fluctuation

Density fluctuation over space:

$$n(\vec{r}) = \frac{1}{V} \int n(\vec{r}) d\vec{r} + \delta n(\vec{r}) = \langle n \rangle + \delta n(\vec{r})$$

 $\langle \delta n \rangle = 0, \, \langle \delta n_p \rangle = 0,$

 $N_{\rm d} = \frac{3}{21/2} \left(\frac{2\pi}{\pi} \right)^{3/2} \int d\vec{r} \, n(\vec{r}) n_p(\vec{r})$

Neutron: Proton:

$$n(\vec{r}) = \langle n \rangle + \delta n(\vec{r}),$$

$$n_p(\vec{r}) = \langle n_p \rangle + \delta n_p(\vec{r}),$$

Strictly speaking, one needs to introduce fluctuation from the beginning in coalescence formulism

$$N_n = \int d\vec{r} \, n = V \langle n \rangle, \, N_p = \int d\vec{r} \, n_p = V \langle n_p \rangle.$$

Approximately:

$$N_{\rm d} = \frac{3}{2^{1/2}} \left(\frac{2\pi}{m_0 T_{\rm eff}}\right)^{3/2} \frac{N_p N_n}{V} = \frac{3}{2^{1/2}} \left(\frac{2\pi}{m_0 T}\right)^{3/2} \int d\vec{r} \left(\langle n \rangle + \delta n(\vec{r})\right) \left(\langle n_p \rangle + \delta n_p(\vec{r})\right) \frac{\text{Cross terms}}{\text{vanish}}$$

$$\sim \int d\vec{r} n(\vec{r}) n_p(\vec{r}) = \frac{3}{2^{1/2}} \left(\frac{2\pi}{m_0 T}\right)^{3/2} \int d\vec{r} \left(\langle n \rangle \langle n_p \rangle + \delta n(\vec{r}) \delta n_p(\vec{r})\right)$$

$$= \frac{3}{2^{1/2}} \left(\frac{2\pi}{m_0 T}\right)^{3/2} \left(N_p \langle n \rangle + \int d\vec{r} \, \delta n(\vec{r}) \delta n_p(\vec{r})\right).$$

シ 上海交通大学 Cluster Yields with density fluctuation

$$\begin{split} N_{^{3}\mathrm{H}} &= \frac{3^{3/2}}{4} \left(\frac{2\pi}{m_{0}T}\right)^{3} \int d\vec{r} \, n(\vec{r})^{2} n_{p}(\vec{r}) & N_{^{3}\mathrm{H}} = \frac{3^{3/2}}{4} \left(\frac{2\pi}{m_{0}T_{\mathrm{eff}}}\right)^{3} \int d\vec{r} \, n(\vec{r})^{2} n_{p}(\vec{r}) & \sim \int d\vec{r} n(\vec{r})^{2} n_{p}(\vec{r}) \\ &= \frac{3^{3/2}}{4} \left(\frac{2\pi}{m_{0}T}\right)^{3} \int d\vec{r} \, (\langle n \rangle + \delta n(\vec{r}))^{2} (\langle n_{p} \rangle + \delta n_{p}(\vec{r})) \\ &= \frac{3^{3/2}}{4} \left(\frac{2\pi}{m_{0}T}\right)^{3} \int d\vec{r} \, \{(\langle n \rangle^{2} + (\delta n(\vec{r}))^{2}) \langle n_{p} \rangle + (2\langle n \rangle \delta n(\vec{r}) + (\delta n(\vec{r}))^{2}) \delta n_{p}(\vec{r})\} \\ &= \frac{3^{3/2}}{4} \left(\frac{2\pi}{m_{0}T}\right)^{3} \{(\langle n \rangle^{2} + \langle (\delta n(\vec{r}))^{2} \rangle) \langle n_{p} \rangle V + \int d\vec{r} \, (2\langle n \rangle \delta n(\vec{r}) + (\delta n(\vec{r}))^{2}) \delta n_{p}(\vec{r})\} \\ N_{\mathrm{d}} &= \frac{3}{2^{1/2}} \left(\frac{2\pi}{m_{0}T}\right)^{3/2} \left(N_{p} \langle n \rangle + \int d\vec{r} \, \delta n(\vec{r}) \delta n_{p}(\vec{r})\right) \\ &= \langle n(\vec{r}) \rangle = \frac{1}{V} \int d\vec{r} n(\vec{r}) \quad \langle (\delta n(\vec{r}))^{2} \rangle = \frac{1}{V} \int d\vec{r} (\delta n(\vec{r}))^{2} \end{split}$$

 $3^{3/2}$ () 3^{3} M M²





³H yield is proportional to the relative density fluctuation!

However, their yields also depend on T and neutron average density <n>.

Let's take ratio to cancel out the T and <n> !





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Observable

The observable we propose to probe density fluctuation is:

$$\mathcal{O}_{p-d-t} = \frac{N_{^{3}\mathrm{H}}N_{p}}{N_{d}^{2}} = g(1+\Delta n)$$

$$g = 4/9 \times (3/4)^{3/2} \approx 0.29$$

$$\Delta n = \frac{\langle (\delta n)^{2} \rangle}{\langle n \rangle^{2}}$$

$$\langle n(\vec{r}) \rangle = \frac{1}{V} \int d\vec{r} (\delta n(\vec{r}))^{2}$$

$$\langle (\delta n(\vec{r}))^{2} \rangle = \frac{1}{V} \int d\vec{r} (\delta n(\vec{r}))^{2}$$

One can see that it has a linear dependence on neutron relative density fluctuation and it has no dependence on T, V, or other parameters.

This is different from what has been done in the measurements of fluctuation of yields of net protons within a specific phase-space in momentum so far.



Correction from correlation

Correlation between density fluctuation of proton and neutron

$$\langle \delta n \delta n_p \rangle = \frac{1}{V} \int d\vec{r} \delta n(\vec{r}) \delta n_p(\vec{r})$$

= $\frac{1}{V} \int d\vec{r} c(\vec{r}) (\delta n(\vec{r}))^2$ with $\delta n_p(\vec{r}) = c(\vec{r}) \delta n(\vec{r})$

One can always express: (a)

$$\langle \delta n \delta n_p \rangle = \alpha \frac{\langle n_p \rangle}{\langle n \rangle} \langle (\delta n)^2 \rangle$$

$$N_{\rm d} = \frac{3}{2^{1/2}} \left(\frac{2\pi}{m_0 T_{\rm eff}}\right)^{3/2} N_p \langle n \rangle (1 + \alpha \Delta n),$$

$$N_{^3\rm H} = \frac{3^{3/2}}{4} \left(\frac{2\pi}{m_0 T_{\rm eff}}\right)^3 N_p \langle n \rangle^2 (1 + (1 + 2\alpha)\Delta n)$$

$$N_{^3\rm He} = \frac{3^{3/2}}{4} \left(\frac{2\pi}{m_0 T_{\rm eff}}\right)^3 N_n \langle n_p \rangle^2 (1 + \Delta n_p + 2\alpha\Delta n)$$

$$N_{^4\rm He} = \frac{1}{2} \left(\frac{2\pi}{m_0 T_{\rm eff}}\right)^{9/2} N_p \langle n_p \rangle \langle n \rangle^2$$

$$\times \left[1 + (1 + 4\alpha)\Delta n + \Delta n_p + \frac{\langle (\delta n \delta n_p)^2 \rangle}{\langle n \rangle^2 \langle n_p \rangle^2}\right]$$

$$\approx g(1 + \Delta n + \mathcal{O}((\alpha\Delta n)^2))$$



Naive expectation



 $\operatorname{Log}(\sqrt{s_{NN}})$



Circumstantial evidence of peak structure



TABLE I: Yields (dN/dy at midrapidity) of p, d, ³He and ³H as well as the yield ratio ³H/³He measured in Pb+Pb collisions at SPS energies [47] together with the derived yield ratio \mathcal{O}_{p-d-t} . The units for E and $\sqrt{s_{NN}}$ are AGeV and GeV, respectively.

E	$\sqrt{s_{NN}}$	centrality	p	d	$^{3}\mathrm{He}$	$^{3}\mathrm{H}/^{3}\mathrm{He}$	$^{3}\mathrm{H}$	$\mathcal{O}_{ ext{p-d-t}}$
20	6.3	0-7%	$46.1 {\pm} 2.1$	$2.094{\pm}0.168$	$3.58(\pm 0.43) \times 10^{-2}$	$1.22{\pm}0.10$	$4.37(\pm 0.64) \times 10^{-2}$	$0.459 {\pm} 0.014$
30	7.6	0-7%	42.1 ± 2.0	$1.379 {\pm} 0.111$	$1.89(\pm 0.23) \times 10^{-2}$	$1.18 {\pm} 0.11$	$2.23(\pm 0.34) \times 10^{-2}$	$0.494{\pm}0.020$
40	8.8	0-7%	41.3 ± 1.1	$1.065 {\pm} 0.086$	$1.28(\pm 0.15) \times 10^{-2}$	$1.16{\pm}0.15$	$1.48(\pm 0.26) \times 10^{-2}$	$0.541 {\pm} 0.022$
80	12.3	0-7%	$30.1 {\pm} 1.0$	$0.543 {\pm} 0.044$	$3.90(\pm 0.50) \times 10^{-3}$	$1.15 {\pm} 0.19$	$4.49(\pm 0.94) \times 10^{-3}$	$0.458 {\pm} 0.038$
158	17.3	0-12%	$23.9{\pm}1.0$	$0.279 {\pm} 0.023$	$1.50(\pm 0.20) \times 10^{-3}$	$1.05{\pm}0.15$	$1.58(\pm 0.31) \times 10^{-3}$	$0.484{\pm}0.037$



Estimation of location of CEP



TABLE I: Locations of the QCD critical point from Lattice QCD and DSE, respectively.

	Lattice	DSE		
(μ_B^E, T^E)	[33] II [34, 35] III [36–39]	I [40] II [41] III [42]		
MeV	(360162) $(285,155)$ $\mu_B^E/T^E > 2$	372,129) (405,127) (504,115)		

M.A. Stephanov, Int. J. Mod. Phys. A20, 4387 (2005).

X.F. Luo and N. Xu, Nucl. Sci. Tech. 28, 112 (2017).

Peak structure and CEP





Peak structure and CEP

AMPT prediction for Trajectories of heavy ion collisions in the QCD phase diagram of temperature and net quark chemical potential.

Lower of the energy is, shorter (smaller) the QGP lifetime (size) is?



L. W. Chen, C. M. Ko, W. Liu, and B.W. Zhang, PoS (CPOD2009) 034 [arXiv:1103.0916].





QCD Critical Endpoint (CEP) Baryon density fluctuation and coalescence production of light nuclei in HIC O = ^{N³H^Np}/_{N²d} as a probe of QCD CEP Summary and Outlook



- $O = \frac{N^3 H^N p}{N^2_d}$ provides a potentially useful probe for QCD critical fluctuations and to locate the position of CEP in QCD phase diagram.
- The extracted relative neutron density fluctuation in central Pb+Pb collisions at CERN SPS energies measured by NA49 collaboration exhibits circumstantially a non-monotonic behavior with a peak at sqrt(s_{NN})=8.8 GeV, suggesting that the CEP in the QCD phase diagram may have been reached in these collisions with its temperature and baryon chemical potential estimated to be about T=144 MeV and μ_B =385 MeV.
- Further investigations from experiments, such as the BES program at RHIC, and theoretical modeling of light nuclei production and its connection to baryon density fluctuations are required.





谢谢! Thanks!

