## OMGT3223 Quiz \#1 Study Guide

The quiz will consist of problem based questions like those in HW\#1. There will be NO multiple choice problems. I will provide a formulae sheet \& extra paper if needed. Time for the quiz will be 1 hour \& 15 minutes. When finished students can exit the class.

For Quiz \#1 these are the areas you are responsible for:

1. Creating probability trees, interpreting probability trees
2. Creating probability tables, interpreting probability tables
3. Understanding \& using Bayes' theorem
4. Interpreting cross over graphs, speaking to the cross over point (aka the point of indifference), discussion risk of choosing one alternative over another in the context of the cross over graph, making a decision or recommending a course of action based on a cross over graph

Attached are some example problems with solutions. Please peruse \& attempt the problems by hand as preparation for the Quiz.

## Example 1

In a university, given a student is a graduate, then there is $60 \%$ chance that he/she is from Asia. And given a student is in undergraduate, then there is a $35 \%$ chance that he/she is from Asia. We also know that there $20 \%$ of students are in graduate school and $80 \%$ of students are in undergraduate school. What is the probability of that a student who is a graduate given he/she is from Asia?

The university's student population consists of graduate students (G) and undergraduate students (UG). Within this population, they could be Asian (A) or non-Asian (NA).

Assumptions:

1. The student population remains unchanged
2. There are no students enrolled in both undergraduate and graduate programs

We are given the following marginal probabilities:
$\mathrm{p}(\mathrm{G})=$ prob. student is a graduate student $=0.20$
$\mathrm{p}(\mathrm{UG})=$ prob. student is an undergraduate student $=0.80$
And the following conditional probabilities:
$\mathrm{p}(\mathrm{A} \mid \mathrm{G})=$ prob. student is Asian given he/she is a graduate $=0.60$
$\mathrm{p}(\mathrm{A} \mid \mathrm{UG})=$ prob. student is Asian given he/she is an undergraduate $=0.35$
We can infer the following:
$\mathrm{p}(\mathrm{NA} \mid \mathrm{G})=$ prob. student is non-Asian given he/she is a graduate $=1-0.60=0.4$
$\mathrm{p}(\mathrm{NA} \mid \mathrm{UG})=$ prob. student is non-Asian given he/she is an undergraduate $=1-0.35=0.65$
Using the general rule of multiplication, the joint probabilities are calculated as follows:

| Prob. of Graduate \& Asian: | $\mathrm{p}(\mathrm{G} \& A)=\mathrm{p}(\mathrm{A} \mid \mathrm{G}) \mathrm{p}(\mathrm{G})=(0.6)(0.2)=0.12$ |
| :--- | :--- |
| Prob. of Graduate \& non-Asian: | $\mathrm{p}(\mathrm{G} \& N A)=\mathrm{p}(\mathrm{NA} \mid \mathrm{G}) \mathrm{p}(\mathrm{G})=(0.4)(0.2)=0.08$ |
| Prob. of Undergraduate \& Asian: | $\mathrm{p}(\mathrm{UG} \& A)=\mathrm{p}(\mathrm{A} \mid \mathrm{UG}) \mathrm{p}(\mathrm{UG})=(0.35)(0.8)=0.28$ |
| Prob. of Undergraduate \& non-Asian: | $\mathrm{p}(\mathrm{UG} \& \mathrm{NA})=\mathrm{p}(\mathrm{NA} \mid \mathrm{UG}) \mathrm{p}(\mathrm{UG})=(0.65)(0.8)=0.52$ |

Thus a joint probability table can be created as follows in Table 1

Table 1 - Joint Probability Table

|  | Graduate | Undergraduate | Marginal <br> Probabilities |
| :--- | :---: | :---: | :---: |
| Asian | 0.12 | 0.28 | 0.40 |
| Non-Asian | 0.08 | 0.52 | 0.60 |
| Marginal Probabilities | 0.20 | 0.80 | 1.00 |

To find the probability the student is a graduate student given he/she is from Asia, Bayes' rule must be employed:

Given Bayes' rule:

$$
p(G \mid A)=\frac{p(G \& A)}{p(A)}
$$

Since we know from the joint probability table, $p(G$ and $A)=0.12$ and $p(A)=0.40$ :

$$
p(G \mid A)=\frac{0.12}{0.40}=0.30
$$

Therefore, the probability the student is a graduate student given he/she is from Asia is $30 \%$.

Figure 1 - Probability Tree


## Example 2

For the safety and health reasons, Japan Airliner Company is considering moving all flights to nonsmoking flights. However, the Board of Directors is going to take a vote and most of them are smokers. The Board of Directors consists of 40 nonsmokers and 60 of smokers. $70 \%$ of the nonsmokers and $35 \%$ of the smokers favor the nonsmoking policy.
a. Construct a probability tree for this problem.
b. Construct a probability table for this problem.
c. Determine the probability given that a Board of Director member opposes the nonsmoking policy that the Board of Director member is a nonsmoker.

Japan Airliner Company is about to conduct a vote amongst its Board of Directors. The Board of Directors consists of smokers (S) and nonsmokers (NS). This Board has to vote in favor (F) or not in favor (NF) for a nonsmoking policy on all its flights.

Assumptions:

1. Each Director can cast one vote.
2. Total number of members in the Board remains unchanged at 100 .

We are given the following marginal probabilities:
$p(S)=$ probability Director is a smoker $=60$ out of $100=0.60$
$p(N S)=$ probability Director is a nonsmoker $=40$ out of $100=0.40$
And the following conditional probabilities:
$\mathrm{P}(\mathrm{F} \mid \mathrm{S})=$ probability Director favors policy given he/she is a smoker $=0.35$
$\mathrm{P}(\mathrm{F} \mid \mathrm{NS})=$ probability Director favors policy given he/she is an nonsmoker $=0.70$
We can infer the following:
$\mathrm{P}(\mathrm{NF} \mid \mathrm{S})=$ probability Director is not in favor given he/she is a smoker $=1-0.35=0.65$
$\mathrm{P}(\mathrm{NF} \mid \mathrm{NS})=$ probability Director is not in favor given he/she is an nonsmoker $=1-0.7=0.30$

Figure 2 - Probability Tree


Thus a joint probability table can be created as follows in Table 2
Table 2 - Joint Probability Table

|  | Smoker | Nonsmoker | Marginal <br> Probabilities |
| :--- | :---: | :---: | :---: |
| Favor | 0.21 | 0.28 | 0.49 |
| Not in favor | 0.39 | 0.12 | 0.51 |
| Marginal Probabilities | 0.60 | 0.40 | 1.00 |

Determine the probability given that a Board of Director member opposes the nonsmoking policy that the Board of Director member is a nonsmoker.

Given Bayes' rule:

$$
p(N S \mid N F)=\frac{p(N S \& N F)}{p(N F)}
$$

Since we know from the joint probability table $p(N S \& N F)=0.12$ and $p(N F)=$ 0.51:

$$
p(N S \mid N F)=\frac{0.12}{0.51} \approx 23.53 \%
$$

7c) Therefore, the probability given that a Director opposes the nonsmoking policy that he/she is a nonsmoker is $23.53 \%$

## Example 3

One hundred college males play varsity football for the University of Free Shoes. Thirty of the players are from local high schools and 70 went to high out of state. Those players from Florida high schools have a $25 \%$ chance of being arrested and charged with a crime while in college and those from out of state have a $15 \%$ chance of being arrested and charged while in college.
a. Construct a probability tree for this problem.
b. Construct a probability table for this problem.
c. What is the probability that a randomly selected University of Free Shoes football player will arrested and charged while in college?
d. Given a player was arrested and charged while in college, what is the probability that they came from a local high school?

There are 100 college males playing varsity football for the University of Free Shoes (UFS). Out of the hundred players, 30 came from local high schools (L) and the remaining 70 from out of state (NL). There is some degree of probability that the players will get charged with a crime (C) and some will not be charged (NC) while in college.

Assumptions:

1. All 100 players will remain on the varsity team.
2. UFS is in the state of Florida.

Given the following:
$p(L)=$ prob. player is from a local high school $=30$ out of $100=0.3$
$\mathrm{p}(\mathrm{NL})=$ prob. player is from out of state $($ non-local $)=70$ out of $100=0.7$
And the following conditional probabilities:
$p(C \mid L)=$ prob. a local player will be charged with a crime $=0.25$
$p(C \mid N L)=$ prob. a non-local player will be charged with a crime $=0.15$
We can infer the following:
$\mathrm{p}(\mathrm{NC} \mid \mathrm{L})=$ prob. a local player will not be charged with a crime $=1-0.25=0.75$
$\mathrm{p}(\mathrm{NC} \mid \mathrm{NL})=$ prob. a non-local player will not be charged with a crime $=1-0.15=0.85$
Joint probabilities:
Prob. of local player \& criminal: $\quad p(L \& C)=p(C \mid L) p(L)=(0.25)(0.3)=0.075$

Prob. of local player \& non-criminal: $p(L \& N C)=p(N C \mid L) p(L)=(0.75)(0.3)=0.225$
Prob. of non-local player \& criminal: $p(N L \& C)=p(C \mid N L) p(N L)=(0.15)(0.7)=0.105$
Prob. of non-local \& non-criminal: $\quad p(N L \& N C)=p(N C \mid N L) p(N L)=(0.85)(0.7)=0.595$
Table 3 - Joint Probability Table

|  | Local player | Nonlocal <br> Player | Marginal <br> Probabilities |
| :--- | :---: | :---: | :---: |
| Criminal | 0.075 | 0.105 | 0.18 |
| Non-Criminal | 0.225 | 0.595 | 0.82 |
| Marginal Probabilities | 0.30 | 0.70 | 1.00 |

Figure 3 - Probability Tree


From the probability table, we see that the probability a football player is arrested and charged while in college is $p(C)=0.18$ and $18 \%$

Using given marginal probabilities and conditional probabilities we can use Bayes’ rule and the joint probability table to calculate the probability it is a player from a local high school given he was arrested and charged while in college $=p(L \mid C)$.

Given Bayes' rule:
Since we know from the joint probability table $p(L \& C)=0.075$ and $p(C)=0.18$ :
Therefore, the probability it is a player from a local high school given he was arrested and charged while in college is approximately $41.67 \%$

## Example 4

Given the following cost-volume break-even graph discuss the following:


Jimmy's boss wants to know which plant to buy from if they plan on using around 200 units on average. What plant would you recommend to Jimmy and his boss? Explain why you chose your answer.

It is easy to see from the graph that Plant I is the lower cost alternative at the 200 units produced mark.

Jimmy's boss now informs him the number of units they plan on using has changed and it could be as high as 455 or as low as 355 . What plant should Jimmy suggest the company buy from? Explain why you chose your answer.

From the graph that Plant I is the lower cost alternative around the 355 units produced mark but at the cross over point of 375 (approx.) Plant II becomes the less costly alternative. So, depending on how one thought they would be producing units, the could pick either plant. But it must be stated in that manner. For example, I believe we will not produce above the 375 cross over point \& therefore it is more economical for us to choose Plant I. However, I do understand we risk paying more with Plant I is we do surpass the cross over point.

