中國地質大學

Scaling properties of multiplicity fluctuations in heavy-ion collisions simulated by AMPT model

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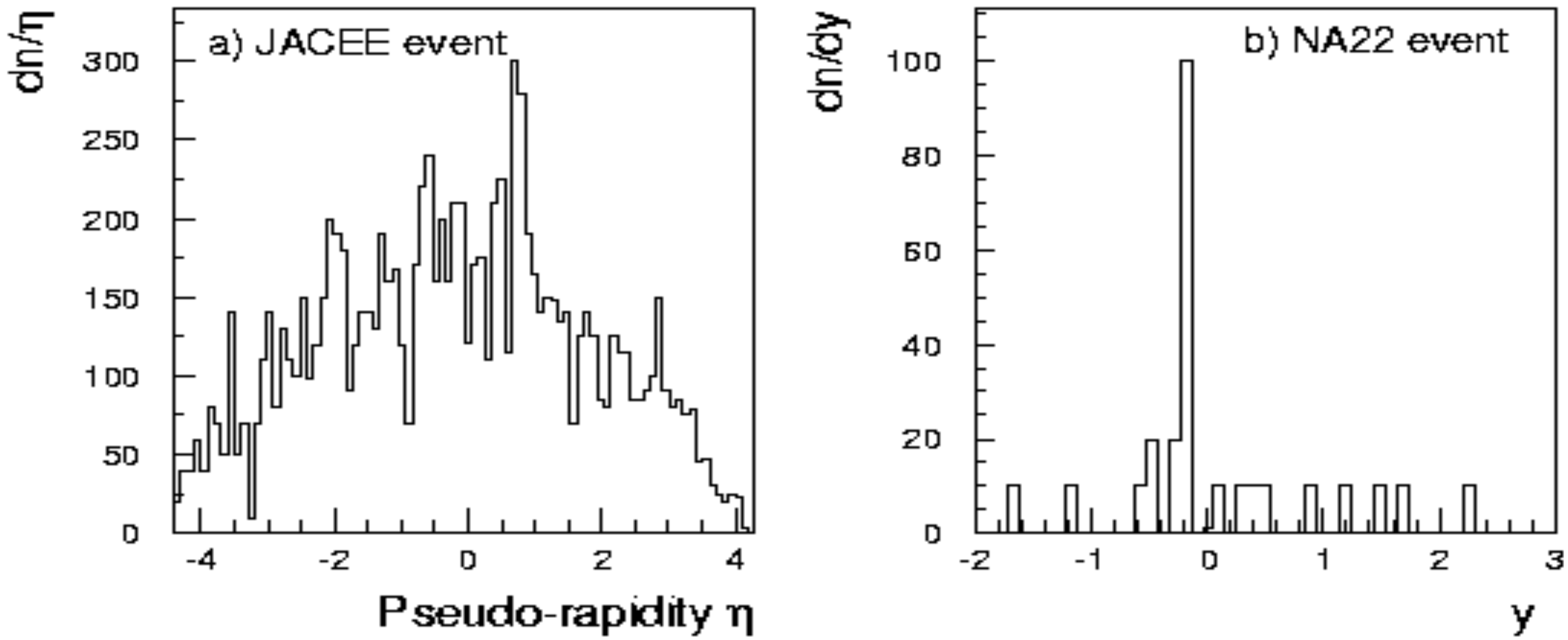
Outline

- Introduction
- Analysis method
- The Results
- Discussion of FM's Scaling
- Summary

1. Introduction

1.1 The non-linear phenomena exist in the high energy collision

- In 1983, JACEE
- NA22



Big local fluctuation implied dynamical factors.

JACEE, PRL 50 (1983) 2062.
NA22, PLB 185 (1987) 200.

1. Introduction

1.2 Bialas：describe the fluctuation with NFM as follow:

$$F_q(M) = \frac{1}{M}\sum_{m=1}^{M}\frac{\langle n_m(n_m-1)\cdots(n_m-q+1)\rangle}{\langle n_m\rangle^q}$$

M=number of partitions in momentum space

where n_m=multiplicity in bin m

$\langle\ldots\rangle$=average over all events

- NFM can characterize the dynamical fluctuations;

A. Bialas, NPB 273 (1986) 703; NPB 308 (1988) 857.

1. Introduction

1.3 Fractal and fluctuation

Fractal is defined

$$C_q(l) \propto l^{\phi_q}$$

- NFM can characterize the dynamical fluctuations;
- Intermittent dynamical fluctuations

$$F_q(M) \propto M^{\phi_q}$$

$\longrightarrow$ power-law scaling

$\longrightarrow$ fractal

W. Ochs, PLB 247 (1990)101.

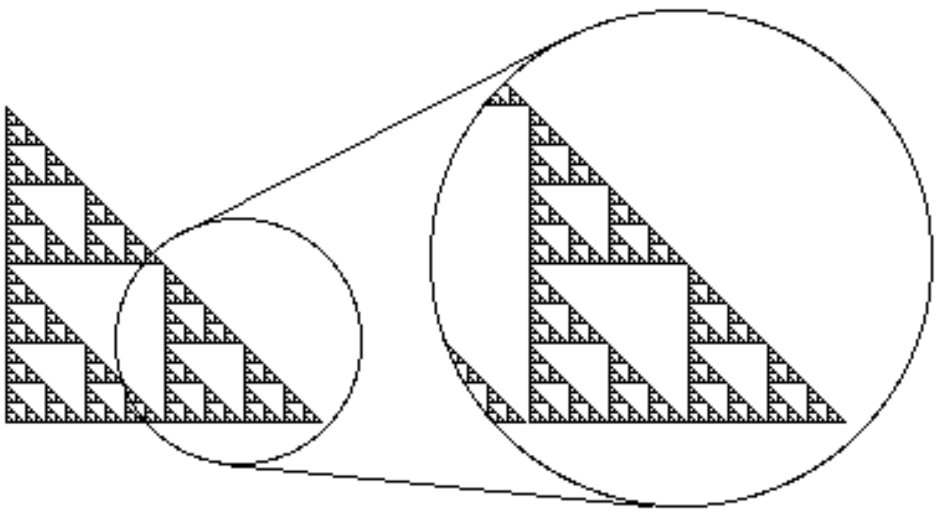
1. Introduction

1.4 Two kinds of fractal in phase space

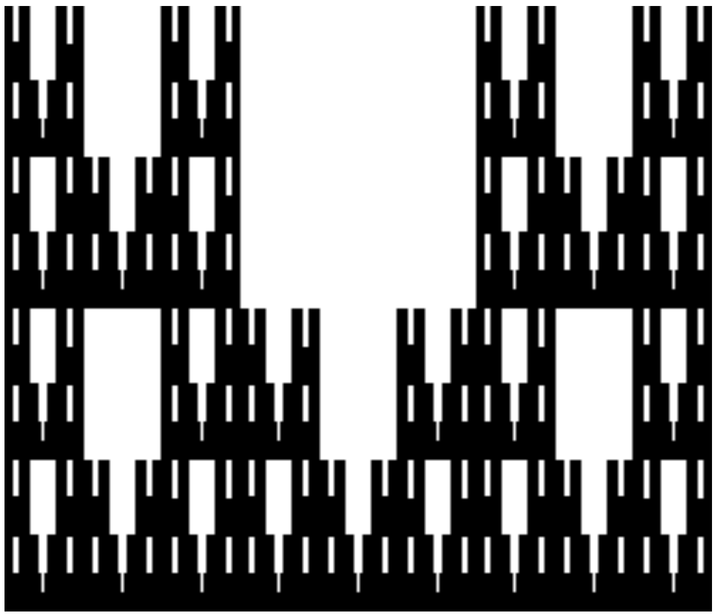
Criterion for existence of fractal

$$F_q(M) \propto M^{\phi_q}$$

The shrinking ratios of two directions ?



Self-similar Fractal

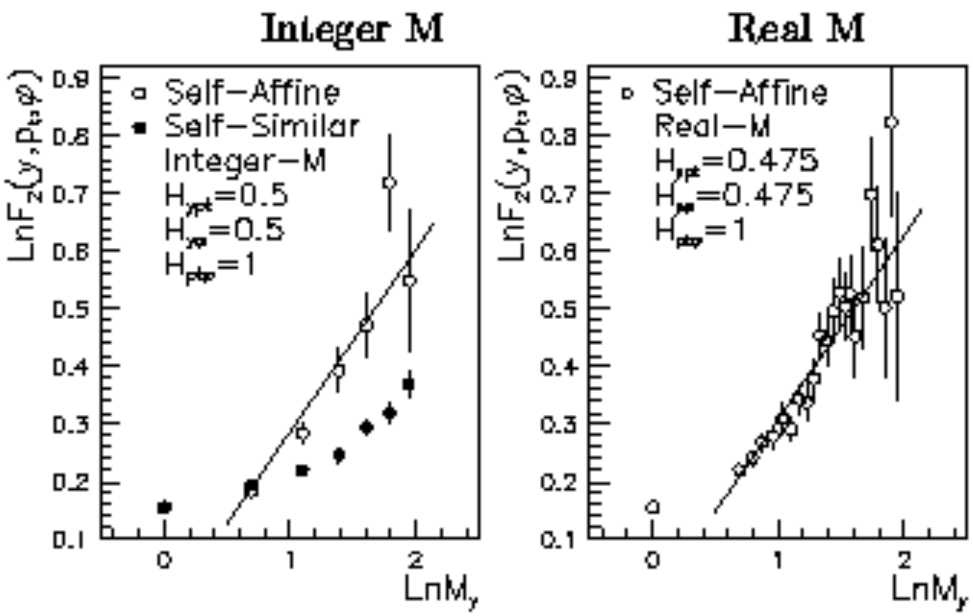


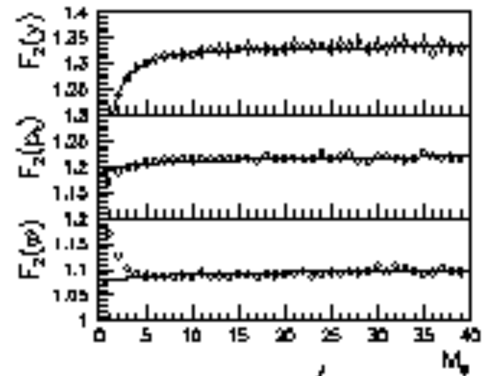
Self-affine Fractal

B.B. Mandelbrot, World Scientific, 1991, p. 11.

1. Introduction

1.5 Hadron-Hadron collision Self-affine Fractal





$$H_{xy} = \ln M_x / \ln M_y, \rightarrow M_\varphi = M_{p_t} = M_y^{1/H_{yp_t}}$$

Integer M: $M_y = 1, 2, ..., 7;\ M_\varphi = M_{p_t} = 1, 4, ..., 49.$

$$\gamma_{p_t} = \gamma_\varphi \neq \gamma_y$$

$$\ln F_2 = c + k \ln M_y$$

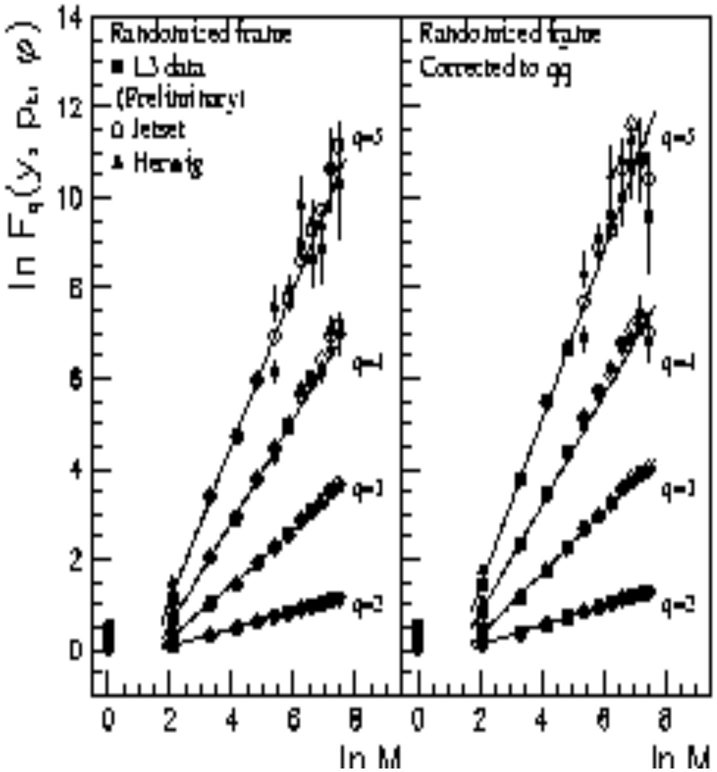
M	k	χ^2/NDF
integer	0.32 ± 0.03	7/4
real	0.32 ± 0.02	19/20

$$\varphi_2 = \frac{k}{\left(1 + \frac{1}{H_{yp_\perp}} + \frac{1}{H_{y\varphi}}\right)}$$

NA22, PLB 382 (1996) 305; B 431 (1998) 451.
Chen,Liu, IJMP A 14 (23) 3687(1999).

1. Introduction

1.6. e^+e^- collision at Z^0 region Self-similar Fractal



Fit using $F_q(M) = b_q M^{\phi_q}$

frame	q	ϕ_q	χ^2/D
Ran.	2	$0.194 \pm 0.003 \pm 0.003$	8/9
	3	$0.598 \pm 0.011 \pm 0.014$	11/9
	4	$1.082 \pm 0.013 \pm 0.018$	8/9
	5	$1.731 \pm 0.024 \pm 0.025$	6/9
to $q\bar{q}$ frame	2	$0.221 \pm 0.003 \pm 0.003$	8/9
	3	$0.685 \pm 0.011 \pm 0.012$	11/9
	4	$1.206 \pm 0.022 \pm 0.026$	7/9
	5	$1.858 \pm 0.028 \pm 0.035$	7/9

Chen,Hu,Liu,Kittel,Metzger(L3 Coll.), World Scientific, Singapore, 2002, p 361.
Chen,Liu,CPL19(9),1271(2002)
Chen,Hu,Kittel,Liu,Metzger,L3 Note, June 2002.

1. Introduction

Experiments has shown that:

The multiplicity multiplicity system in h-h collisions is self-affine fractal

The multiplicity system in e^+e^- collisions at Z^0 region is self-similar fractal

So:

Does fractal exist in heavy ion collisions?

Which kind of fractal?

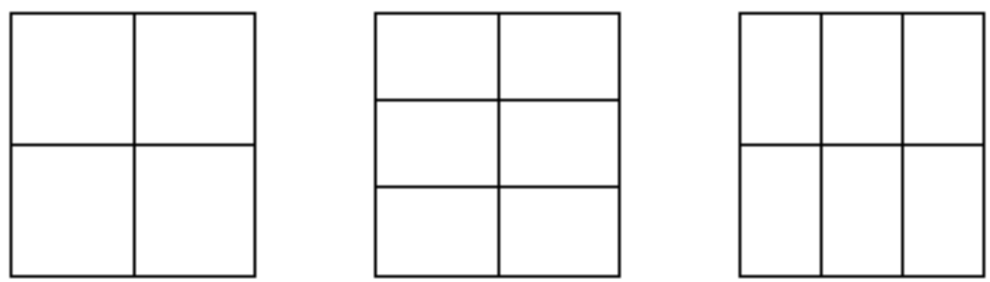
2. Analysis method

2.1 Isotropic or anisotropic analysis

- Strict power-law scaling exists in highly dimensional phase space.

It depends on how to partition phase space.

e.g., in 2-dimension (x,y) $\quad M = M_x M_y$



(Y.-F. Wu and L.-S. Liu, Phys. Rev. Letts. 70,3197, 1993)

(a) power-law scaling when $M_x = M_y$

$\Longleftrightarrow$ isotropic dynamical fluctuations

$\Longleftrightarrow$ self-similar fractal

(b,c) power-law scaling when $M_x \neq M_y$

$\Longleftrightarrow$ an-isotropic dynamical fluctuations

$\Longleftrightarrow$ self-affine fractal

Wu, Liu, PRL.70,3197,1993.

2. Analysis method

2.2 Criterion for fractal in phase space

Hurst exponent:

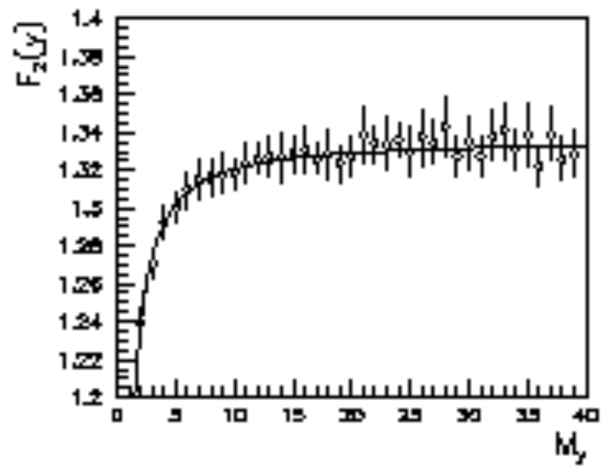
$$H_{xy} = \frac{\ln M_x}{\ln M_y}$$

If projected onto 1 dimension

$$F_2(M_i) = A_i + B_i M_i^{-\gamma_i} \qquad (i = x, y)$$

Then,

$$H_{xy} = \frac{1 + \gamma_y}{1 + \gamma_x}$$



Thus, $\text{isotropy} \Longleftrightarrow H_{xy} = 1 \Longleftrightarrow \gamma_x = \gamma_y$

$\text{anisotropy} \Longleftrightarrow H_{xy} \neq 1 \Longleftrightarrow \gamma_x \neq \gamma_y$

Hurst, Trans. Amer. Soc. Civil Eng. 116 (1951) 770.
Wu, Zhang, Liu, PRD 51 (1995) 6576.

2. Analysis method

2.3 Analysis step

(1) Fractal types of judgment

Calculated the 1-D NFM, fitting to the saturation index

$$F_2(M_i) = A_i + B_i M^{-\gamma_i} \quad (i = y, p_t, \varphi)$$

If $\gamma_i = \gamma_j, \quad (i, j = y, p_t, \varphi)$

Then, $H_{ab} = 1$, the system is the self-simlar fractal。

Otherwise, it is a **self-affine fractal**.

(2) NFM analysis of 2-D or 3-D for isotropic or anisotropic

3. Results of NFM Analysis

3.1 1-D NFM Analysis

1-D NFM tends to saturate,

Saturation exponent: $\gamma_y = \gamma_\varphi \approx \gamma_{p_t}$

Identical Hurst exponents:

$H_{yp_t} = 0.94 \pm 0.05, \quad H_{\varphi p_t} = 0.94 \pm 0.06, \quad H_{y\varphi} = 1.00 \pm 0.04$

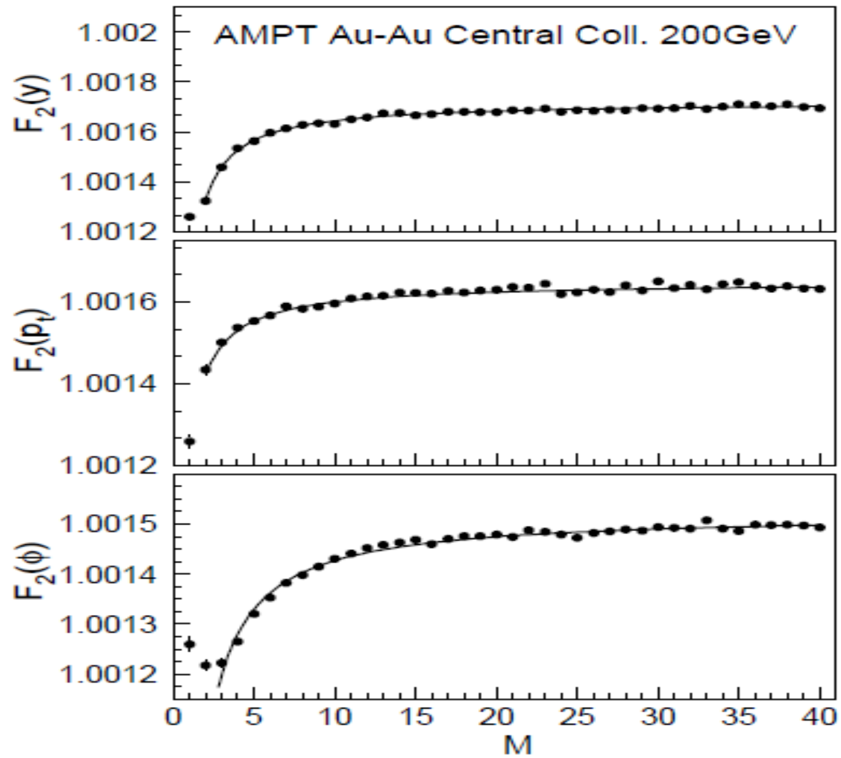


Fig.1. 1-D NFM: $F_2 \propto M$ (q=2).

Tab.1. Fitting parameter for 1-D NFM (q=2).

variable	A	B	γ	χ^2/DF
y	1.001720±0.000003	0.0008±0.00003	1.03±0.04	39/37
p_t	1.001655±0.000004	0.0004±0.00003	0.91±0.07	56/37
φ	1.001521±0.000004	0.0010±0.00006	1.03±0.05	51/36

Xie, Chen, NPA 920 (2013) 33

3. Results of NFM Analysis

3.2 2-D NFM Analysis

Fitting formula：

$$\ln F_q = c + \phi_q \ln M$$

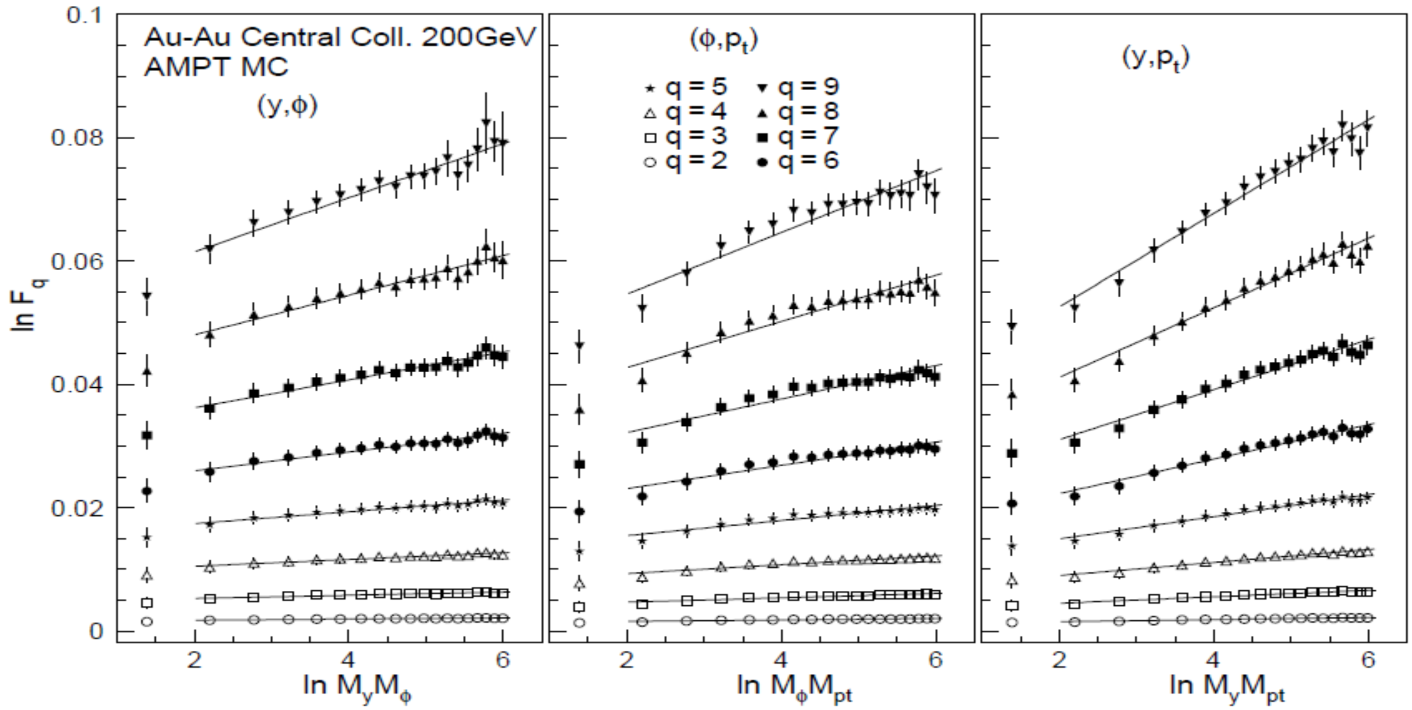


Fig.2. The logarithm distribution of 2-D NFM, i.e. $\ln F_q \propto \ln M \quad (q = 2-9)$

Show good Scaling → **Conclusion: Self-Similar Fractal**

3. Results of NFM Analysis

3.3 3-D NFM Analysis

Fitting formula：

$$\ln F_q = c + \phi_q \ln M$$

Tab.2. Fitting parameter for 3-D NFM

q	c	ϕ_q	χ^2/DF
2	0.0009±0.0004	0.0003±0.0001	0.2/8
3	0.0027±0.0006	0.0008±0.0001	0.7/8
4	0.0055±0.0009	0.0016±0.0002	0.8/8
5	0.0091±0.0012	0.0027±0.0002	0.6/8
6	0.0136±0.0014	0.0041±0.0003	2/8
7	0.0186±0.0019	0.0058±0.0004	4/8
8	0.0242±0.0023	0.0079±0.0005	8/8
9	0.0304±0.0028	0.0104±0.0006	10/8

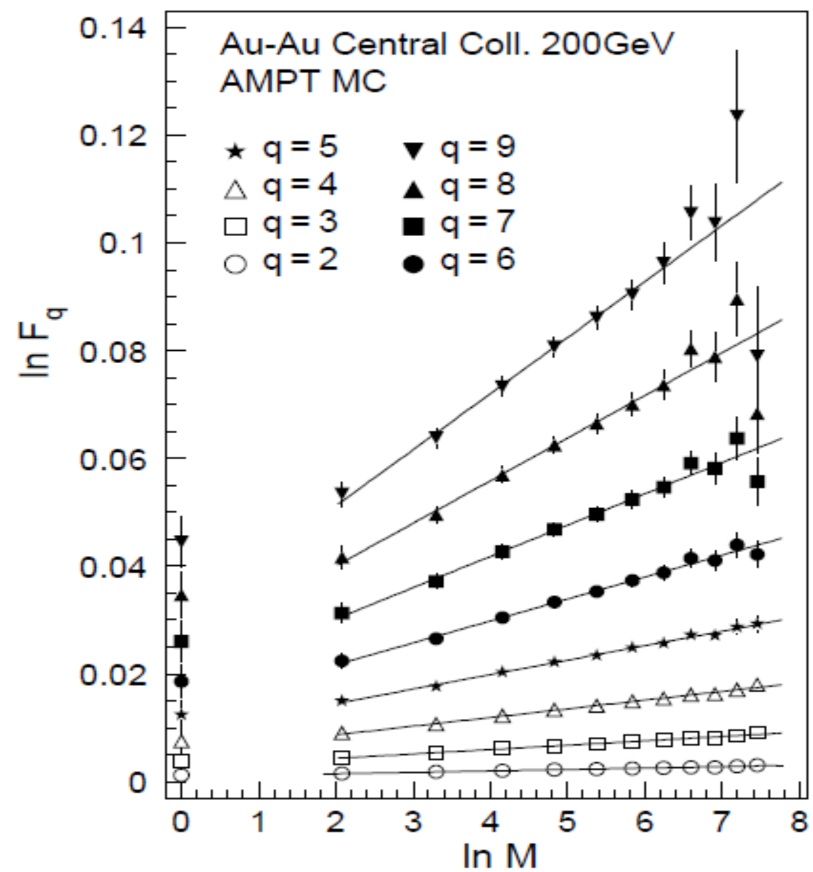


Fig.3. The logarithm distribution of 3-D NFM: $\ln F_q \propto \ln M$

Show good Scaling → **Conclusion: Self-Similar Fractal**

4. Discussion of FM's Scaling

4.1 Effect Fluctuation Strength

$$\alpha_{eff} = \sqrt{\frac{6\ln 2}{q}(1-D_q)} = \sqrt{\frac{6\ln 2}{q}\frac{\phi_q}{q-1}}$$

Tab.3. Effect Fluctuation Strength

q	α_{eff} Au-Au	$\pi^+(K^+)p$ [25, 27]	e^+e^- [17]
2	0.025±0.004	0.356±0.011	0.635±0.010
3	0.024±0.001	0.408±0.011	0.644±0.013
4	0.024±0.001	0.496±0.012	0.612±0.009
5	0.024±0.001	0.572±0.017	0.600±0.008
6	0.024±0.001	*	*
7	0.024±0.001	*	*
8	0.024±0.001	*	*
9	0.025±0.001	*	*

Fluctuation in heavy ion collision is larger than those in h-h collisions and e+e- collisions.

→ QGP

Liu,Fu,Wu,PLB444(1998)563..

NA22, PLB431(1998)451.

Chen (L3 Coll.), World Scientific, Singapore, 2002, p 361

4. Discussion of FM's Scaling

4.2 Ginzburg-Landau phase transition indices

$$\ln F_q = c + \beta_q \ln F_2 \qquad \beta_q = \phi_q/\phi_2 \propto (q-1)^{\nu} \qquad \nu_0 = 1.304$$

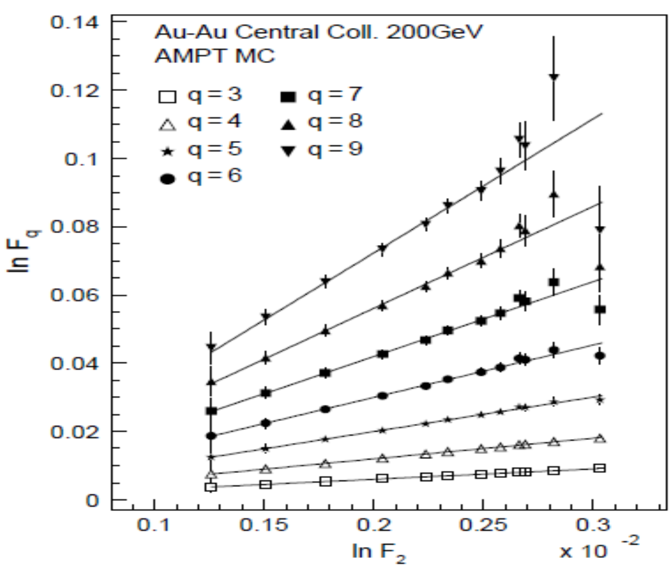


Fig.4. The distribution of 3-D NFM: $\ln F_q \propto \ln F_2$

Scaling property is checked again.

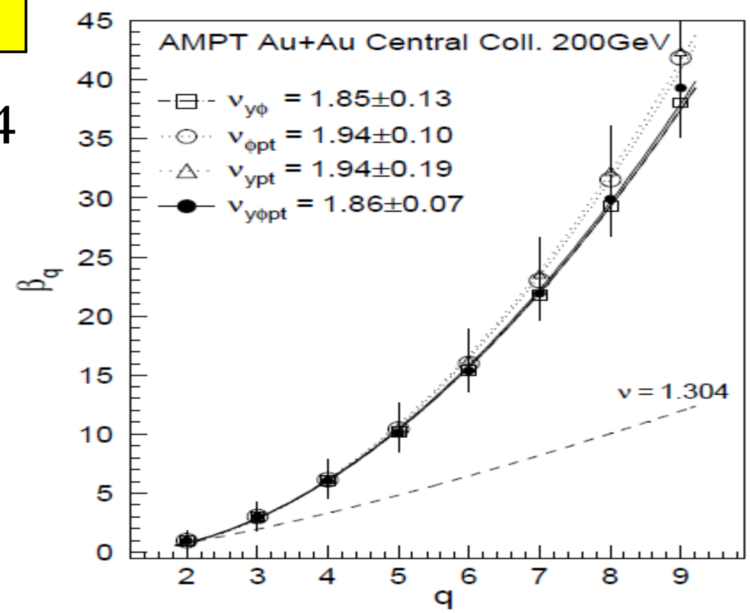


Fig.5. Parameter ν are obtained when fitting the relation

(1) $\nu_{2D} \approx \nu_{3D}$.

(2) $\nu_{y\varphi p_t} = 1.86 \pm 0.13 > 1.304$

Rudolph,PRL69(1992)1741.
Rudolph,PRD47(1993)2773.

4. Discussion of FM's Scaling

4.3 Transverse momentum dependence of fluctuation

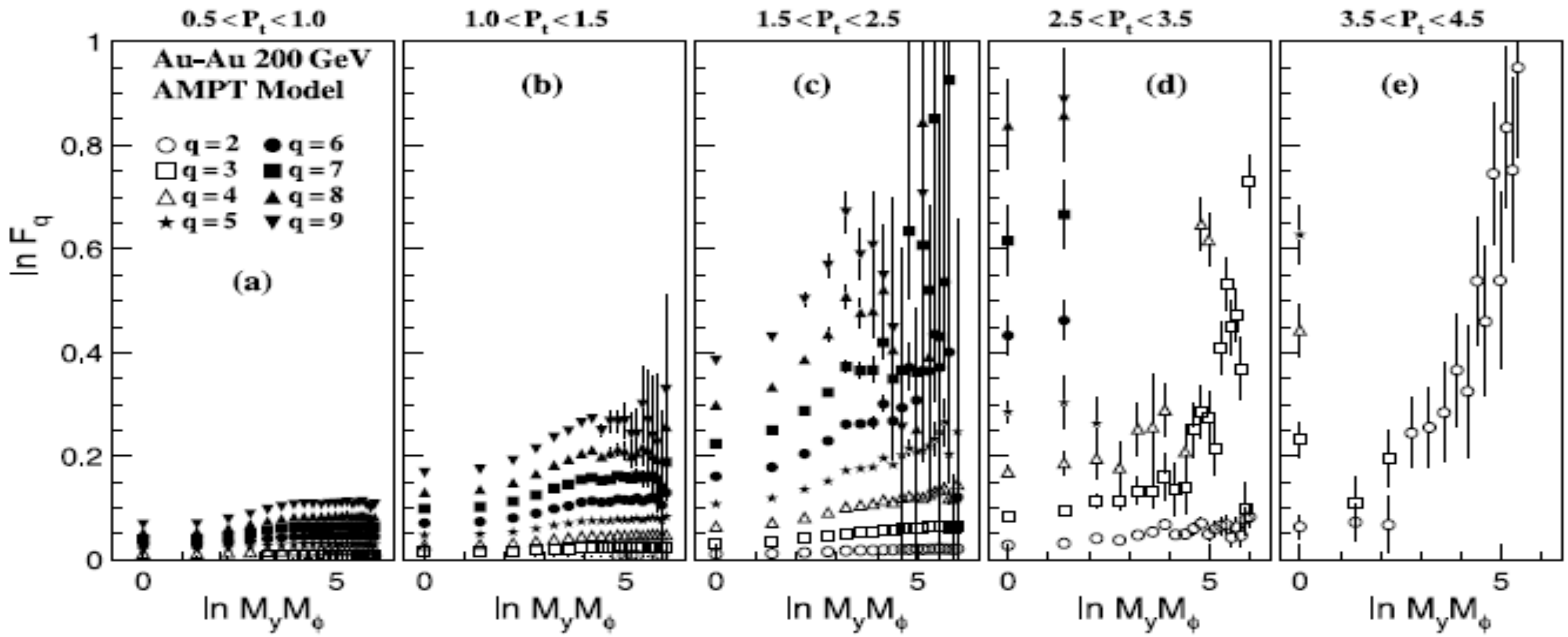


Fig.6. The loglog distribution of 2-D NFM F_q in the (y,φ) plane at various p_t intervals

(1) Expectation: $\nu_{p_t\in(0.5,1.0)} < \nu_{p_t\in(3.5,4.5)}$

(2) Indeed, Fluctuation ↑ with p_t ↑

4. Discussion of FM's Scaling

4.3 Transverse momentum dependence of fluctuation

2-D NFM on (y,φ) plan (M=1)

$$f_q(y,\varphi)=\frac{\langle n_m(n_m-1)\cdots(n-q+1)\rangle}{\langle n\rangle^q}$$

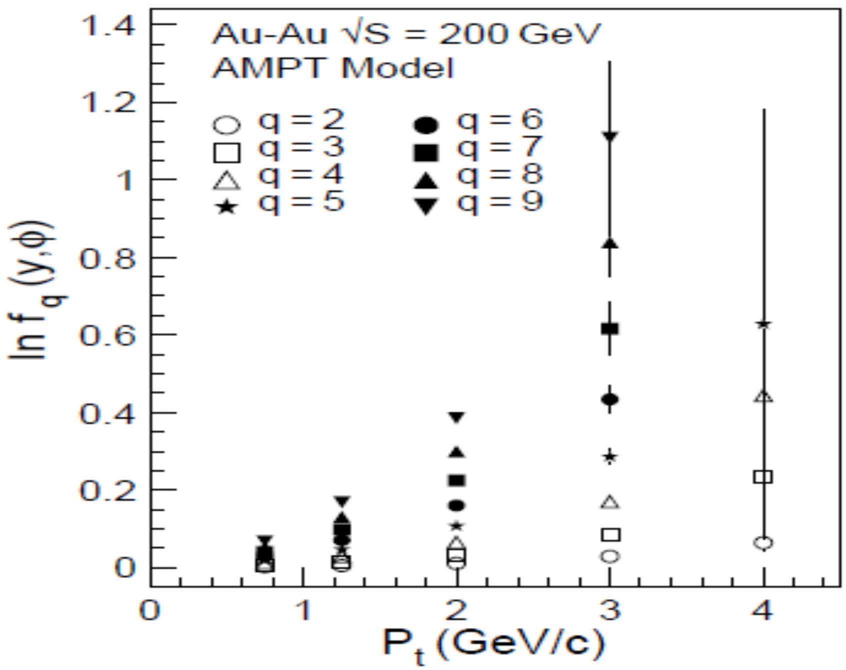


Fig.7. The distribution of 2-D NFM $\ln f_q(y,\varphi)$ as a function of p_t, with partition number M=1

$f_q(y,\varphi)$ ⇧ rapidly with p_t ⇧ .

⇩

Fluctuation in high p_t is actually much larger than in low p_t .

Summary

1. We study fractal properties of multiplicity in Au-Au collision at 200GeV by AMPT model, using NFM analysis method. It is shown that is the self-similar fractal.

2. The effect fluctuation strengths in heavy ion collisions is much smaller than those in h-h collisions and electron-positron collisions, which may be the signal of QGP.

3. When checking the scaling property again with the Ginzburg-Landau method, the parameter ν can be obtained. Our results $\nu_{y\varphi p_t} = 1.86 \pm 0.13$ is larger than v =1.304 derived in Ginzburg-Landau type of phase transition. It is shown that AMPT would do not include the phase transition.

4. We also explored the intermittency and fluctuation in dependence on the transverse momentum. The result shows that the factorial moment, as well as intermittency or fluctuations, increases rapidity with the increasing of transverse momentum pt.


Thanks for your attention!