SCALING RELATIONS OF SPEED, ACCELERATION, AND STRENGTH IN COLLEGIATE AMERICAN FOOTBALL PLAYERS

ABSTRACT. The different positions on an American football team have different requirements in terms of speed, acceleration, and strength. Coaches have to find the right compromise between how heavy a player needs to be (since mass is a factor in bringing down the opponent), how fast that player can accelerate, and how fast he can run the 40-yard dash. Our results are to offer a systematic and quantitative approach towards this compromise. We recorded the shuttle run times and the 40-yard (37 m) dash times of more than 80 collegiate football players. We also recorded their weights and the weights that they could power clean. The power laws that result from allometric scaling lead to simple formulae that connect the involved variables and account well for the observed performances. A new and simple mathematical framework is used to process and present the data. Our results can aid coaches in interpreting the athlete's performance measures in common tests and determining the best compromise between desired position-specific performance and body mass.

Key words: American football, scaling, strength, speed, acceleration, control analysis

1. INTRODUCTION

There are large body size differences between players on American football teams, more so than on soccer, rugby, or basketball teams [1]. The different positions have different requirements in terms of running speed, agility, and brute force. Players are commonly divided into three groups [2]. The (i) *lineman*'s most important task is to push against a lineman of the opposing team. A lineman usually weighs more than 110 kg. The quarterbacks and running backs are (ii) *combo* players. They should be able to maneuver fast as well as be sufficiently sized to handle a physical confrontation. The combo player's

Matthew P. Milner
Department of Mechanical Science and Engineering, University of Illinois Urbana-Champaign, Urbana, USA
Robert C. Hickner
Department of Nutrition, Food, and Exercise Sciences, Institute of Sports Sciences and Medicine, College of Human Sciences, Florida State University, Tallahassee, USA
Department of Biokinetics, Exercise, and Leisure Sciences, College of Health Sciences, University of KwaZulu-Natal, Westville, S. Africa
Department of Kinesiology, College of Health & Human Performance, East Carolina University, Greenville, USA
Martin Bier
Department of Physics, East Carolina University, Greenville, USA
Faculty of Mechanical Engineering and Institute of Mathematics and Physics, UTP University of Technology and Life Sciences, Bydgoszcz, Poland

weight typically falls into the intermediate category, i.e. 90 to 110 kg. The (iii) *skill* positions require the ability to accelerate, to sprint, and to maneuver quickly. Wide receivers and defensive backs, whose weights are generally below 90 kg, fall into this category. Just the fact that the different aptitudes and player positions go hand-in-hand with different body masses already suggests the veracity of allometric scaling. In several publications it has indeed been reported that scaling relationships can quantitatively account for the output of American football players in weightlifting exercises [3].

In this article we will examine the extent to which allometric scaling can account for players' output in the power clean, the shuttle run, and the 40-yard (37 m) dash. We will also present methods to quantify and interpret the predictions of allometric scaling. Good running speed over distances up to 10 m is essential for all positions in American football. A player's ability on such a short sprint is commonly tested in a so-called shuttle run. For players in certain positions it is also important to be able to run fast over distances of up to 40 meters or more. There appears to be a correlation between an athlete's playing level and his performance in power clean, shuttle run, and 40-yard dash [4]. However, it can be difficult to formulate expectations and goals for players based on the data acquired in standard performance tests. Apart from providing insight and understanding, our results could help in providing guidelines and setting targets.

2. METHODS

2.1. Experimental Approach to the Problem

Power clean data, times on the 40-yard (37 m) sprint, and times on the 20-yard (18 m) shuttle run were recorded. This study was approved by the East Carolina University Institutional Review Board (UMCIRB 13-000411).

2.2. Subjects

Eighty-four Division I collegiate football players were tested in the power clean, the 37 m sprint, and the 18 m shuttle run. These exercises and the recording of the results are part of the regular training regimen for the East Carolina University football team. The subjects' mean mass was 107.3±21.5 kg.

2.3. Procedures

2.3.1. Athlete Training Program

The testing of all athletes took place during the final two weeks of the two-month training cycle as a part of the team's normal training and testing procedure. Over the entire training period athletes trained five days per week with an

even split between pushing and pulling strength-training exercises, for both upper and lower body muscle groups. Each day consisted of lifting weights in addition to agility and speed drills. Athletes were weighed during each of the last two weeks of the period. The final weight used in our calculations was the average of those two values.

2.3.2. Power Clean Strength Measurement

In the "power clean" the barbell is brought in one smooth motion from the ground to the shoulders. The lifter is to keep his feet side-by-side. This lift is similar to a sprint in that it is an explosive and coordinated effort involving hips, knees, and upper body. Prior to the assessment, each athlete was instructed to do 5 jump shrugs and 3 full squat cleans. To warm up for the maximal effort attempt, they next followed a procedure where they power cleaned increasing percentages of their estimated maximal lift. The athletes were allowed to use both a belt and wrist straps, if they desired. No safety spotters were used. The athletes were allowed to increase their weight based on what they felt they could achieve. Testing continued until the athlete achieved his maximum. We took the maximal weight that an athlete could power clean and then the result was divided by the athlete's body mass.

2.3.3. Running Measurements

To prepare for both the 40-yard sprint and the 20-yard shuttle run, there was a dynamic warm-up followed by some submaximal efforts at both the sprint and shuttle run. In the 40-yard sprint, the athlete was instructed to get into a 3-point stance; the feet were approximately a shoulder-width apart and the hand was placed wherever the athlete felt most comfortable. Timing was done by hand using stopwatches. Two independent timers were used for each trial and were instructed to start the clock at the first sign of movement; the clock was stopped when the sprinter’s hips crossed the 40-yard mark. The average time of the two timers was used as the final time for each trial, with each athlete being allowed three trials.

The shuttle run was initiated from a three-point stance facing perpendicular to the direction of initial travel. Each person being tested was allowed to start in either direction. The test consisted of a five-yard sprint in one direction, a 180-degree reversal of direction and a sprint of ten yards, and then again a turn of 180 degrees and a five-yard sprint back to the start-finish line. Coaches monitored each end of the shuttle run to ensure that each athlete touched the line as direction was changed. If the line was not touched with the hand, the trial did not count. The timer was started at the first movement and stopped when the runner’s hips crossed the finish line. Each trial was hand timed with a stop watch by one coach and athletes were given three trials.

2.4. Control Analysis

Control Analysis [5, 6] provides a good framework to interpret and discuss our results. Four decades ago this approach proved fruitful in the analysis of metabolic pathways. Let x be a variable that we change by a small amount δx. The relative change of x then equals $\delta x/x$. Now suppose we have a function $y = f(x)$. Near $x = x_0$, we have $\delta y = f'(x_0)\,\delta x$, where $f'(x_0)$ denotes the derivative of $f(x)$ at $x = x_0$. It is easy to derive that the relative changes of y and x are related through:

$$\frac{\delta y}{y} \approx \frac{f'(x_0)x_0}{f(x_0)}\frac{\delta x}{x} \tag{1}$$

This is the central formula in Control Analysis. A good way to think about this is as follows: if x changes by 1% (i.e. $\delta x/x \approx 0.01$), then the corresponding change of y amounts to $f'(x_0)x_0/f(x_0)$ percent.

3. RESULTS

3.1. Allometric Scaling Theory

The idea behind allometric scaling is that a larger athlete is simply a scaled up version of a smaller athlete. This assumption leads to straightforward predictions for the output of an athlete as a function of his or her size. Because the athletes on a football team are all similarly well trained, the scaling assumption may be more valid among them than among arbitrary members of the general population.

We let L be a measure of any *length* on the athlete's body. Muscle strength can be assumed to be proportional to the cross sectional area, A, of the involved muscle. For any *area* A we have $A \propto L^2$. The symbol $\propto$ stands for "proportional to," i.e. $A \propto L^2$ means $A = C \times L^2$, where C is any constant. For *volumes* we have $V \propto L^3$. So a person that is 10% taller will lift 20% more (as $1.1^2 \approx 1.2$) and, since mass is proportional to volume, weigh 30% more ($1.1^3 \approx 1.3$). The theory thus predicts that the weight W that a weightlifter can lift will increase with his mass M as $W \propto L^2 \propto (M^{1/3})^2 = M^{2/3}$. It is easy to check that world records for different weight classes indeed follow this simple power law with remarkable accuracy [7].

In a shuttle run the athlete accelerates for the entire duration of the run (see Fig. 1). If we start from standstill and assume the acceleration constant, then the acceleration a, the time t, and the covered distance d, are connected through the kinematic relation $d = \frac{1}{2}at^2$. As d is fixed, we find $t \propto a^{-1/2}$. The acceleration that an athlete generates can be estimated with Newton's Second Law, $F = Ma$. Here F is the generated force. With $M \propto L^3$ and $F \propto L^2$, we find $a \propto L^{-1}$.

How an athlete's maximal sprinting speed scales with size is a little more complicated. Several considerations appear relevant. When running close to 10 m/s, a significant part of the generated force and energy goes into overcoming air resistance. The air resistance is proportional to the sprinter's frontal surface area, which scales like L^2. As muscle strength also scales with L^2, no net scale dependence for the speed v ensues. Part of the sprinter's effort also goes into repeatedly lifting the foot from the ground (where it is standing still) and next accelerating it forward to the running speed v. So at every step a kinetic energy $\frac{1}{2}mv^2$ has to be generated, where m is the mass of the foot. The foot is accelerated through a force F ($\propto L^2$) that operates over a step length s ($\propto L$). The resulting product, $F \times s$, is the work that leads to kinetic energy $\frac{1}{2}mv^2$. With $Fs = \frac{1}{2}mv^2$, we have in terms of scale $L^2 L \propto L^3 v^2$, i.e. $v \propto L^0$. So we again find a top speed that is scale independent. Results in competitive sprinting appear to bear out this scale independence. The sizes of Olympic sprinters vary widely: Usain Bolt (personal best on 100 m: 9.58) is 1.95 m tall, while Olusoji Fasuba (personal best on 100 m: 9.85) is 1.65 m tall. Also in the animal world it is found that there is little variation in top speed as animal weights vary over 5 orders of magnitude [8].

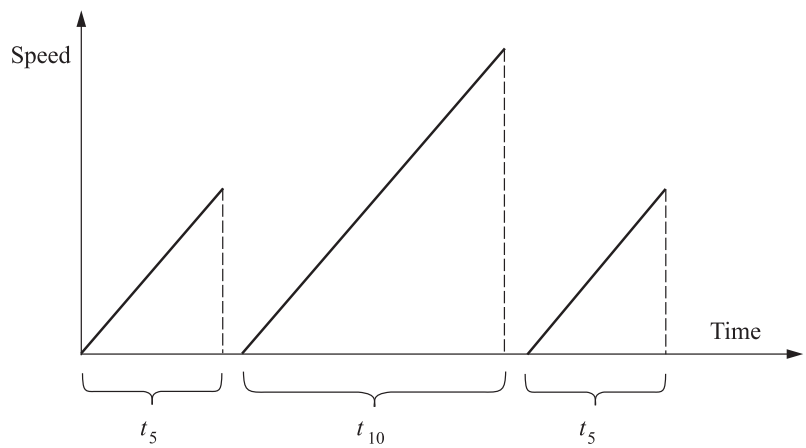

Fig. 1. In the shuttle run the athlete runs 5 yards (4.6 m) to the left, 10 yards to the right, and then again 5 yards to the left. The figure depicts how we approximate the speed profile. The three segments each start from standstill and are run with constant acceleration. This assumption allows us to derive an acceleration value from the time in which the run is completed (see text)

Fig. 1 shows our simple approximation for the speed profile of the shuttle run. The first 5-yard segment is covered in a time t_5. The 10-yard segment is covered in t_{10}. The second 5-yard segment is again covered in t_5. We assume a constant acceleration at each segment and we approximate the stopping and turning as instantaneous. The acceleration, a, corresponds to the slope of the

line in each segment. The distance covered on each segment can be identified with the area under the corresponding triangle. We thus have $5 = ½\, a\, t_5^2$ and $10 = ½\, a\, t_{10}^2$. For the total time in which the shuttle run is covered we have $t_{tot} = 2t_5 + t_{10}$. Next, in terms of the acceleration *a*, this is easily derived to be $t_{tot} = (2/\sqrt{a})(\sqrt{10} + \sqrt{5})$. This formula gives us a straightforward expression for the athlete's acceleration, *a*, in terms of the total shuttle run time t_{tot}: $a = 4\left((\sqrt{10} + \sqrt{5})/t_{tot} \right)^2$.

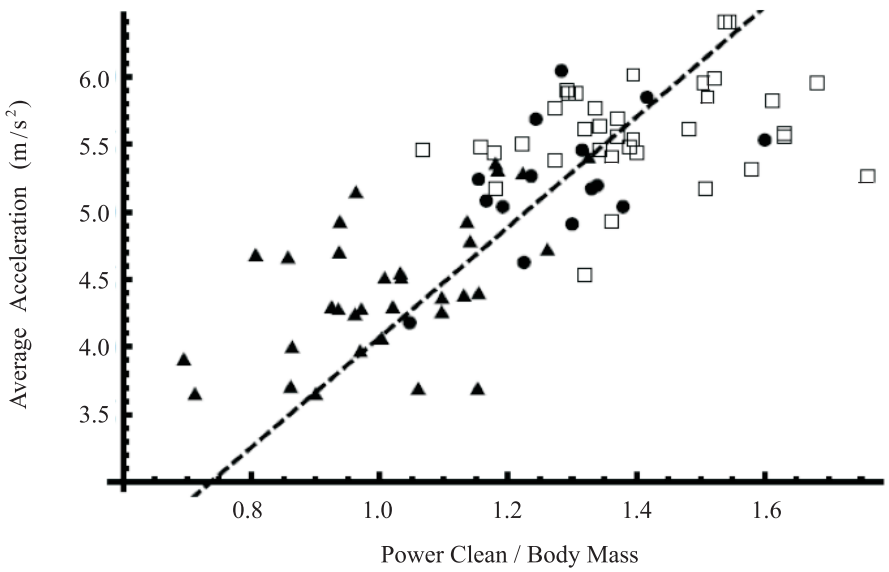

Fig. 2. A scatter chart where the horizontal coordinate represents, for each player, the ratio of the power cleaned weight and the body mass. The vertical coordinate represents the acceleration in the shuttle run. These quantities are expected to scale identically with body size. Triangles indicate linemen, circles are combo players, and squares are skill players. We indeed see athletes get smaller and lighter as we move from the left bottom to the right top of the data set. The theoretically predicted straight line through the origin appears to cover the data well. The correlation coefficient of the data is $R = 0.76$

Fig. 2 is a scatter chart where each symbol in the graph stands for one particular athlete. For each athlete the horizontal coordinate represents the ratio *W/M* of the weight *W* that he can "power clean" and his body mass *M*. The vertical coordinate gives the acceleration in the shuttle run. Both *W/M* and the acceleration *a* scale like L^{-1}, and a straight line through the origin is predicted to fit the points. This is indeed what Fig. 2 shows. The points corresponding to the larger athletes are concentrated at the left bottom. The points corresponding to the smaller lighter athletes are found near the right top.

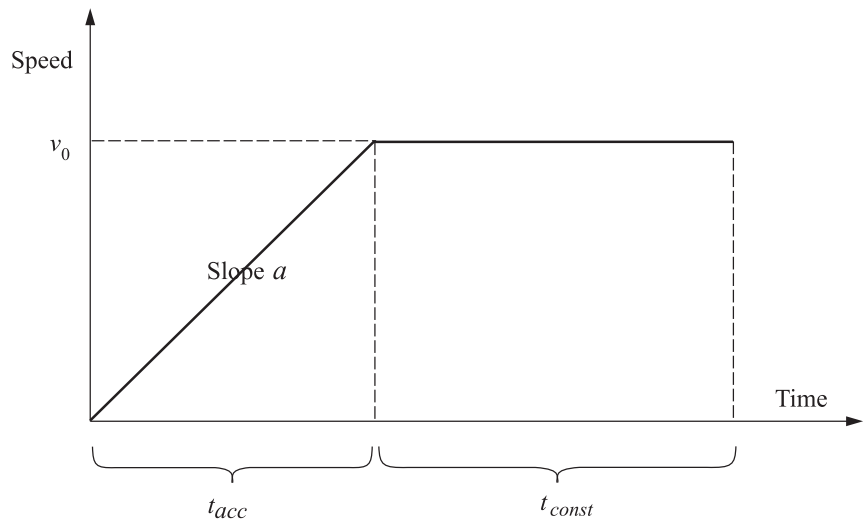

Fig. 3. We model the s_{tot} = 40 yards (37 m) dash as a constant acceleration, "a," until the top speed, v_0, is reached. After that, acceleration is zero and v_0 is maintained. With this assumption, the total time, t_{tot}, and the average speed, v_{avg}, for the entire run can be readily expressed in terms of a and v_0. In the text we derive: $t_{tot} = s_{tot}/v_0 + ½\, v_0/a$

The sprint over s_{tot} = 40 yards (= 37 m) is part acceleration ($a \propto L^{-1}$) and part running at a constant speed that does not scale ($v_0 \propto L^{\circ}$). For the world's top 100 m sprinters, the evolution of the speed as the race progresses has been measured, plotted, and analyzed [9, 10]. The actual speed profile does, of course, not contain a corner (after which acceleration stops and constant speed is maintained) as in Fig. 3. But our profile, as depicted in Fig. 3, is a very close approximation and gives rise to very simple formulae. An average football player runs the 40-yard sprint in about 5 s. At a top speed of $v_0 \approx 10$ m/s and an acceleration of $a \approx 4$ m/s^2, the acceleration phase of the run lasts about $t_{acc} = v_0/a \approx 2.5$ s and extends over $s_{acc} = ½\, at_{acc}^2 = ½\, v_0^2/a \approx 13$ m. The remaining 2.5 s and 24 m is covered at the constant speed, v_0. Next we let s_{tot} be the full 40 yards (37 m) and we let s_{const} be the distance within that 40 yards that is run at constant speed. So we have $s_{tot} = s_{acc} + s_{const}$. Obviously, we also have $t_{tot} = t_{acc} + t_{const}$ for the times to cover these distances. In terms of the formulae we have $s_{const} = s_{tot}$-$½\, v_0^2/a$ and $t_{const} = (s_{tot}$-$½\, v_0^2/a)/v_0 = s_{tot}/v_0$ -$½\, v_0/a$. This leads to a total time, t_{tot}, to cover the total distance, s_{tot}: $t_{tot} = t_{acc} + t_{const} = s_{tot}/v_0 + ½\, v_0/a$. Here the first term scales like L^0 and the second term scales like L^1. So we can express the total time as $t_{tot} = p + qL$, where p and q are constants. With the indicated numerical values of speed and acceleration, the qL^1 term is found to make up about 20% of the total time, t_{tot}. In Fig. 4 the y-axis gives the average

speed over the entire 40 yard run, i.e. $v_{avg} = s_{tot}/t_{tot}$. We thus come to the following expected scale dependence for the average speed: $v_{avg} = 1/(p + qL)$.

In Fig. 4 the quantity represented on the *x*-axis is the same as in Fig. 2 and scales like $x = L^{-1}$. The theory predicts that the points follow a curve that goes like $y = 1/(p + q/x) = Ax/(B + x)$, where *A* and *B* are again constants. The function $y = Ax/(B + x)$ is the well-known Michaelis-Menten curve that is discussed in any textbook on Biochemistry [11]. For small *x*, there is a linear increase of *y* with *x*. But then the curve flattens and as $x \to \infty$ a horizontal asymptote at the saturation level $y = A$ is approached. For $x = B$ the value for *y* is at half the saturation level. In our case the saturation level would be represented by an infinite acceleration and an ensuing immediate reaching of $v_0 = 10$ m/s. We would then also have $v_{avg} = v_0$. With the scale-dependent acceleration part making for 20% of the total time, we are at 80% of the saturation level. Fig. 4 shows how the Michaelis-Menten curve is indeed a good approximation.

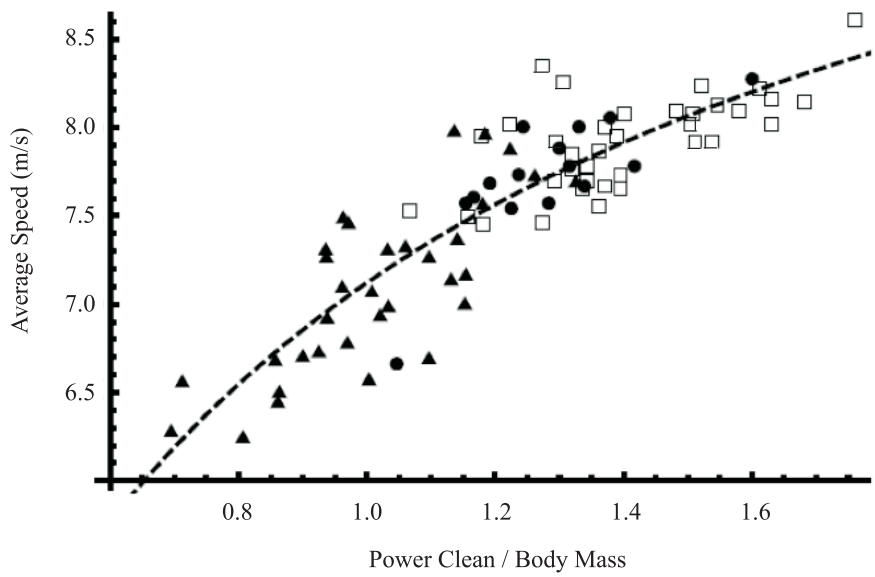

Fig. 4. A scatter diagram with horizontally the ratio *W/M* (*W* is the maximal power clean weight and *M* is the body mass) and vertically the average speed on the 40-yard (37 m) dash. There is good agreement between the data and the theoretically derived curve. Again, triangles indicate linemen, circles are combo players, and squares are skill players. The lighter, faster-accelerating athletes are again observed to cluster at the right top of the graph and are closer to the saturation level of about 10 m/s

4. CONCLUSIONS AND DISCUSSION

The *x*-coordinate in Fig. 2 and 4 denotes the ratio of the power clean weight and the body mass. This quantity scales as L^2/L^3. With $L \propto m^{1/3}$, we thus have $x = Cm^{-1/3}$, where *m* denotes the body mass. Not knowing the exact value of the constant *C* is not an issue in relating the relative changes. With Eq. (1) it is easily derived that $\delta x/x = -(1/3)\,\delta m/m$. So an athlete that "bulks up" and increases his body mass by 1% can expect his power-clean/body-mass ratio to decrease by about 0.3%. Scaling theory and Fig. 2 tell us that also his acceleration is expected to go down by 0.3%.

For the aforementioned Michaelis-Menten relation, $y = Ax/(B + x)$, it is easily derived that:

$$\frac{\delta y}{y} = \frac{1}{1 + x/B}\frac{\delta x}{x} \tag{2}$$

As was observed before, our athletes on the 40-yard dash are at about 80% of the saturation level given by the horizontal asymptote. With $x/(x + B) \approx 0.8$, we have $x/B \approx 4$ and $1/(1 + x/B) \approx 0.2$. Using $\delta x/x = -(1/3)\,\delta m/m$, we see:

$$\frac{\delta y}{y} = -(1/3)\,\frac{1}{1 + Dm^{-1/3}}\frac{\delta m}{m} \tag{3}$$

where D is a constant that is set through $Dm^{-1/3} = x/B$. This equation tells us that a 1% body mass increase leads to a 40-yard average speed that is about 0.07% smaller. But the $Dm^{-1/3}$ term in the above formula implies that this percentage will be higher for heavier athletes (larger *m*) and lower for the lighter athletes (smaller *m*).

Let two variables, *X* and *Y*, be connected through a simple power law, $Y = \alpha X^{\beta}$. With Eq. (1) we find that the relative changes of X and Y are related through $\delta Y/Y = \beta\,\delta X/X$. So the proportionality constant "α" cancels out and we see how Y changes by β% if *X* changes by 1%. If, like in the 40-yard sprint, the result is the sum of two power-law dependencies, then there is no longer a fixed percentage that is valid for the entire range. As we see in Eq. 3, the percentage by which the average speed changes in response to a 1% change in body mass is no longer fixed. It depends on the body mass itself.

Our data and analysis leads to several obvious and applicable implications. In a confrontation on the field between football players of opposing teams, momentum is often a key issue. A player's momentum is the product of body mass and speed. A player needs acceleration and speed to either outrun or catch another player. He next needs momentum to either ward off or bring down the opposing player. In balancing the acceleration, speed, strength, and body mass of their players, coaches have generally followed their instincts and intuitions. In this article we have identified relations between acceleration, speed, strength, and body mass. These relations should allow for a more quantitative and reasoned approach.

We can now also quantify some of the likely consequences of anatomic changes. Suppose a player increases his body mass from 125 kg to 130 kg, i.e. a 4% increase. If we treat this as just a scaling up, then we can predict the consequences. The acceleration, which is important for the ability to catch or escape an opposing player, will be reduced by about (1/3) × 4%, i.e. a little more than one percent. His top speed will be unaffected, so his momentum (mv_0) and kinetic energy ($\frac{1}{2}mv_0^2$), which are measures for the effectiveness in a confrontation, will go up by 4%. The time in which he is expected to cover his 40-yard dash will go up by a small 0.3%.

There is an apparent strong correlation (R = 0.76) between how much a player can power clean and his acceleration on the field. Of two players with the same top running speed, the player with the stronger power-clean lift is expected to accelerate to top speed faster. Imagine a defensive linebacker and a running back, each having a mass of 100 kg. A confrontation between two such players is common in a football game: after a running back gets the ball, he may start accelerating while a nearby linebacker may start accelerating at the same time in an attempt to tackle him. Assume that one of these two players power cleans 150 kg and the other power cleans 140 kg. This 7% difference translates into also a 7% difference in acceleration. At accelerations of $a \approx 5.5$ m/s^2, that difference amounts to $\Delta a \approx 0.4$ m/s^2. The acceleration to top speed lasts about $t = 2$ seconds. In these two seconds the better power-clean lifter is expected to gain a considerable $\Delta s = \frac{1}{2}\,\Delta a\, t^2 \approx 0.8$ m on his opponent.

Our interpretation in terms of Control Analysis can give coaches a quantitative and scientific way to assess their players and to formulate goals and expectations in the context of their expected playing positions on the team.

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