Critical exponents near a bifurcation point in noise-induced escape over a fluctuating barrier

Martin Bier
Department of Physics, East Carolina University, Greenville, North Carolina 27858, USA
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If a bistable system is brought closer to the bifurcation point at $A=A_c$, the rate of noise-activated escape to the other stable equilibrium increases as $\ln k \propto -|A-A_c|^{3/2}$. Recently, Dykman, Golding, and Ryvkine derived that upon the addition of an oscillating force, the exponent equals 2 in an intermediate-frequency regime [Phys. Rev. Lett. **92**, 080602 (2004); Phys. Rev. E **69**, 061102 (2004)]. In this paper we study the critical exponents near a bifurcation point for an analytically solvable case; that of a piecewise linear well with an imposed dichotomous fluctuation. It is shown how an exponent 2 emerges when a parameter is changed to move a system from nonadiabaticity to adiabaticity. It is, furthermore, shown that the power law breaks down for low barriers. Finally, it is explained why fundamental differences occur between smooth and piecewise linear setups.

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I. INTRODUCTION

Bistability is a frequently occurring phenomenon throughout nature. A huge body of literature has been devoted to the theoretical study of bistability and many authoritative textbooks carry a chapter on this topic [1,2].

Figure 1(a) shows an arbitrary, generic bifurcation curve. Along the vertical dotted line the energy profile looks like Fig. 1(b): x_{s_1} and x_{s_2} are the stable fixed points, and x_u is the unstable fixed point. In order for the system to cross over from x_{s_1} to x_{s_2}, an activation barrier of height E has to be mounted. If the dotted line is brought further to the right, the critical value A_c is approached. At $A=A_c$, the points x_{s_1} and x_u merge, and the system is no longer bistable.

When A comes closer to A_c, i.e., $A_c-A=\delta A\to 0$, the barrier can become sufficiently small for noise to be able to induce a barrier crossing. Particularly in a microscopically small system, the barrier height E may be only a few k_BT's high (where k_B is the Boltzmann constant and T is the absolute temperature; k_BT is then the average amount of energy in Brownian motion). The escape rate k over a barrier of height E follows the proportionality $k\propto \exp[-E/k_BT]$. In the remainder of this paper, I will take k_BT as the unit of energy, so $k\propto\exp[-E]$. We will work in the overdamped realm and the diffusion coefficient will be scaled to $D=1$. Because of Einstein's fluctuation-dissipation theorem, i.e., $\beta=k_BT/D$, this implies $\beta=1$ for the coefficient of friction.

A bistable system that is miniaturized to a nanometer scale is going to be more susceptible to k_BT-induced switchovers. This actually sets a soon-to-be-reached limit to the size to which computer hardware can be shrunk [3].

In many contexts in which bistability occurs, researchers have tried to derive and measure at what rate noise-induced escape from a stable state occurs. Already in the 1970s, and working in the context of Josephson junctions, Kurkijärvi derived that as $\delta A\to 0$, the activation energy goes to zero as $E\propto(\delta A)^{3/2}$ [4]. This means that the noise-induced escape rate k out of x_{s_1} follows $\ln k\propto -(\delta A)^{3/2}$. Below I will first show a short, but general, derivation of this critical exponent of $3/2$.

In a sufficiently small neighborhood around the bifurcation point $A=A_c$, the bifurcation curve approaches a parabola, i.e, there is a quadratic dependence $(x_{s_1}-x_u)^2\propto\delta A$ [cf. Fig. 1(a)]. We thus have $(x_{s_1}-x_u)\propto\sqrt{\delta A}$. Figure 2(a) shows how the quadratic situation around $A=A_c$ in the bifurcation diagram translates into a cubic energy profile [like $E(x)\approx -x^3+x^2$] in the neighborhood around x_{s_1} and x_u. The force due to this profile equals $F(x)=-(d/dx)E(x)$ and is again quadratic [Fig. 2(b)]. Obviously, the force driving a particle from x_u back to x_{s_1} is largest at the midpoint between x_u and x_{s_1}, i.e., at the bottom of the parabola in Fig. 2(b). For

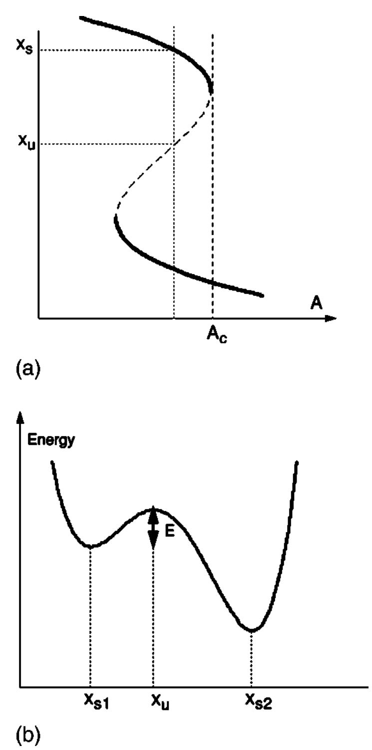

FIG. 1. (a) A generic bifurcation curve for a bistable system where the solid part of the curve indicates a stable attractor and the dashed part of the curve indicates the unstable repellor. At $A=A_c$ there is a bifurcation point. (b) The energy profile along the dotted vertical line of part (a).

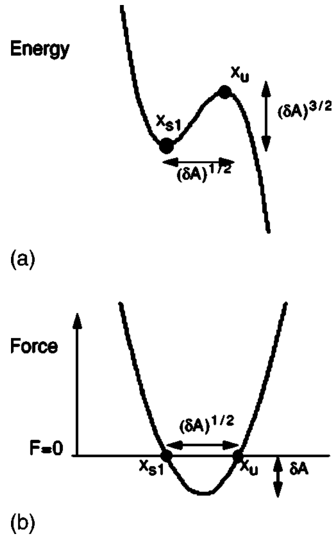

FIG. 2. (a) A part of the energy profile depicted in Fig. 1(b). If the bifurcation curve is quadratic in a neigborhood around $A=A_c$, where $\delta A=|A-A_c|$ is small, then the distance between x_{s_1} and x_u is proportional to $\sqrt{(\delta A)}$, the height of the activation barrier is proportional to $(\delta A)^{3/2}$, and (b) the maximal force driving a particle towards x_{s_1} scales like δA.

a quadratic profile, this maximum force is proportional to δA. The energy that has to be overcome for a barrier crossing can be derived as $E=-\int_{x_{s_1}}^{xu} F dx$. The area that is enclosed by a parabola $f(x)=bx^2-a$ (where $a>0$ and $b>0$) and the x axis runs from $x=-\sqrt{a/b}$ to $x=\sqrt{a/b}$ and equals $4a^{3/2}/(3b)$. Identifying a with δA, it is obvious that the activation energy is proportional to $(\delta A)^{3/2}$. We thus find for the escape rate $\ln k \propto -(\delta A)^{3/2}$.

II. THE MODULATED BARRIER

What if an oscillating or fluctuating force is added to the setup of Fig. 2? Such an addition leads to a varying height of the activation barrier. Dykman, Golding, and Ryvkine derived that the critical exponent equals 3/2 in the limits of high and low frequency. But, surprisingly, they found an intermediate regime where the exponent equals 2 [5,6]. The derivation of Dykman, Golding, and Ryvkine involves a mathematically intricate asymptotic analysis. Below I will present simpler arguments in the context of a related setup. A piecewise linear well (Fig. 3) with a Markovian dichotomous fluctuation allows for analytical derivation of the escape rate. The two cases, i.e., piecewise linear with imposed fluctuation (this paper), and smooth with imposed oscillation (Refs. [5,6]), exhibit a similar decrease of the escape rate when the imposed modulation is speeded up from the adiabatic regime to the nonadiabatic regime. This is not surprising: introducing corners in systems with diffusion is generally of little consequence as diffusion tends to "smooth out" corners. However, in later sections of this paper we will see that fundamental differences arise between smooth and piecewise linear when it comes to the behavior of their critical exponents.

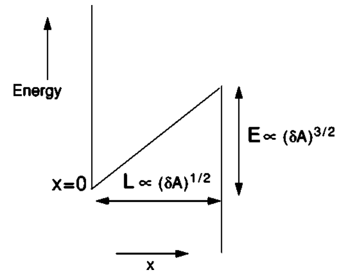

FIG. 3. A piecewise linear setup with one straight segment between a reflecting and an absorbing barrier. This profile mimics the differentiable energy profile in Fig. 2(a). For this piecewise linear profile, a Brownian particle's mean first passage time τ over the barrier can be analytically evaluated. Analytic evaluation is also possible when the barrier fluctuates between E_+ and E_-.

In the *low-frequency regime*, escape over the barrier occurs before there is any significant variation of the barrier profile. Let $\tau(E)$ be the mean first passage time over a stationary barrier of height E. The eventual mean first passage time τ over the modulated barrier is obtained by averaging the $\tau(E)$'s over all barrier heights. In case of an oscillation we thus have $\tau=(1/T)\int_{t=0}^{T}\tau(E)dt \propto \int_{t=0}^{T}\exp[E(t)]dt$, where T is the period of the oscillation. For a fluctuation between a finite number of barrier heights we get $\tau \propto \Sigma_i[\exp(E_i)]w_i$, where w_i represents the average fraction of time spent in the ith state. τ goes up exponentially with the barrier height E. So, generally, τ will be dominated by the highest values of E, i.e., $\tau \approx C\exp[E_{max}]$ with C being a constant. E_{max} scales like $(\delta A)^{3/2}$. $\ln\tau$ will therefore scale like $(\delta A)^{3/2}$. For the escape rate, we have $k=1/\tau$ (see Ref. [7] for a rigorous derivation of this). We thus eventually still get $\ln k \propto -(\delta A)^{3/2}$ in the low-frequency regime. This approximation applies when the period T of the oscillation or fluctuation is much larger than the escape time t_{esc} over the barrier at maximum height, i.e., $T \gg t_{esc}$.

When a potential well as in Fig. 2(a) changes shape, it takes time for a population of Brownian particles to redistribute and attain a new Boltzmann equilibrium. So there is an "adjustment" time t_{adj} for the escape rate when the energy profile changes. The *high-frequency regime* occurs when the imposed fluctuation or oscillation is so fast that the escape rate out of the well can never adjust to the shape of the potential at a particular instant. Instead, the population of Brownian particles will simply "feel" and adjust to the average potential. We get $\tau \propto \exp\langle E\rangle$ and thus $\ln k \propto -(\delta A)^{3/2}$ for the escape rate if $T \ll t_{adj}$.

The *intermediate-frequency regime* occurs when the period of the imposed oscillation or fluctuation is in between the two characteristic time scales t_{adj} and t_{esc}, i.e., $t_{adj}<T<t_{esc}$. In the low-frequency regime ($T>t_{esc}$) escape took place before any appreciable change in barrier height occurred. We, therefore, had to integrate the escape time over the distribution of barrier heights to obtain the average escape time. In the intermediate-frequency regime we face the reverse situation: the barrier height now changes much faster than the escape time t_{esc}. But the change of the barrier height is still slow enough for the system to remain adiabatic. So eventually we have a variable escape rate $k(E)$.

We obtain the average escape rate k over the modulated barrier by integrating $k(E)$ over the distribution of barrier heights: $k=(1/T)\int_{t=0}^{T}k(t)dt$. As $k\propto\exp[-E]$, we have $k\propto(1/T)\int_{t=0}^{T}\exp[-E(t)]dt$. Because of the minus sign, it is now the minimum barrier height that will dominate. We thus get $k\approx C\exp[-E_{min}]$ for the entire modulated system. It is in this intermediate-frequency regime that the escape rate is highest. Again we expect a critical exponent of 3/2. This result hinges on $k\propto\exp[-E]$ and thus requires that E_{min} is larger than a few kT.

III. THE PIECEWISE LINEAR CASE

Setups like the one in Fig. 2(a) can be well approximated by a piecewise linear setup as in Fig. 3. The piecewise linearity introduces corners. However, for problems involving diffusion, the introduction of corners does not generally lead to qualitatively different behavior. The setup of Fig. 3 allows for the analytical evaluation of the mean first passage time τ over the barrier for a particle that is in $x=0$ at $t=0$. The mean first passage time is [8–10]

$$\tau=\frac{e^{E}-1-E}{E^{2}}L^{2}. \tag{1}$$

A delta-function-like distribution at $x=0$ will rapidly evolve towards a Boltzmann equilibrium inside the well. It is only with such a stationary equilibrium distribution inside the well that there is a constant escape rate k. With an equilibrium distribution inside the well the escape rate can be derived to be [11,12]

$$k=\frac{E^{2}(1-e^{-E})}{e^{E}-2E-e^{-E}}L^{-2}. \tag{2}$$

This formula also shows how $\ln k\propto -E$ actually loses validity when E gets too close to unity (i.e., when E gets too close to the noiseband).

To study the escape over a modulated barrier, we let the barrier height fluctuate between $E_{+}=E_{0}+\Delta E$ and $E_{-}=E_{0}-\Delta E$. In each of these two states there is a constant rate γ, the flipping rate, at which transition to the other state occurs. So the average dwelling time in each state is $1/\gamma$. As the initial condition we again take $x=0$ at $t=0$, and an equal distribution over the two barrier heights, i.e., $P(E_{+})=P(E_{-})=1/2$ at $t=0$. The resulting system still allows for analytical evaluation of the mean first passage time τ [8–10]. Figure 4(a) shows $\log_{10}\tau$ as a function of $\log_{10}\gamma$. The four curves are for different values of the parameters E_0, ΔE, and L (see below). Each of the four curves exhibits an intermediate-frequency plateau (where $t_{adj}<1/\gamma<t_{esc}$) where the mean first passage time τ is minimal. In 1993, Doering and Gadoua discovered this plateau and they termed the enhanced escape "resonant activation" [9]. In the resonant activation regime, the Brownian particles in the well can actually "take advantage" of an oscillation or fluctuation. In the high- or low-frequency regime, escape is equally likely over the E_{+} and E_{-} barrier. But at resonant activation, the dwelling time is short enough so no escape occurs from the E_{+} well and long enough for adjustment to occur on the E_{-} profile and have an enhanced escape rate for some time.

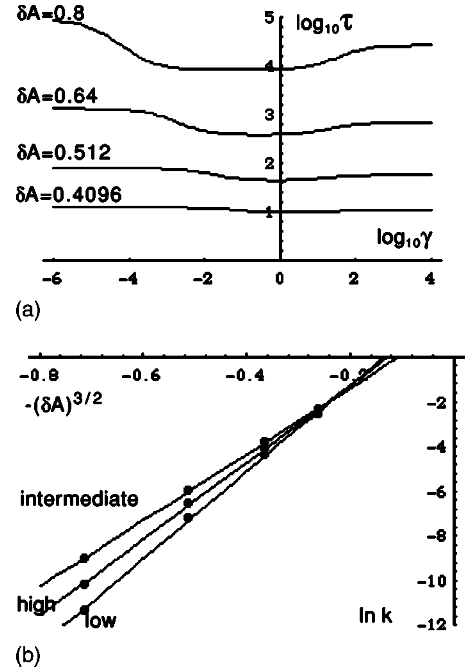

FIG. 4. (a) A log-log plot for the mean first passage time τ as a function of the rate of the imposed fluctuation γ. The amplitude of the barrier height fluctuation is small (15%) relative to the barrier height itself (cf. Fig. 3). The four curves correspond to different values of δA. Exact values are given in the text. δA indicates the distance to the bifurcation point. Each curve shows three distinct plateaus for low, intermediate, and high frequency. (b) The logarithm of the escape rate $\ln k(=-\ln\tau)$ appears to linearly depend on $(\delta A)^{3/2}$ for low, intermediate, and high frequency, thus affirming the 3/2 critical exponent derived in this paper.

Next we will study the transition from the nonadiabatic high-frequency regime to the adiabatic intermediate-frequency regime. On the average potential we have for the normalized probability distribution $\rho(x)=(E_0/L)\exp[-E_0x/L]$. Here a small term (order $\exp[-E_0]$) in the normalization factor has been neglected. For this Boltzmann distribution, the deterministic force towards $x=0$ is balanced out by a diffusive force towards $x=L$. When the barrier flips down to E_{-}, immediately after the flip (before adiabatic adjustment), there is a net force of $(E_0-E_{-})/L$ pushing the entire Boltzmann distribution towards $x=L$. Since the coefficient of friction β equals unity, the speed with which the distribution moves towards $x=L$ also equals $(E_0-E_{-})/L$. The average time in the E_{-} state equals $1/\gamma$. If $1/\gamma$ is short enough for the distribution not to change shape appreciably, the fraction of the distribution between $x=L-(E_0-E_{-})/(\gamma L)$ and $x=L$ will be "pushed over the edge," i.e., absorbed at $x=L$. We call this fraction $\Delta\rho$ and we find

$$\begin{aligned}\Delta\rho&=\frac{E_0}{L}\int_{x=L-(E_0-E_{-})/(\gamma L)}^{L}\exp\left[-E_0\frac{x}{L}\right]dx\\&=\exp\left[-E_0+E_0\left(\frac{E_0-E_{-}}{L^{2}}\right)\left(\frac{1}{\gamma}\right)\right]-\exp[-E_0].\end{aligned} \tag{3}$$

We neglect the latter term. The eventual escape rate k will be proportional to $\Delta\rho$. So with $E \propto (\delta A)^{3/2}$ and $L \propto (\delta A)^{1/2}$ we derive

$$\ln k \approx -\lambda_1(\delta A)^{3/2} + \lambda_2(\delta A)^2\gamma^{-1}. \qquad (4)$$

Here λ_1 and λ_2 represent positive proportionality constants. The method in the above paragraph becomes inaccurate for large $1/\gamma$. This is because the change of the shape of the distribution can no longer be neglected for larger dwelling times. Numerically Eq. (4) turns out to be a poor fit to the curves in Fig. 4(a). It follows the "drop-off" from the high-frequency regime to the intermediate-frequency regime only a very small part of the way.

However, Eq. (4) does have a few features that are worth pointing out. It is obvious that for high values of γ, the exponent 3/2 will dominate. As γ gets smaller and moves toward $1/t_{adj}$, we move toward the intermediate-frequency regime and the exponent 2 will become more significant. An exponent 2 thus plays a brief role in the regime around $\gamma \approx 1/t_{adj}$, where the transition between high and intermediate frequency takes place. Equation (4) predicts the right proportionality for t_{adj}. Assume that the transition between the regimes occurs at $\gamma = \gamma^*$, when the two terms in (4) have some fixed ratio. This leads to $\gamma^* \propto (\delta A)^{1/2}$, which implies that $t_{adj} \propto (\delta A)^{-1/2}$. In other words, for a smaller well, the adjustment time is larger; when δA is made smaller, the dynamics slows down. This somewhat counterintuitive result was also deduced by Dykman, Golding, and Ryvkine [5,6]. It has also been derived that t_{adj} equals the time it takes a particle to slide down deterministically from the barrier top to the bottom of the well [11,12]. In Fig. 3 it can be seen that for decreasing δA, the force driving the particle to the bottom ($\propto \delta A$) decreases faster than the width of well $[\propto (\delta A)^{1/2}]$. So the downslide time is indeed proportional to $(\delta A)^{-1/2}$.

It would be tempting but erroneous to identify the exponent 2 in Eq. (4) with the exponent 2 that Dykman, Golding, and Ryvkine derived as the dominant critical exponent in an intermediate-frequency regime. The $(\delta A)^2$ that Dykman, Golding, and Ryvkine derived does not carry the γ^{-1} frequency factor and, moreover, comes with a negative proportionality constant.

IV. COMPARISON WITH NUMERICAL RESULTS

The two inflection points in each of the curves in Fig. 4(a) mark the transitions from low to intermediate ($\gamma_{L\to I}$) and from intermediate to high ($\gamma_{I\to H}$) frequency. The left inflection points correspond to the aforementioned escape time, $\gamma_{L\to I} \approx 1/t_{esc}$. From $t_{esc} \propto \exp[E]$ and $E \propto (\delta A)^{3/2}$ we get $\log_{10}\gamma_{L\to I} \propto -\log_{10}(\delta A)$. The minus in the latter formula correctly predicts that the left inflection point moves further left for increasing values of δA. The inflection points on the right correspond to the escape rate's adjustment time, i.e., $\gamma_{I\to H} \approx 1/t_{adj}$. With $t_{adj} \propto (\delta A)^{-1/2}$ we derive $\log_{10}\gamma_{I\to H} \propto \log_{10}(\delta A)$. This formula implies that the right inflection point moves further right for increasing δA; a prediction that is again borne out by Fig. 4(a). All in all, we see that for $\delta A \to 0$, the mean first passage time τ approaches zero and the resonant activation regime narrows.

Figure 4(a) shows $\log_{10}\tau$ vs $\log_{10}\gamma$ for $E_0 = 20(\delta A)^{3/2}$, $\Delta E = 3(\delta A)^{3/2}$, and $L = 2(\delta A)^{1/2}$. From top to bottom, the four curves correspond to $\delta A = 0.8$, $\delta A = 0.64$, $\delta A = 0.512$, and $\delta A = 0.4096$. The curves in Fig. 4(a) were evaluated for an initial condition at $x = 0$ and the barrier having an equal probability to be in either E_+ or E_-, i.e., $P_+(t=0) = P_-(t=0) = 1/2$. Technically, a constant transition rate k only makes sense in case of a stationary Boltzmann distribution. We therefore took our parameter values such that the mean first passage time τ is large (>10); much larger than the time it takes to establish a Boltzmann distribution [12]. This means that $k = 1/\tau$, and therefore $\ln k = -\ln \tau$, is a good approximation. Figure 4(b) depicts $\ln k$ vs $(\delta A)^{3/2}$ for low-, intermediate-, and high-frequency plateaus. The data points are found to be on straight lines, thus affirming the above-derived value of 3/2 for the critical exponent in all of the three regimes.

Next we take $E_0 = \Delta E$, i.e., a fluctuation between zero height and a finite height. The above approach breaks down because $t_{adj} \to \infty$ when the barrier height is zero. A rough estimate of $\ln k$ in the intermediate-frequency regime can be obtained as follows. If $E_0 + \Delta E$ is sufficiently large, a Brownian particle will be close to the reflecting barrier after adiabatic adjustment when the barrier is up. When the barrier is flat, the average time to diffuse the distance L to the absorbing end of the interval equals $\frac{1}{2}L^2$. This corresponds to a rate $k \approx 2L^{-2}$ when the barrier is down. With $L \propto (\delta A)^{1/2}$, we obtain for the average k for the entire time-varying system: $\ln k \propto -\ln(\delta A)$. Figure 5(a) shows $\log_{10}\tau$ versus $\log_{10}\gamma$ for $E_0 = \Delta E = 12(\delta A)^{3/2}$ and $L = 3(\delta A)^{1/2}$. The four curves again correspond to $\delta A = 0.8$, $\delta A = 0.64$, $\delta A = 0.512$, and $\delta A = 0.4096$. In this case, the 3/2 power law is still expected to be valid for the low- and high-frequency plateaus. Figure 5(b) shows that this is indeed the case. For the intermediate-frequency regime, Fig. 5(c) shows that $\ln k$ no longer follows a power law, but is, instead, proportional to the predicted $-\ln(\delta A)$.

V. DISCUSSION

We studied a piecewise linear barrier with an imposed dichotomous fluctuation of the barrier height. There are three distinguishable plateaus when we plot the logarithm of the mean first passage time versus the logarithm of the imposed fluctuation rate. We have seen, theoretically and numerically, that the critical exponent equals 3/2 on each of these plateaus as long as the minimal barrier height is still high enough. Over the past decades, researchers have generally operated with the guiding assumption that there are no fundamental qualitative differences between a piecewise linear potential and an everywhere differentiable potential if the setup involves diffusion. Diffusion is supposed to smooth things out anyway. Also, the difference between a fluctuation (with instantaneous flips) and a smooth harmonic oscillation was supposed not to be of major consequences in the presence of diffusion. But in this paper we have identified a realm where piecewise linear and smooth profiles are very distinct. It appears that they exhibit very different behavior when it comes to their critical exponents.

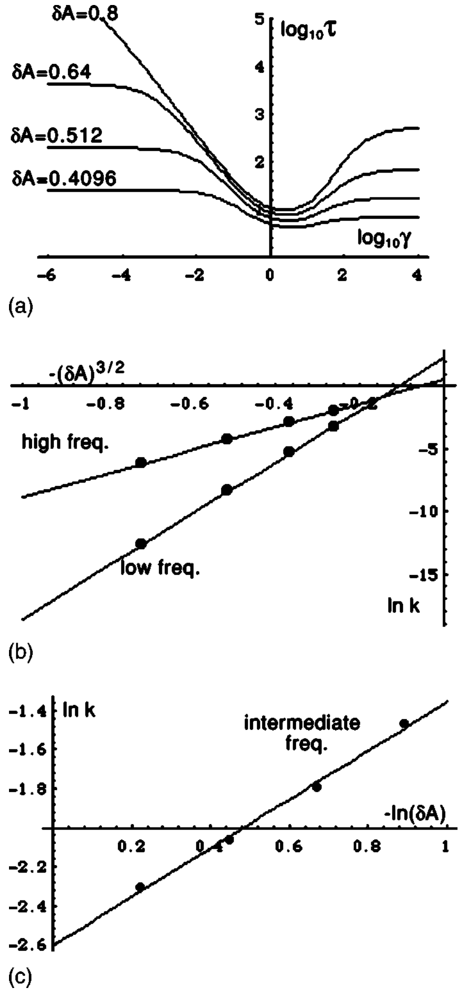

FIG. 5. (a) A log-log plot of the mean first passage time τ as a function of the rate of the imposed fluctuation γ when the linear barrier of Fig. 3 fluctuates between a finite height and zero. The four curves correspond to different values of δA. δA indicates the distance to the bifurcation point. Exact values for barrier height and width are given in the text. Again each curve shows three distinct plateaus for low, intermediate, and high frequency. (b) The logarithm of the escape rate $\ln k(=-\ln \tau)$ appears to linearly depend on $(\delta A)^{3/2}$ in the low- and high-frequency regimes. (c) In the intermediate-frequency, "resonant activation," regime, $\ln k$ increases linearly with $-\ln \delta A$.

In their asymptotic analysis, Dykman, Golding, and Ryvkine [5,6] follow the approach of the stable and the unstable point (i.e., x_{s_1} and x_u in Figs. 1 and 2) as the oscillation is in progress. Imagine a particle at the midpoint x_* between x_{s_1} and x_u in Fig. 2. Let F denote the force driving this particle back to x_{s_1} and let $t=t_*$ be the time during the oscillation at which x_{s_1} and x_u are closest. If the oscillation is harmonic, we can expand around the point of closest approach as follows:

$$F = F_0 + \alpha\vartheta^2 + \beta\varepsilon^2. \tag{5}$$

Here $\vartheta=t-t_*$, $\varepsilon=x-x_*$, α and β are positive constants, and F_0 is a term proportional to δA. Equation (5) is at the center of the analysis of Dykman, Golding, and Ryvkine, and the quadratic approach in ϑ and ε is crucial. There is no equivalent of Eq. (5) in our setup; fluctuation instead of oscillation gets rid of ϑ^2 and piecewise linearity does away with ε^2.

Our Eqs. (3) and (4) originate from a different line of reasoning. The "2" exponent appears in the formula, but, unlike Dykman, Golding, and Ryvkine, we do not find a significant area in parameter space where the "2" power dominates. Furthermore, the analysis of Dykman, Golding, and Ryvkine still applies when x_{s_1} and x_u are merged at the extremum of the oscillation (i.e., $F_0=0$) or even when the slope of the potential is negative for a brief moment (i.e., $F_0<0$). In the system that we investigated, we find that power laws break down when the minimum barrier height goes to zero. When the minimum barrier height is zero in our system, a simple approximation predicts that in the intermediate-frequency regime $\ln k$ does not follow a power law [i.e., $\propto -(\delta A)^{3/2}$] but, instead, exhibits a logarithmic dependence [i.e., $\propto -\ln(\delta A)$]. Numerical observation agrees with this approximation. The overall conclusion is that flips and corners change the way in which a system responds to parameter changes. The presence of diffusion does not take away from this fact. Taking a finite ΔE and $E_0=0$, we have also evaluated cases for our piecewise linear setup where the minimum barrier height is negative. The resulting graphs are not shown in this paper, but it appears that in such a situation, the intermediate-frequency regime exhibits neither a power law nor a logarithmic law. No simple approximate law can be observed or derived for this case.

The above theory describes setups where the barriers are sufficiently low to allow for noise-induced escape, but also sufficiently high above the noiseband to allow for the approximations we used. A great many protein-regulated reactions have activation barriers between 5 and 10 k_BT, which is precisely where the above theory applies. In such environments, oscillating or fluctuating electric fields or concentrations can modulate the barrier height. As biophysicists are dealing with smaller and smaller systems, critical exponents may soon become a part of their experimental reality. The critical exponents we derived could, furthermore, characterize the behavior of many bistable systems in the nanotechnological realm. Many circumstances are imaginable where δA corresponds to a parameter that can be controlled. The discussed escape corresponds to the system making a "switch." Noise-induced switches will become more common as technology pushes further into the molecular realm. Critical exponents are important in characterizing "switching behavior" and a good theoretical understanding of such exponents will be imperative.

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